Chapter 10. Transmission Lines

- **Transmission line**: carrier to transmit electric energy and signals from one point to another

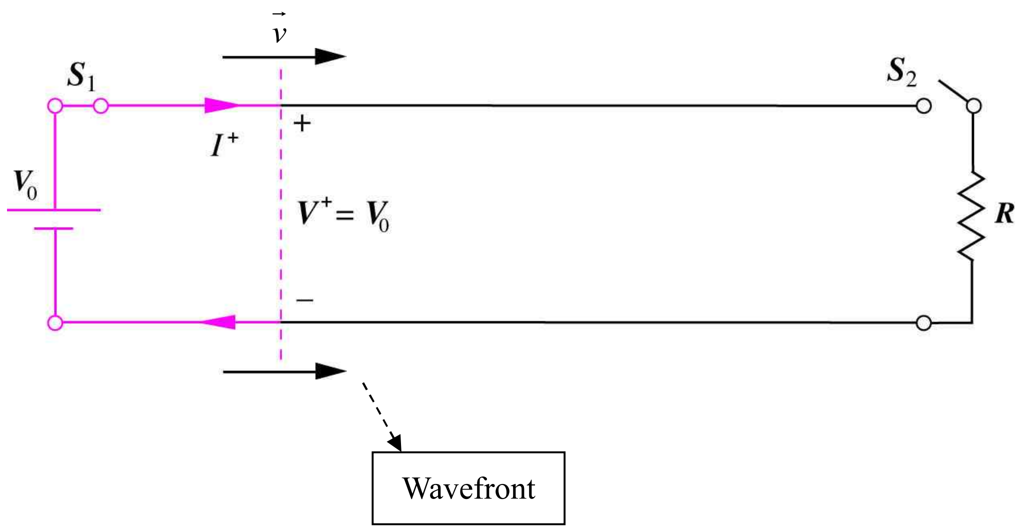
  Ex.] Source ←*→ Load, Transmitter ←*→ Receiver,
  Computer ←*→ Computer,
  Hydroelectric generating plant ←*→ Home
  Audio System ←*→ Speaker,
  Cable service provider ←*→ TV, Device ←*→ Device

- 1) **Lumped elements** (R, L, C, etc.) ignores a time delay to traverse the elements ➔ Per-unit-distance basis

  2) **Distributed elements** (transmission line) can't be ignored the time delay to traverse the elements ➔ Over-unit-distance basis

10.1 Physical Description of Transmission Line Propagation

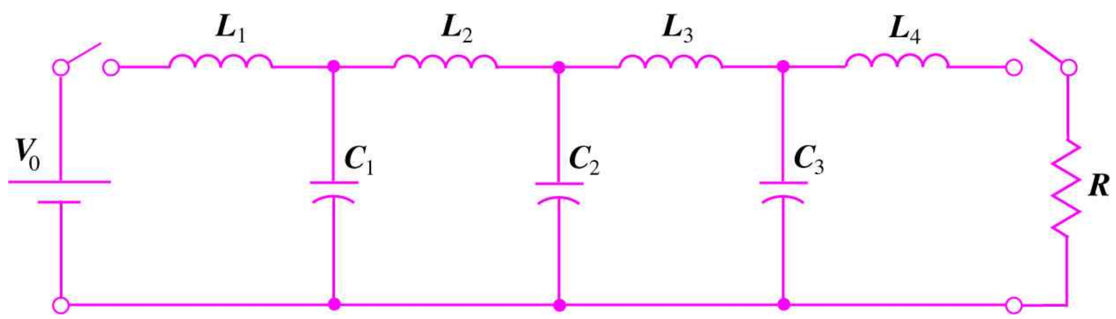
• Lossless basic transmission line circuit



- Assume the switch S_1 is closed at $t = 0$
- At $t = 0^+$, the source voltage (V_0) doesn't instantaneously appear everywhere on the line but begins to travel from the battery toward the load (R) at a certain velocity (v).
- Wavefront: instantaneous boundary between the sections of the line to be charged or not

• Lumped-element model of transmission line

Assumptions: $L_1 = L_2 = L_3 = L_4$ [H/m] and $C_1 = C_2 = C_3$ [F/m]



S_1 On $\rightarrow$ Current through $L_1 \rightarrow$ Charging on C_1

$\rightarrow$ Current through $L_2 \rightarrow$ Charging on C_2 $\rightarrow$ $\cdots$

$\Leftrightarrow$ Wavefront moving from the left to right

S_2 On $\rightarrow$ Discharging on C_3 (Dissipating by R)

$\rightarrow$ Discharging on C_2 (Dissipating by R) $\rightarrow$ $\cdots$

: **<u>Pulse-forming network</u>** (dissipating on R)

Wave velocity: $v = 1 / (LC)^{0.5}$ (to be proved in later.)

• Analyzing approaches

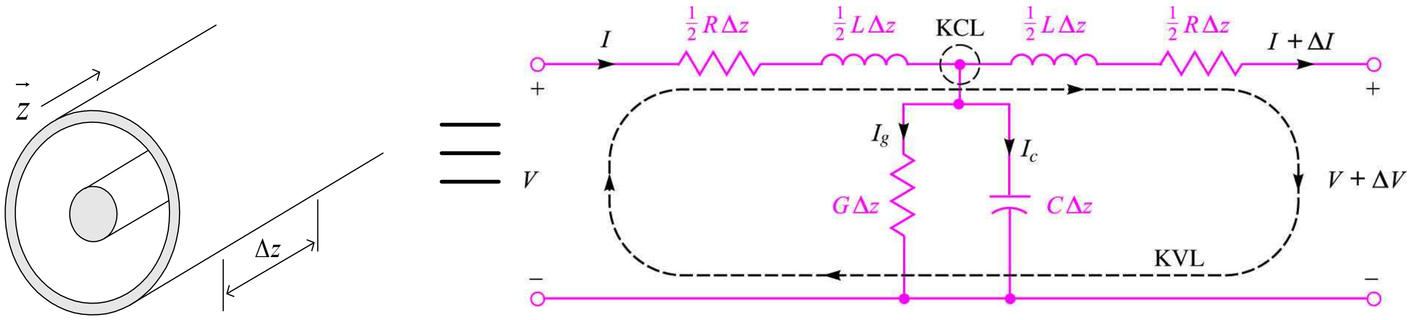
1) Maxwell's equations: electromagnetic field method
(complete & accurate)

Ex.] Wave power, velocity, ⋯

2) Circuit equations: equivalent voltage and current analysis method
(intuitive & inaccurate)

10.2 The Transmission Line Equations

• **Lumped-element model** of a short transmission line section **with losses** (length : Δz)



R: series resistance per unit length [Ω/m]

← due to finite conductor conductivity σ_c

L: series inductance per unit length [H/m]

G: shunt conductance per unit length [Ω^{-1}/m]

← due to finite dielectric conductivity σ_d

C: shunt capacitance per unit length [F/m] ← due to ε_r

R, G: power loss components $(= f\,(\text{freq.}))$

- 전송선로의 미소길이에 대하여 등가 회로 model을 구하고, 이것으로부터 전압, 전류 방정식을 유도한 후 그 결과식들이 field 이론에서 구한 기본 방정식들(예: wave equation)과 성질이 같다면 uniform plane wave의 전달경로가 전송선로와 같음을 보일 수 있게 됨.

- Propagation direction: $\vec{a}_z$

- KVL

$$V = \frac{1}{2}RI\Delta z + \frac{1}{2}L\frac{\partial I}{\partial t}\Delta z + \frac{1}{2}L\left(\frac{\partial I}{\partial t} + \frac{\partial \Delta I}{\partial t}\right)\Delta z \qquad \leftarrow L = \frac{\Phi}{I} \;\&\; V = \frac{d\Phi}{dt}$$

$$+\frac{1}{2}R(I + \Delta I)\Delta z + (V + \Delta V) \qquad (1)$$

$$(V + \Delta V) - V = -\frac{1}{2}RI\Delta z - \frac{1}{2}L\frac{\partial I}{\partial t}\Delta z - \frac{1}{2}L\left(\frac{\partial I}{\partial t} + \frac{\partial \Delta I}{\partial t}\right)\Delta z - \frac{1}{2}R(I + \Delta I)\Delta z$$

$$\frac{\Delta V}{\Delta z} = -\frac{1}{2}RI - \frac{1}{2}L\frac{\partial I}{\partial t} - \frac{1}{2}L\left(\frac{\partial I}{\partial t} + \frac{\partial \Delta I}{\partial t}\right) - \frac{1}{2}R(I + \Delta I)$$

$$= -\left(RI + L\frac{\partial I}{\partial t} + \frac{1}{2}L\frac{\partial \Delta I}{\partial t} + \frac{1}{2}R\Delta I\right) \qquad (2)$$

Applying $\Delta I = \frac{\partial I}{\partial z}\Delta z$ and $\Delta V = \frac{\partial V}{\partial z}\Delta z$ (3)

$$\frac{\partial V}{\partial z} = -\left\{\left(RI + L\frac{\partial I}{\partial t}\right) + \frac{1}{2}L\frac{\partial}{\partial t}\left(\frac{\partial I}{\partial z}\Delta z\right) + \frac{1}{2}R\frac{\partial I}{\partial z}\Delta z\right\}$$

$$= -\left\{\left(RI + L\frac{\partial I}{\partial t}\right) + \frac{\Delta z}{2}\frac{\partial}{\partial z}\left(RI + L\frac{\partial I}{\partial t}\right)\right\} = -\left(1 + \frac{\Delta z}{2}\frac{\partial}{\partial z}\right)\left(RI + L\frac{\partial I}{\partial t}\right) \qquad (4)$$

As $\Delta z \to 0$,

$$\frac{\partial V}{\partial z} = -\left(RI + L\frac{\partial I}{\partial t} \right) \qquad (5)$$

• KCL at the mid-point using the symmetric property

$$I = I_g + I_c + (I + \Delta I) \qquad \leftarrow Q = CV \;\;\&\;\; I = \frac{dQ}{dt}$$

$$= G\Delta z\left(V + \frac{\Delta V}{2} \right) + C\Delta z\frac{\partial}{\partial t}\left(V + \frac{\Delta V}{2} \right) + (I + \Delta I) \qquad (6)$$

$$(I + \Delta I) - I = -G\Delta z\left(V + \frac{\Delta V}{2} \right) - C\Delta z\frac{\partial}{\partial t}\left(V + \frac{\Delta V}{2} \right)$$

$$\frac{\Delta I}{\Delta z} = -G\left(V + \frac{\Delta V}{2} \right) - C\frac{\partial}{\partial t}\left(V + \frac{\Delta V}{2} \right) \quad \leftarrow \Delta V = \frac{\partial V}{\partial z}\Delta z$$

$$= -G\left(V + \frac{\Delta z}{2}\frac{\partial V}{\partial z} \right) - C\frac{\partial}{\partial t}\left(V + \frac{\Delta z}{2}\frac{\partial V}{\partial z} \right)$$

$$= -\left\{ GV\left(1 + \frac{\Delta z}{2}\frac{\partial}{\partial z} \right) + C\frac{\partial V}{\partial t}\left(1 + \frac{\Delta z}{2}\frac{\partial}{\partial z} \right) \right\}$$

$$= -\left(1 + \frac{\Delta z}{2}\frac{\partial}{\partial z} \right)\left(GV + C\frac{\partial V}{\partial t} \right) \qquad (7)$$

As $\Delta z \to 0$

$$\frac{\partial I}{\partial z} = -\left(GV + C\frac{\partial V}{\partial t} \right) \qquad (8)$$

• Telegraphist's equations: Eqs. (5) and (8)

$$\frac{\partial V}{\partial z} = -\left(RI + L\frac{\partial I}{\partial t} \right) \qquad (5)$$

$$\frac{\partial I}{\partial z} = -\left(GV + C\frac{\partial V}{\partial t} \right) \qquad (8)$$

• ∂(Eq. (5))/∂z and ∂(Eq. (8))/∂t:

$$\frac{\partial^2 V}{\partial z^2} = -R\frac{\partial I}{\partial z} - L\frac{\partial^2 I}{\partial t \partial z} \tag{9}$$

$$\frac{\partial^2 I}{\partial z \partial t} = -G\frac{\partial V}{\partial t} - C\frac{\partial^2 V}{\partial t^2} \tag{10}$$

• Eq. (9) ← Eqs. (8) and (10):

$$\frac{\partial^2 V}{\partial z^2} = R\left(GV + C\frac{\partial V}{\partial t}\right) + L\left(G\frac{\partial V}{\partial t} + C\frac{\partial^2 V}{\partial t^2}\right)$$

$$= LC\frac{\partial^2 V}{\partial t^2} + (LG + RC)\frac{\partial V}{\partial t} + RGV \tag{11}$$

• ∂(Eq. (5))/∂t and ∂(Eq. (8))/∂z:

$$\frac{\partial^2 V}{\partial z \partial t} = -R\frac{\partial I}{\partial t} - L\frac{\partial^2 I}{\partial t^2} \tag{*}$$

$$\frac{\partial^2 I}{\partial z^2} = -G\frac{\partial V}{\partial z} - C\frac{\partial^2 V}{\partial t \partial z} \tag{**}$$

• Eq. (**) ← Eqs. (5) and (*):

$$\frac{\partial^2 I}{\partial z^2} = G\left(RI + L\frac{\partial I}{\partial t}\right) + C\left(R\frac{\partial I}{\partial t} + L\frac{\partial^2 I}{\partial t^2}\right)$$

$$= LC\frac{\partial^2 I}{\partial t^2} + (LG + RC)\frac{\partial I}{\partial t} + RGI \tag{12}$$

• **General wave equations**: Eqs. (11) and (12)
(V와 I의 방정식이 공간과 시간에 대한 함수로 표현됨)

10.3 Lossless propagation

- Lossless propagation: (input) power isn't dissipated or otherwise deviated as the wave travels down the transmission line.

  ➔ $R = G = 0$

- Lossless general wave equation:

$$\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2} \qquad (13) \leftarrow \text{Eq.}(11)$$

➔ General solution:

$$V(z,t) = f_1\left(t - \frac{z}{v}\right) + f_2\left(t + \frac{z}{v}\right) = V^+ + V^- \qquad (14)$$

where v: wave velocity

f_1, f_2: forward and backward z-directional travel functions

V^+, V^-: forward and backward voltage wave components

Ex.] As t is ↑, z must be ↑ for $f_1(0)$.

➔ Positive (or forward) z-direction traveling

As t is ↑, z must be ↓ for $f_2(0)$.

➔ Negative (or backward) z-direction traveling

- From Eq. (14),

$$\frac{\partial f_1}{\partial z} = \frac{\partial f_1}{\partial (t - z/v)}\frac{\partial (t - z/v)}{\partial z} = -\frac{1}{v}f_1' \qquad (15)$$

$$\frac{\partial f_1}{\partial t} = \frac{\partial f_1}{\partial (t - z/v)}\frac{\partial (t - z/v)}{\partial t} = f_1' \qquad (16)$$

$$\frac{\partial^2 f_1}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial f_1}{\partial z}\right) = \frac{1}{v^2}f_1'' \quad \text{and} \quad \frac{\partial^2 f_1}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial f_1}{\partial t}\right) = f_1'' \qquad (17)$$

• Eq. (13) ← Eq. (17):

$$\frac{1}{v^2} f_1'' = LC f_1'' \quad (18)$$

$$v = \frac{1}{\sqrt{LC}} \quad \text{: wave velocity} \quad (19)$$ ← page 2

• Lossless voltage and current equations along the transmission line:

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t} \quad (20) \quad \leftarrow \text{Eq. (5)}$$

$$\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t} \quad (21) \quad \leftarrow \text{Eq. (8)}$$

• Eq. (20) ← Eq. (14):

$$\frac{\partial I}{\partial t} = -\frac{1}{L}\frac{\partial V}{\partial z} \quad \leftarrow \text{Eq. (14)}$$

$$= -\frac{1}{L}\frac{\partial}{\partial z}\left[f_1\left(t-\frac{z}{v}\right) + f_2\left(t+\frac{z}{v}\right)\right] = \frac{1}{Lv}(f_1' - f_2') \quad (22)$$

• Integrate Eq. (22) over time:

$$I(z,t) = \int \frac{\partial I}{\partial t} dt = \frac{1}{Lv}\left[f_1\left(t-\frac{z}{v}\right) - f_2\left(t+\frac{z}{v}\right)\right] = I^+ + I^- \quad (23) \Leftrightarrow I = \frac{V}{Z}$$

where Lv: characteristic impedance of lossless line

➔ $(f_1 + f_2)$가 voltage wave이고, $(f_1 - f_2)$가 current wave이므로

• Characteristic impedance (Z_0): a ratio of voltage to current waves

$$Z_0 = Lv = L\sqrt{\frac{1}{LC}} = \sqrt{\frac{L}{C}} \quad (24)$$

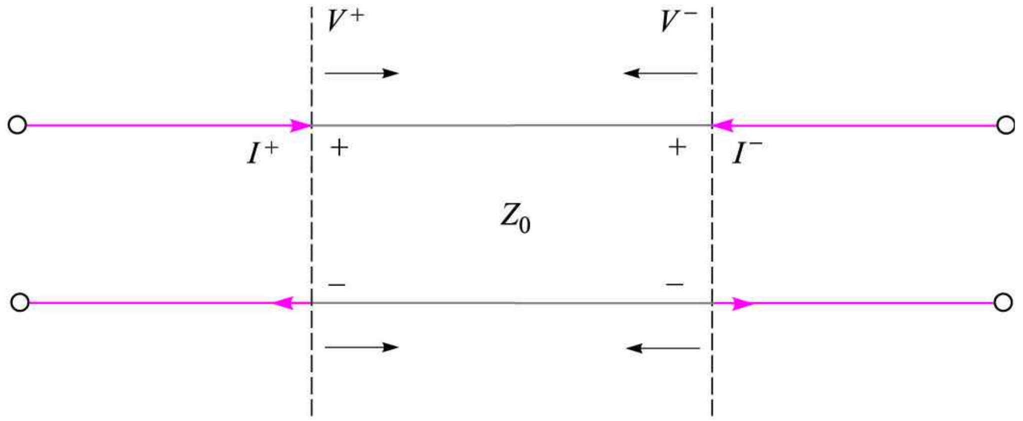
• By comparing Eqs. (14) and (23),

$$V^+ = Z_0 I^+ \quad (25a)$$

$V^- = -Z_0 I^-$ (25*b*)

where minus (-) means backward propagating.

• Current directions in waves having positive and negative voltages:



10.4 Lossless Propagation of Sinusoidal Voltages

• Any transmitted signal can be composed into a discrete or continuous summation of sinusoids.

➔ Fourier transformation (time domain ↔ frequency domain)

• Sinusoidal voltage function:

$$V(z,t) = f_1\left(t - \frac{z}{v}\right) + f_2\left(t + \frac{z}{v}\right) = V^+ + V^- \qquad (14)$$

Consider a specific example:

$f = \omega/2\pi$, $f_1 = f_2 = V_0\cos(\omega t + \phi)$, $t \rightarrow t \pm z/v_p$

$$\Rightarrow V(z,t) = |V_0|\cos[\omega(t \pm z/v_p) + \phi] = |V_0|\cos[\omega t \pm \beta z + \phi] \qquad (26)$$

For sine wave, $\phi = \pi/2$

In case of $\phi = 0$,

$$V_f(z,t) = |V_0|\cos(\omega t - \beta z) \quad \text{(forward } z \text{ propagation)} \qquad (27a)$$

$$V_b(z,t) = |V_0|\cos(\omega t + \beta z) \quad \text{(backward } z \text{ propagation)} \qquad (27b)$$

➔ Real instantaneous forms of transmission-line voltage
V_f or $V_b = f(\omega t, \beta z)$

where $|V_0|$: value of V at $z = 0$ and $t = 0$

ω: angular frequency [rad/sec]

β: spatial frequency [rad/m]

• Phase constant (β):

$$\beta \equiv \frac{\omega}{v_p} \qquad (28) \quad \text{: [rad/sec]/[m/sec]=[rad/m]}$$

• Instantaneous voltage in case of $t = 0$,

$$V_f(z,0) = V_b(z,0) = |V_0|\cos(\beta z) \qquad (29)$$

➔ Simple periodic function with distance λ

Requirement of wavelength (λ): $\beta\lambda = 2\pi$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega / v_p} = \frac{v_p}{f} \qquad (30)$$

10.5 Complex Analysis of Sinusoidal Waves

• Expressing sinusoidal waves as complex function:
usability of the evaluation and visualization of signal phase

- Euler identity: $e^{\pm jx} = \cos(x) \pm j\sin(x)$ (32)

  ➔ $\cos(x) = \mathrm{Re}[e^{\pm jx}] = (e^{jx} + e^{-jx}) / 2 = e^{jx}/2 + c.c.$ (33a)

  $\sin(x) = \pm\mathrm{Im}[e^{\pm jx}] = (e^{jx} - e^{-jx}) / 2j = e^{jx}/2j - c.c.$ (33b)

  where $j = (-1)^{0.5}$

  $c.c.$: complex conjugate of the preceding term

$$V(z,t) = |V_0|\cos[\omega(t \pm z/v_p) + \phi] = |V_0|\cos[\omega t \pm \beta z + \phi] \quad (26)$$

$$= \frac{1}{2}\underbrace{|V_0|e^{j\phi}}_{V_0} e^{\pm j\beta z} e^{j\omega t} + c.c. \quad (34)$$

where $V_0 = (|V_0|e^{j\phi})$: complex amplitude of the wave

- Complex instantaneous voltage:

$$V_c(z,t) = V_0 e^{\pm j\beta z} e^{j\omega t} \quad (35)$$

- Phasor voltage:

$$V_s(z) = V_0 e^{\pm j\beta z} \quad (36)$$

  ➔ sinusoidal steady-state condition ($\neq f(t)$)

- Real instantaneous voltage:

$$V(z,t) = |V_0|\cos[\omega t \pm \beta z + \phi] = \mathrm{Re}[V_c(z,t)] = \frac{1}{2}V_c + c.c. \quad (37a)$$

- Phasor voltage:

$$V(z,t) = |V_0|\cos[\omega t \pm \beta z + \phi] = \mathrm{Re}[V_s(z)e^{j\omega t}] = \frac{1}{2}V_s e^{j\omega t} + c.c. \quad (37b)$$

(sinusoidal instantaneous voltage: $V(z, t) = \mathrm{Re}[V_s(z)e^{j\omega t}]$)

[Ex. 10.1] Two voltage waves having equal frequencies and amplitudes propagate in opposite directions on a lossless transmission line.

➔ Phasor form:

$$V_{sT}(z) = V_0 e^{-j\beta z} + V_0 e^{+j\beta z} = 2V_0\cos(\beta z)$$

Real instantaneous form:

$$V(z,t) = \mathrm{Re}[(V_0 e^{-j\beta z} + V_0 e^{+j\beta z})e^{j\omega t}] = \mathrm{Re}[2\frac{V_0(e^{-j\beta z} + e^{+j\beta z})}{2}e^{j\omega t}]$$

$$= \mathrm{Re}[2V_0\cos(\beta z)e^{j\omega t}] = 2V_0\cos(\beta z)\cos(\omega t)$$

➔ $V(z, t) = f(z, t)$

10.6 Transmission Line Equations and Their Solutions in Phasor Form

• Real instantaneous voltage of general wave equations:

$$\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2} + (LG+RC)\frac{\partial V}{\partial t} + RGV \qquad (11)(38)$$

$$\frac{\partial V(z,t)}{\partial t} = \frac{\partial}{\partial t}\mathrm{Re}[V_0 e^{j\beta z}e^{j\omega t}] = \frac{\partial}{\partial t}\mathrm{Re}[V_0 e^{j(\omega t+\beta z)}] = \frac{\partial}{\partial t}[V_0\cos(\omega t+\beta z)]$$

$$= -\omega V_0\sin(\omega t+\beta z) = j\omega V_0\cdot\{j\sin(\omega t+\beta z)\}$$

$$= \mathrm{Re}[j\omega V_0 e^{j(\omega t+\beta z)}] = \mathrm{Re}[j\omega V_0 e^{j\beta z}e^{j\omega t}] = \mathrm{Re}[j\omega V_S e^{j\omega t}]$$

$$\therefore\ \frac{\partial}{\partial t} \underset{\text{instaneous}}{\overset{\text{phasor}}{\leftrightarrow}} j\omega$$

• Phasor voltage of the general wave equations:

$$\frac{d^2 V_s}{dz^2} = -\omega^2 LCV_s + j\omega(LG+RC)V_s + RGV_s \quad (39)$$

$$= \underbrace{(R+j\omega L)}_{Z}\underbrace{(G+j\omega C)}_{Y}V_s = \gamma^2 V_s \qquad (40)$$

where Z, Y: net series impedance and shunt admittance of transmission line as per-unit-distance

• Propagation constant:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{ZY} = \alpha + j\beta \quad (41)$$

• General solution of Eq. (40):

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \quad (42a)$$

• Phasor current of general wave equations:

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z} \quad (42b)$$

• Real instantaneous current:

$$I(z,t) = |I_0| \cos(\omega t \pm \beta z + \xi) = \frac{1}{2}\underbrace{\left(|I_0| e^{j\xi}\right)}_{I_0} e^{\pm j\beta z} e^{j\omega t} + c.c. = \frac{1}{2} I_s(z) e^{j\omega t} + c.c. \quad (43)$$

• Eqs. (5) and (8) ← Eqs. (37b) and (43)

$$\frac{\partial V}{\partial z} = -\left(RI + L\frac{\partial I}{\partial t}\right) \quad \Rightarrow \frac{dV_s}{dz} = -\left(R + j\omega L\right) I_s = -ZI_s \quad (44a)$$

$$\frac{\partial I}{\partial z} = -\left(GV + C\frac{\partial V}{\partial t}\right) \quad \Rightarrow \frac{dI_s}{dz} = -\left(G + j\omega C\right) V_s = -YI_s \quad (44b)$$

• Eqs. (44a) and (44b) ← Eqs. (42a) and (42b)

$$-\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{\gamma z} = -Z(I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}) \quad (45)$$

• Line characteristic impedance:

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \frac{Z}{\gamma} = \frac{Z}{\sqrt{ZY}} = \sqrt{\frac{Z}{Y}} \qquad (46) \quad \leftarrow \ (40)$$

$$= \sqrt{\frac{R + j\omega L}{G + j\omega C}} = |Z_0| e^{j\theta} \qquad (47)$$

[Ex. 10.2] Lossless line length l = 0.8 [m], f = 600 [MHz],

Line parameters: L = 0.25 [uH/m], C = 100 [pF/m]

➔ Because of lossless line, $R = G = 0$.

Characteristic impedance:

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.25\times 10^{-6}}{100\times 10^{-12}}} = 50[\Omega]$$

Since $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} = j\omega\sqrt{LC}$,

$$\alpha = 0$$

$$\beta = 2\pi \times (600\times 10^6)\times\sqrt{(0.25\times 10^{-6})(100\times 10^{-12})} = 18.85 \ \text{[rad/m]}$$

$$v_p = \frac{\omega}{\beta} = \frac{2\pi(600\times 10^6)}{18.85} = 2\times 10^8 \ \text{[m/s]}$$

10.7 Low-Loss Propagation

- Eq. (42a) $\leftarrow$ Eq. (41):

$$V_s(z) = V_0^+ e^{-\alpha z} e^{-j\beta z} + V_0^- e^{\alpha z} e^{j\beta z} \qquad (48)$$

Magnitude attenuation

- Real instantaneous voltage:

$$V(z,t) = \mathrm{Re}[V_s(z) e^{j\omega t}] = V_0^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_0^- e^{\alpha z} \cos(\omega t + \beta z) \qquad (49)$$

Forward-propagating wave

Backward-propagating wave

Attenuation coefficient: α [Np/m]

Phase constant: β $(= \omega / v_p)$ [rad/m]

- Lossless case: $R = G = 0$, $\alpha = 0$, $\gamma = j\beta = j\omega(LC)^{0.5}$,

  $v_p = 1 / (LC)^{0.5}$

- Low loss case: $R << \omega L$, $G << \omega C$ (but $R \neq 0$ and $G \neq 0$)

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \qquad (41)$$

$$= j\omega\sqrt{LC}\left[\left(1 + \frac{R}{j\omega L}\right)^{1/2}\left(1 + \frac{G}{j\omega C}\right)^{1/2}\right] \qquad (50)$$

$$\leftarrow \sqrt{1+x} \cong 1 + \frac{x}{2} - \frac{x^2}{8} + \cdots \quad (x << 1) \qquad (51)$$

$$\gamma \cong j\omega\sqrt{LC}\left[\left(1 + \frac{R}{j2\omega L} + \frac{R^2}{8\omega^2 L^2} + \cdots\right)\left(1 + \frac{G}{j2\omega C} + \frac{G^2}{8\omega^2 C^2} + \cdots\right)\right] \qquad (52)$$

Since $R << \omega L$, $G << \omega C$, RG^2, R^2G and R^2G^2 can be negligible,

$$\gamma = \alpha + j\beta \cong j\omega\sqrt{LC}\left[\left(1+\frac{R}{j2\omega L}+\frac{R^2}{8\omega^2 L^2}\right)\left(1+\frac{G}{j2\omega C}+\frac{G^2}{8\omega^2 C^2}\right)\right]$$

$$\cong j\omega\sqrt{LC}\left[1+\frac{1}{j2\omega}\left(\frac{R}{L}+\frac{G}{C}\right)+\frac{1}{8\omega^2}\left(\frac{R^2}{L^2}-\frac{2RG}{LC}+\frac{G^2}{C^2}\right)\right] \quad (53)$$

$$\alpha = j\omega\sqrt{LC}\left[\frac{1}{j2\omega}\left(\frac{R}{L}+\frac{G}{C}\right)\right]=\frac{1}{2}\left(R\sqrt{\frac{C}{L}}+G\sqrt{\frac{L}{C}}\right) \quad (54a)$$

$$\beta = \omega\sqrt{LC}\left[1+\frac{1}{8\omega^2}\left(\frac{R^2}{L^2}-\frac{2RG}{LC}+\frac{G^2}{C^2}\right)\right]=\omega\sqrt{LC}\left[1+\frac{1}{8}\left(\frac{R}{\omega L}-\frac{G}{\omega C}\right)^2\right]$$

$$=\omega\sqrt{LC}\left[1+\frac{1}{8}\left(\frac{G}{\omega C}-\frac{R}{\omega L}\right)^2\right] \quad (54b)$$

- Phase velocity: $v_p = \omega/\beta$

  Signal velocity: $v_g = d\omega / d\beta$ ← signal distortion

  ➔ If signal is distortionless, $\beta = \omega(LC)^{0.5}$ (← $R = G = 0$)

- Characteristic impedance in case of low-loss line:

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{j\omega L(1+\frac{R}{j\omega L})}{j\omega C(1+\frac{G}{j\omega C})}} \cong \sqrt{\frac{L}{C}}\left[\frac{1+\frac{R}{j2\omega L}+\frac{R^2}{8\omega^2 L^2}}{1+\frac{G}{j2\omega C}+\frac{G^2}{8\omega^2 C^2}}\right] \quad (55)$$

$$\cong \sqrt{\frac{L}{C}}\left[\left(1+\frac{R}{j2\omega L}+\frac{R^2}{8\omega^2 L^2}\right)\left(1-\frac{G}{j2\omega C}-\frac{G^2}{8\omega^2 C^2}\right)\right] \leftarrow \frac{1}{1+x}\cong 1-x$$

$$\cong \sqrt{\frac{L}{C}}\left[1+\frac{1}{2\omega^2}\left\{\frac{1}{4}\left(\frac{R^2}{L^2}-\frac{G^2}{C^2}\right)-\frac{RG}{2LC}\right\}+\frac{j}{2\omega}\left(\frac{G}{C}-\frac{R}{L}\right)\right] \quad (56)$$

[Ex. 10.3] Transmission line with $G = 0$, $R \neq 0$ and $R << \omega L$

Characteristic impedance:

$$Z_0 = \sqrt{\frac{L}{C}}\left[1+\frac{1}{2\omega^2}\left\{\frac{1}{4}\left(\frac{R^2}{L^2}-\frac{G^2}{C^2}\right)-\frac{RG}{2LC}\right\}+\frac{j}{2\omega}\left(\frac{G}{C}-\frac{R}{L}\right)\right]\Bigg|_{G=0,R<<\omega L}$$

$$\cong \sqrt{\frac{L}{C}}\left[1+\frac{j}{2\omega}\left(-\frac{R}{L}\right)\right] \cong |Z_0|e^{j\theta}$$

where $|Z_0| \cong \sqrt{L/C}$ and $\theta = \tan^{-1}(-R/2\omega L)$

10.8 Power Transmission and The Use of Decibels in Loss Characterization

• Consider the forward-propagating voltage and current term with some phase shifting.

$$P(z,t) = V(z,t)I(z,t) = |V_0||I_0|e^{-2\alpha z}\cos(\omega t-\beta z)\cos(\omega t-\beta z+\theta) \quad (57)$$

• Time-averaged power <P>:

$$\langle P\rangle = \frac{1}{T}\int_0^T |V_0||I_0|e^{-2\alpha z}\cos(\omega t-\beta z)\cos(\omega t-\beta z+\theta)dt \quad (58)$$

$$= \frac{1}{2T}\int_0^T |V_0||I_0|e^{-2\alpha z}[\cos(2\omega t-2\beta z+\theta)+\cos(\theta)]dt \quad (59)$$

$$= \frac{1}{2}|V_0||I_0|e^{-2\alpha z}\cos(\theta) = \frac{1}{2}\frac{|V_0|^2}{|Z_0|}e^{-2\alpha z}\cos(\theta) \ \ \text{[W]} \quad (60)$$

where $T = 2\pi/\omega$

• Another method to obtain the instantaneous power phasor voltage: $V_s(z) = V_0 e^{-\alpha z} e^{-j\beta z}$ (61)

phasor current: $I_s(z) = I_0 e^{-\alpha z} e^{-j\beta z} = \dfrac{V_0}{Z_0} e^{-\alpha z} e^{-j\beta z}$ (62)

where $Z_0 = |Z_0| e^{j\theta}$

$$\langle P \rangle = \frac{1}{2} \mathrm{Re}\{V_s I_s^*\} \qquad (63) \text{ : time averaged power}$$

$$= \frac{1}{2} \mathrm{Re}\left\{ V_0 e^{-\alpha z} e^{-j\beta z} \frac{V_0^*}{|Z_0| e^{-j\theta}} e^{-\alpha z} e^{+j\beta z} \right\}$$

$$= \frac{1}{2} \mathrm{Re}\left\{ \frac{V_0 V_0^*}{|Z_0|} e^{-2\alpha z} e^{j\theta} \right\} = \frac{1}{2} \frac{V_0 V_0^*}{|Z_0|} e^{-2\alpha z} \cos\theta \qquad (64)(60)$$

$$\langle P(z) \rangle = \frac{1}{2} \frac{V_0 V_0^*}{|Z_0|} \cos\theta \cdot e^{-2\alpha z} = \frac{1}{2} \frac{V_0 V_0^*}{|Z_0|} e^{-2\alpha z} \cos\theta \bigg|_{z=0} e^{-2\alpha z} = \langle P(0) \rangle e^{-2\alpha z} \qquad (65)$$

➔ $V(z, t)$ and $I(z, t) = f(e^{-\alpha z})$

$\langle P \rangle = f(e^{-2\alpha z})$

- Power decreasing ratio:

$$\frac{\langle P(z) \rangle}{\langle P(0) \rangle} = e^{-2\alpha z} = 10^{-\kappa \alpha z} \qquad (66)$$

Setting $\alpha z = 1$

$$e^{-2} = 10^{-\kappa} \quad \Rightarrow \quad \kappa = \log_{10}(e^2) = 0.869 \qquad (67)$$

$$\text{Power loss (dB)} = 10\log_{10}\left[\frac{\langle P(0) \rangle}{\langle P(z) \rangle}\right] = 10\log_{10}(10^{\kappa \alpha z}) = 10(\kappa \alpha z)$$

$$= 8.69 \alpha z \qquad (68)$$

$$\text{Power Loss (dB)} = 10\log_{10}\left[\frac{\left.\frac{1}{2}\frac{|V_0|^2}{|Z_0|}e^{-2\alpha z}\cos\theta\right|_{z=0}}{\frac{1}{2}\frac{|V_0|^2}{|Z_0|}e^{-2\alpha z}\cos\theta}\right]$$

$$= 20\log_{10}\left[\frac{|V_0(0)|}{|V_0(z)|}\right] \qquad (69)$$

where $|V_0(z)| = |V_0(0)|e^{-\alpha z}$

[Ex. 10.4] Insertion loss of line with a 20 m length: 2.0 dB

a) Power fraction: $\dfrac{\langle P(20)\rangle}{\langle P(0)\rangle} = 10^{-0.2} = 0.63$

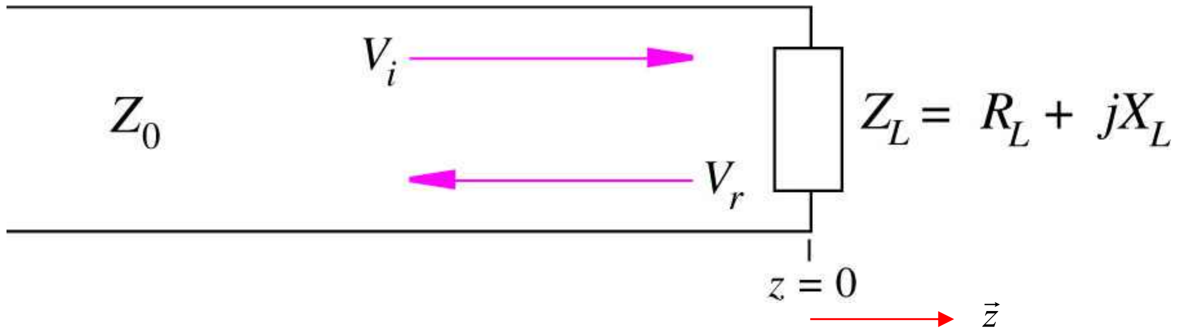
b) Power fraction at the midpoint: $10^{-0.1} = 0.79$

c) Exponential attenuation coefficient:

$$\alpha = \frac{2.0\,\text{dB}}{(8.69\,\text{dB/Np})(20\,\text{m})} = 0.012\,[\text{Np/m}]$$

10.9 Wave Reflection at Discontinuities

• Voltage wave reflection from a complex load impedance



Characteristic impedance of line: Z_0

Load impedance at $z = 0$: $Z_L = R_L + jX_L$

• (Phase formed) Incident voltage wave on the load:

$$V_i(z) = V_{0i} e^{-\alpha z} e^{-j\beta z} \qquad (70a)$$

Reflected voltage wave from the load:

$$V_r(z) = V_{0r} e^{+\alpha z} e^{+j\beta z} \qquad (70b)$$

Phasor voltage at the load (@$z = 0$):

$$V_L = V_{0i} + V_{0r} \qquad (71)$$

• Phasor current through the load:

$$I_L = I_{0i} + I_{0r} = \frac{1}{Z_0}[V_{0i} - V_{0r}] = \frac{V_L}{Z_L} = \frac{1}{Z_L}[V_{0i} + V_{0r}] \qquad (72)$$

• Reflection coefficient (Γ):

$$\Gamma \equiv \frac{V_{0r}}{V_{0i}}$$

$$\frac{V_{0i}}{Z_0}\left(1-\frac{V_{0r}}{V_{0i}}\right)=\frac{V_{0i}}{Z_L}\left(1+\frac{V_{0r}}{V_{0i}}\right) \qquad \leftarrow (72)$$

$$\frac{Z_L}{Z_0}(1-\Gamma)=(1+\Gamma) \qquad \Gamma\left(1+\frac{Z_L}{Z_0}\right)=\frac{Z_L}{Z_0}-1$$

$$\Gamma=\frac{\frac{Z_L}{Z_0}-1}{\frac{Z_L}{Z_0}+1}=\frac{Z_L-Z_0}{Z_L+Z_0}=|\Gamma|e^{j\phi_r} \qquad (73)$$

• With Eq. (71) and (73)

$$V_L=V_{0i}+V_{0r}=V_{0i}+\Gamma V_{0i} \qquad (74)$$

• Transmission coefficient (τ):

$$\tau \equiv \frac{V_L}{V_{0i}}=\frac{V_{0i}+\Gamma V_{0i}}{V_{0i}}=1+\Gamma=1+\frac{Z_L-Z_0}{Z_L+Z_0}=\frac{2Z_L}{Z_L+Z_0}=|\tau|e^{j\phi_t} \qquad (75)$$

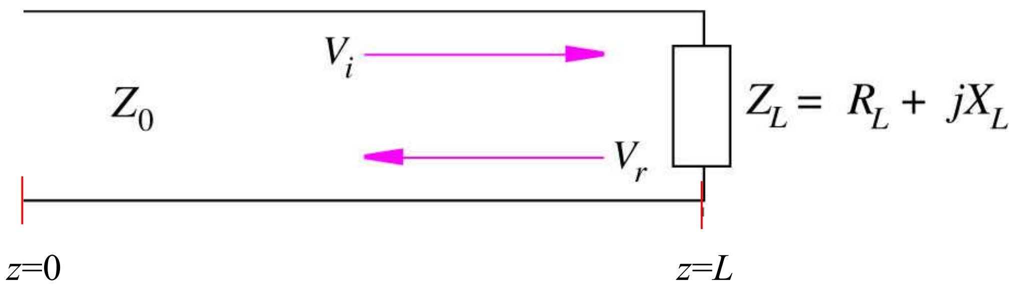
• Circuit design goal: No reflection at load

➔ $\Gamma = 0$ ➔ $Z_L = Z_0$

➔ (Impedance) Matching !!

• Incident power with Eq. (64), $z = L$ and line input $z = 0$:

$$\langle P_i \rangle=\frac{1}{2}\mathrm{Re}\left\{\frac{V_0V_0^*}{|Z_0|}e^{-2\alpha L}e^{j\theta}\right\}=\frac{1}{2}\frac{|V_0|^2}{|Z_0|}e^{-2\alpha L}\cos\theta \qquad (76a)$$



Reflected power:

$$\langle P_r \rangle = \frac{1}{2}\mathrm{Re}\left\{\frac{V_{0r}V_{0r}^*}{|Z_0|}e^{-2\alpha L}e^{j\theta}\right\} = \frac{1}{2}\mathrm{Re}\left\{\frac{(\Gamma V_0)(\Gamma^* V_0^*)}{|Z_0|}e^{-2\alpha L}e^{j\theta}\right\}$$

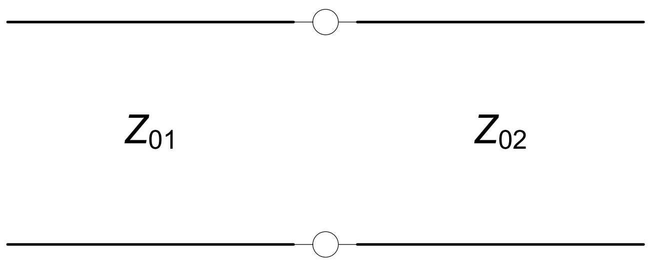
$$= |\Gamma|^2 \frac{1}{2}\frac{|V_0|^2}{|Z_0|}e^{-2\alpha L}\cos\theta = \langle P_i \rangle |\Gamma|^2 \qquad (76b)$$

Reflected and transmitted power fraction at the load:

$$\frac{\langle P_r \rangle}{\langle P_i \rangle} = \Gamma\Gamma^* = |\Gamma|^2 \qquad (77a)$$

$$\frac{\langle P_t \rangle}{\langle P_i \rangle} = 1 - |\Gamma|^2 \qquad (77b)$$

- Reflection coefficient at the connection of two semi-infinite lines:



$$\Gamma = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} \qquad (78)$$

[Ex. 10.5] Lossless line characteristic impedance: 50 Ω

Load impedance: $Z_L = 50 - j75\ \Omega$

Incident Power: 100 mW

➔ Reflection coefficient and transmitted power:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - j75 - 50}{50 - j75 + 50} = \frac{-j75}{100 - j75} = 0.36 - j0.48 = 0.6e^{-j0.93}$$

$$\langle P_t \rangle = \left(1 - |\Gamma|^2\right)\langle P_i \rangle = [1 - (0.6)^2](100) = 64\ \text{mW}$$

[Ex. 10.6] Two lossy lines are jointed end-to-end.

Line 1: $l_1 = 10$ m, loss rate = 0.2 dB/m

Line 2: $l_2 = 15$ m, loss rate = 0.1 dB/m

Reflection coefficient at the junction: Γ = 0.3

$\langle P_i \rangle = 100$ mW

➔ (a) dB loss of the joint:

$$L_j\ \text{(dB)} = 10\log_{10}\left(\frac{1}{1 - |\Gamma|^2}\right) = 10\log_{10}\left(\frac{1}{1 - 0.09}\right) = 0.41\ \text{dB}$$

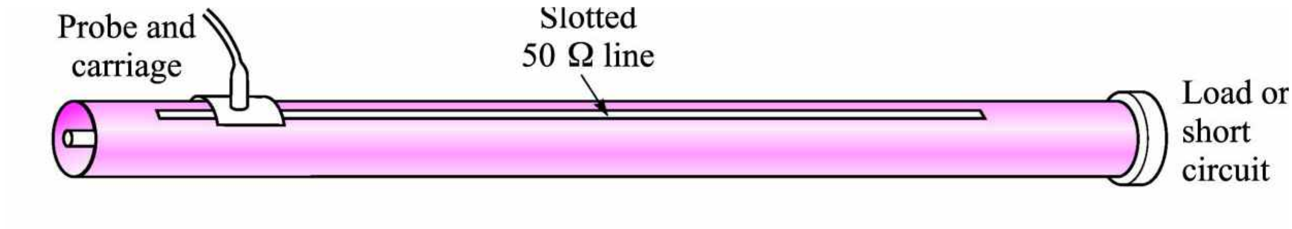
Total loss of the link:

$$L_t\ \text{(dB)} = (0.2)(10) + 0.41 + (0.1)(15) = 3.91\ \text{dB}$$

(b) Output power: $P_{\text{out}} = 100 \times 10^{-0.391} = 41$ mW

10.10 Voltage Standing Wave Ratio

• Coaxial slotted line
- Ability to measure voltage amplitudes according to the position.
- Ruler
- Measuring items
  1) Maximum & minimum voltages
  2) Distance between the maximum (or minimum) and the next maximum (or minimum) voltage point



• Voltage standing wave ratio (VSWR):
A ratio of maximum voltage amplitude to minimum voltage amplitude

• Case study
1) Case 1: The slotted line is terminated by a matched impedance.
- No reflected wave voltage
- Same voltage amplitude in spite of different instantaneous voltage
2) Case 2: The slotted line is terminated by an open or short circuit.
- Reflected wave voltage
- Different voltage amplitude

• Derivation of the (total) voltage at the specific point

- Load position: $z = 0$
- All measuring point $z < 0$
- Input voltage wave amplitude: V_0
- Line impedance: Z_0 (real), load impedance: Z_L (complex)
- Total phasor voltage:

$$V_{sT}(z) = V_0 e^{-j\beta z} + \Gamma V_0 e^{j\beta z} \tag{79}$$

- Complex reflection coefficient

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\phi} \tag{80}$$

Ex.] $Z_L = 0$ or Z_L (in case of only real value) $< Z_0 \quad \rightarrow \phi = \pi$

Z_L (in case of only real value) $> Z_0 \quad \rightarrow \quad \phi = 0$

$$\begin{aligned}
V_{sT}(z) &= V_0 e^{-j\beta z} + \Gamma V_0 e^{j\beta z} = V_0\left(e^{-j\beta z} + |\Gamma| e^{j(\beta z+\phi)}\right) \\
&= V_0 e^{j\phi/2}\left(e^{-j\beta z}e^{-j\phi/2} + |\Gamma| e^{j\beta z}e^{j\phi/2}\right) \qquad (81) \\
&= V_0\left(1-|\Gamma|\right)e^{-j\beta z} + V_0 e^{j\phi/2}\left(e^{-j\beta z}e^{-j\phi/2} + |\Gamma| e^{j\beta z}e^{j\phi/2}\right) - V_0\left(1-|\Gamma|\right)e^{-j\beta z} \\
&= V_0\left(1-|\Gamma|\right)e^{-j\beta z} + V_0 e^{-j\beta z} + V_0|\Gamma| e^{j(\beta z+\phi)} - V_0 e^{-j\beta z} + V_0|\Gamma| e^{-j\beta z} \\
&= V_0\left(1-|\Gamma|\right)e^{-j\beta z} + V_0|\Gamma| e^{j(\beta z+\phi)} + V_0|\Gamma| e^{-j\beta z} \\
&= V_0\left(1-|\Gamma|\right)e^{-j\beta z} + V_0|\Gamma| e^{j\phi/2}\left(e^{j\beta z}e^{j\phi/2} + e^{-j\beta z}e^{-j\phi/2}\right) \qquad (82) \\
&= V_0\left(1-|\Gamma|\right)e^{-j\beta z} + 2V_0|\Gamma| e^{j\phi/2}\cos(\beta z + \phi/2) \qquad (83)
\end{aligned}$$

- Real instantaneous form:

$$V(z,t) = \mathrm{Re}[V_{sT}(z)e^{j\omega t}]$$
$$= \underbrace{V_0\left(1-|\Gamma|\right)\cos(\omega t - \beta z)}_{\text{traveling wave}} + \underbrace{2|\Gamma|V_0\cos(\beta z + \phi/2)\cos(\omega t + \phi/2)}_{\text{standing wave}} \quad (84)$$

Standing wave: the reflected or back-propagated wave interferes with an equivalent portion of the incident wave to form a standing wave.

• Phasor-formed incident voltage with Eq. (79) and (80):

$$V_{sT}(z) = V_0\left(e^{-j\beta z} + |\Gamma|e^{j(\beta z+\phi)}\right) \quad (85)$$

Location of the minimum voltage amplitude:

$$-\beta z_{\min} = \beta z_{\min} + \phi + \pi + 2m\pi \qquad (m = 0,\ 1,\ 2,\ \cdots)$$

$$z_{\min} = -\frac{1}{2\beta}\{\phi + (2m+1)\pi\} \quad (86)$$

Minimum voltage amplitude:

$$V_{sT}(z_{\min}) = V_0(1-|\Gamma|) \quad (87)$$

In Eq. (84),

$$\omega t - \beta z_{\min} = \omega t - \beta \times \left(-\frac{1}{2\beta}\right)\{\phi + (2m+1)\pi\} = \omega t + \frac{\phi}{2} + \frac{1}{2}(2m+1)\pi$$

$$\beta z_{\min} + \frac{\phi}{2} = \beta \times \left(-\frac{1}{2\beta}\right)\{\phi + (2m+1)\pi\} + \frac{\phi}{2} = -\frac{1}{2}(2m+1)\pi$$

Instantaneous voltage at the minimum point from Eq. (84)

$$V(z_{\min},t) = \underbrace{V_0\left(1-|\Gamma|\right)\cos(\omega t-\beta z)}_{\text{traveling wave}} + \underbrace{2|\Gamma|V_0\cos(\beta z+\phi/2)\cos(\omega t+\phi/2)}_{\text{standing wave}}\Bigg|_{z=z_{\min}} \quad (84)$$

$$= V_0\left(1-|\Gamma|\right)\cos\left\{\omega t+\frac{\phi}{2}+\frac{1}{2}(2m+1)\pi\right\} + 2|\Gamma|V_0\cos\{(2m+1)\pi/2\}\cos(\omega t+\phi/2)$$

$$= V_0\left(1-|\Gamma|\right)\left\{\cos(\omega t+\phi/2)\cos\{(2m+1)\pi/2\} - \sin(\omega t+\phi/2)\sin\{(2m+1)\pi/2\}\right\}$$

$$+ 2|\Gamma|V_0\cos\{(2m+1)\pi/2\}\cos(\omega t+\phi/2)$$

$$= \pm V_0\left(1-|\Gamma|\right)\sin(\omega t+\phi/2) \quad (88)$$

• Location of the maximum voltage amplitude:

$$-\beta z_{\max} = \beta z_{\max} + \phi + 2m\pi \qquad \left(n=0,\ \pm 1,\ \pm 2,\cdots\right)$$

$$z_{\max} = -\frac{1}{2\beta}(\phi+2m\pi) \quad (89)$$

Maximum voltage amplitude:

$$V_{sT}(z_{\max}) = V_0(1+|\Gamma|) \quad (90)$$

In Eq. (84),

$$\omega t - \beta z_{\max} = \omega t - \beta\times\left(-\frac{1}{2\beta}\right)(\phi+2m\pi) = \omega t + \frac{\phi}{2} + m\pi$$

$$\beta z_{\max} + \frac{\phi}{2} = \beta\times\left(-\frac{1}{2\beta}\right)(\phi+2m\pi) + \frac{\phi}{2} = -m\pi$$

Instantaneous voltage at the maximum point from Eq. (84)

$$V(z_{\max},t) = \underbrace{V_0\left(1-|\Gamma|\right)\cos(\omega t-\beta z)}_{\text{traveling wave}} + \underbrace{2|\Gamma|V_0\cos(\beta z+\phi/2)\cos(\omega t+\phi/2)}_{\text{standing wave}}\Bigg|_{z=z_{\max}} \quad (84)$$

$$= V_0\left(1-|\Gamma|\right)[\cos\left\{\omega t+\frac{\phi}{2}+m\pi\right\} + 2|\Gamma|V_0\cos(-m\pi)\cos(\omega t+\frac{\phi}{2})$$

$$= V_0\left(1-|\Gamma|\right)[\cos(\omega t+\frac{\phi}{2})\cos(m\pi)+\sin(\omega t+\frac{\phi}{2})\sin(m\pi)] + 2|\Gamma|V_0\cos\left(\omega t+\frac{\phi}{2}\right)\cos(m\pi)$$

$$= V_0\left(1+|\Gamma|\right)\cos\left(\omega t+\frac{\phi}{2}\right)\cos(m\pi) = \pm V_0\left(1+|\Gamma|\right)\cos\left(\omega t+\frac{\phi}{2}\right) \quad (91)$$

* Case studies

$Z_L > Z_0$ & (Z_L, Z_0 : real) $\Rightarrow$ Γ : real positive $\Rightarrow$ $\phi = 0$

$\Rightarrow$ Voltage maximum at the load

$\Rightarrow$ Voltage maxima are found at $z_{\max} = -m\pi/\beta = -m\lambda/2$

$Z_L = 0$ $\Rightarrow$ Γ = -1 $\Rightarrow$ $\phi = \pi$

$\Rightarrow$ Voltage maxima are found at

$$z_{\max} = -\frac{1}{2\beta}(\phi+2m\pi)\Bigg|_{\phi=\pi} = -\frac{\pi}{2\beta}, -\frac{3\pi}{2\beta}, -\frac{5\pi}{2\beta}, \cdots \leftarrow \frac{1}{\beta} = \frac{\lambda}{2\pi}$$

$$\overset{\text{or}}{=} -\frac{\lambda}{4}, -\frac{3\lambda}{4}, \cdots$$

Distance between the minima: $\lambda/2$

$$z_{\min} = -\frac{1}{2\beta}\{\phi+(2m+1)\pi\} \quad (86) \quad \leftarrow \frac{1}{\beta} = \frac{\lambda}{2\pi}$$

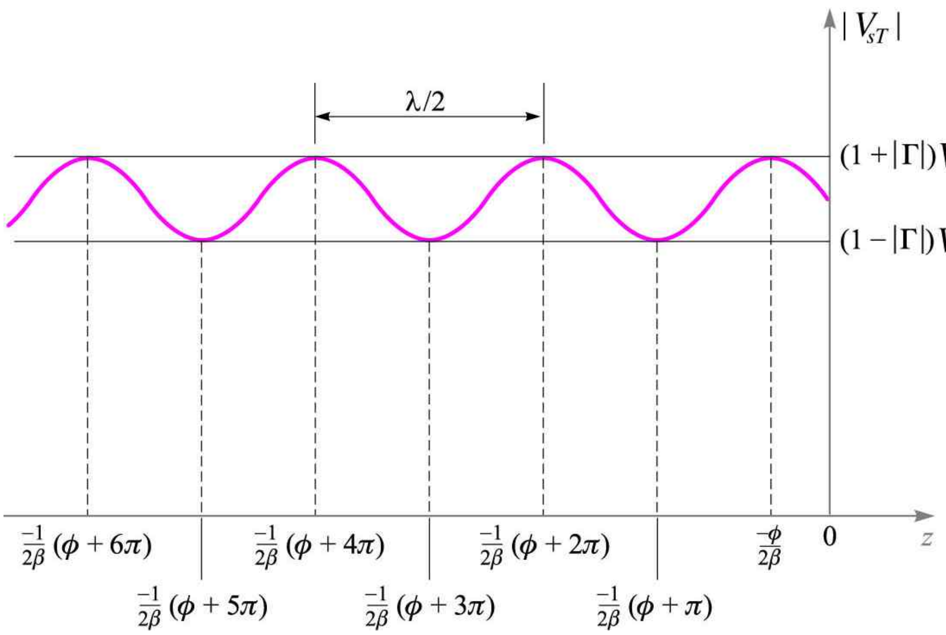
$$= -\frac{\phi}{2\beta} - \frac{(2m+1)\pi}{2\beta} = -\frac{\phi}{2\beta} - \frac{(2m+1)\lambda}{4}$$

$Z_L = 0 \quad \Rightarrow \quad \Gamma = -1 \quad \Rightarrow \quad \phi = \pi$

$\Rightarrow$ The 1st voltage minimum is found at

$$z_{\min} = -\frac{1}{2\beta}\{\phi + (2m+1)\pi\}\Big|_{\phi=\pi,\, m=-1} = 0$$

$$Z_L < Z_0 \quad \Rightarrow \quad \phi = \pi \quad \Rightarrow \quad z_{\min} = -\frac{1}{2\beta}\{\phi + (2m+1)\pi\}\Big|_{\phi=\pi,\, m=-1} = 0$$



• Voltage standing wave ratio (VSWR):

$$s \equiv \frac{V_{sT}(z_{\max})}{V_{sT}(z_{\min})} = \frac{1+|\Gamma|}{1-|\Gamma|} \qquad (92)$$

[Ex. 10.7] 50 Ω slotted line measurement

- VSWR (s) = 5, successive voltage maximum spacing = 15 cm
- Distance of the 1st maximum from load = 7.5 cm

➔ $\lambda/2 = 0.15$, $\lambda = 0.3$

$f = c / \lambda = (3\times10^8)/0.3 = 1\times10^9 = 1$ GHz

7.5 cm $\leftrightarrow \lambda/4 \leftrightarrow$ Voltage minimum occurs at the load.

$\leftrightarrow Z_L < Z_0$

$\leftrightarrow \phi = \pi$

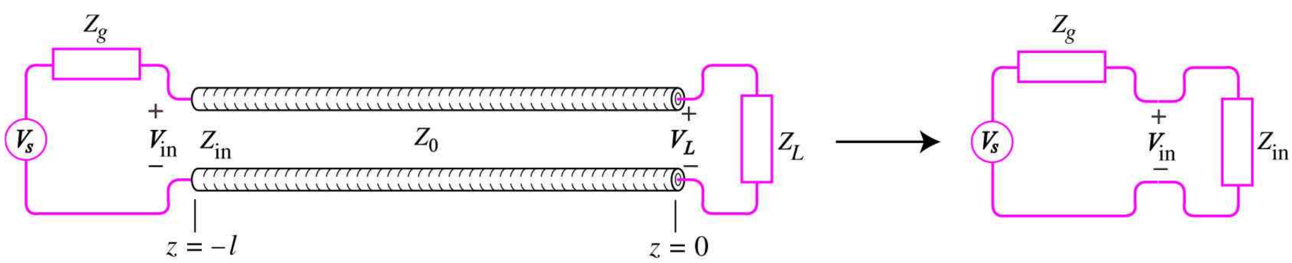
$$|\Gamma| = \frac{s-1}{s+1} = \frac{5-1}{5+1} = \frac{2}{3}$$

$$\Gamma = -\frac{2}{3} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_L = \frac{1}{5} Z_0 = 10 \ \Omega$$

10.11 Transmission Lines of Finite Length

• Finite-length transmission line configuration and its equivalent circuit



• Total voltage in the line

$$V_{sT}(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \qquad (93)$$

Where V_0^+: sum of all individual forward waves

V_0^-: sum of all individual backward waves

• Total current in the line:

$$I_{sT}(z) = I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z} \tag{94}$$

• Wave impedance $Z_W(z)$: a ratio of total phasor voltage to total phasor current at $z = -l$

$$Z_w(z) \equiv \frac{V_{sT}(z)}{I_{sT}(z)} = \frac{V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}}{I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z}} \quad (95) \leftarrow V_0^- = \Gamma V_0^+, I_0^+ = V_0^+ / Z_0, I_0^- = -V_0^- / Z_0$$

$$= \frac{V_0^+ e^{-j\beta z} + \Gamma V_0^+ e^{j\beta z}}{(V_0^+ e^{-j\beta z} - \Gamma V_0^+ e^{j\beta z}) / Z_0} = Z_0 \left[\frac{e^{-j\beta z} + \Gamma e^{j\beta z}}{e^{-j\beta z} - \Gamma e^{j\beta z}} \right] \quad (96) \leftarrow \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= Z_0 \left[\frac{(Z_L + Z_0) e^{-j\beta z} + (Z_L - Z_0) e^{j\beta z}}{(Z_L + Z_0) e^{-j\beta z} - (Z_L - Z_0) e^{j\beta z}} \right]$$

$$= Z_0 \left[\frac{Z_L (e^{j\beta z} + e^{-j\beta z}) - Z_0 (e^{j\beta z} - e^{-j\beta z})}{Z_0 (e^{j\beta z} + e^{-j\beta z}) - Z_L (e^{j\beta z} - e^{-j\beta z})} \right]$$

$$= Z_0 \left[\frac{Z_L (e^{j\beta z} + e^{-j\beta z}) / 2 - jZ_0 (e^{j\beta z} - e^{-j\beta z}) / j2}{Z_0 (e^{j\beta z} + e^{-j\beta z}) / 2 - jZ_L (e^{j\beta z} - e^{-j\beta z}) / j2} \right]$$

$$= Z_0 \left[\frac{Z_L \cos(\beta z) - jZ_0 \sin(\beta z)}{Z_0 \cos(\beta z) - jZ_L \sin(\beta z)} \right] \quad (97) \leftarrow z = -l$$

$$= Z_0 \left[\frac{Z_L \cos(\beta l) + jZ_0 \sin(\beta l)}{Z_0 \cos(\beta l) + jZ_L \sin(\beta l)} \right] \quad (98)$$

$$= Z_0 \left[\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right]$$

• In case of $\beta l = \frac{2\pi}{\lambda}\frac{m\lambda}{2} = m\pi \qquad (m = 0,\ 1,\ 2,\ \cdots)$,

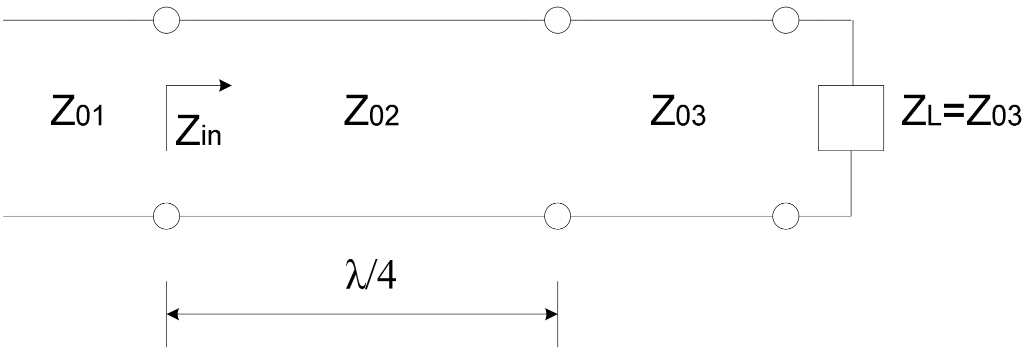
$$Z_{\text{in}}(l = m\lambda/2) = Z_L \qquad (99)$$

➔ 부하(load)에서 $m\lambda/2$ 떨어진 지점에서의 입력 임피던스는 부하 임피던스와 같음.

• In case $\beta l = \frac{2\pi}{\lambda}(2m+1)\frac{\lambda}{4} = (2m+1)\frac{\pi}{2} \qquad (m = 0,\ 1,\ 2,\ \cdots)$,

$$Z_{\text{in}}(l = \lambda/4) = \frac{Z_0^2}{Z_L} \qquad (100)$$

• Joining two lines having different characteristic impedances



Input impedance to line 2:

$$Z_{\text{in}} = Z_{02}\frac{Z_{03}\cos(\beta_2 l) + jZ_{02}\sin(\beta_2 l)}{Z_{02}\cos(\beta_2 l) + jZ_{03}\sin(\beta_2 l)} \qquad (101) \quad \leftarrow l = \frac{\lambda}{4},\ \beta l = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

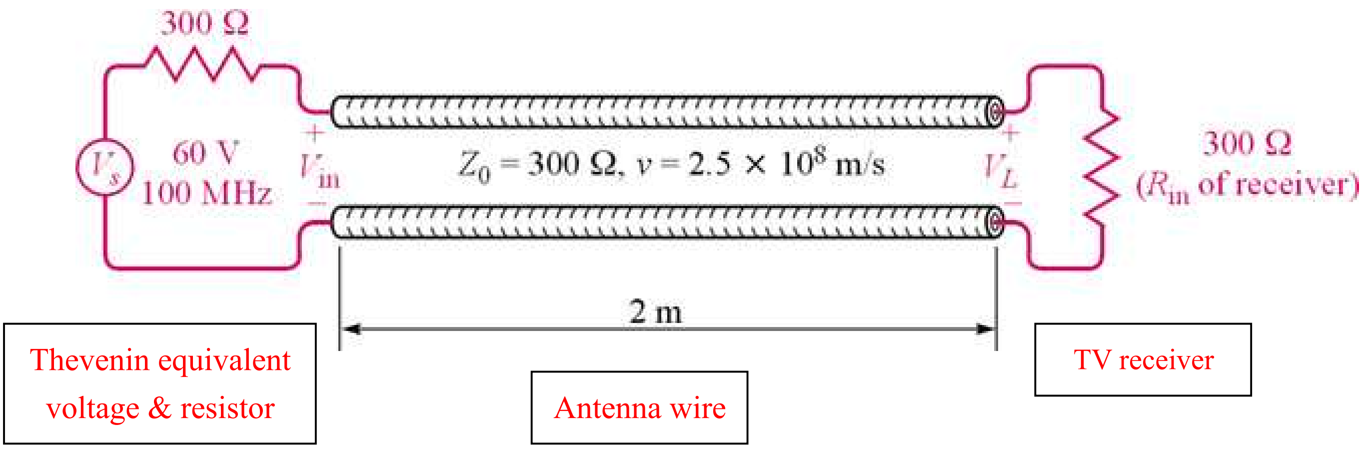
$$= \frac{Z_{02}^2}{Z_{03}} \qquad (102)$$

Quarter-wave matching:

$$Z_{02} = \sqrt{Z_{01}Z_{03}} \quad (@\, l = (2m+1)\lambda/4) \qquad (103)$$

10.12 Some Transmission Line Examples

[Ex. 1]



• $R_s = R_C = R_L \quad \Rightarrow \quad \text{Matched} \quad \Rightarrow \quad \Gamma = 0 \quad \Rightarrow \quad S = 1$

• $\lambda = \dfrac{v}{f} = \dfrac{2.5\times10^8}{100\times10^6} = 2.5 \;\; [\text{m}]$ (wavelength)

• $\beta = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{2.5} = 0.8\pi \; [\text{rad/m}]$ (phase constant)

$\alpha = 0$ (lossless assumption)

• $\beta l = 0.8\pi \times 2 = 1.6\pi \; [\text{rad}]$ (electric length)

$$\Leftrightarrow \quad \frac{1.6\pi}{2\pi}\times 360^\circ = 288^\circ \qquad \Leftrightarrow \quad \frac{1.6\pi}{2\pi}\times\lambda = 0.8\lambda$$

• $V_s = 60\cos 2\pi 10^8 t \quad [\text{V}]$

$$V_{\text{in}} = \frac{300}{300+300} V_s = 30\cos 2\pi 10^8 t \quad [\text{V}]$$

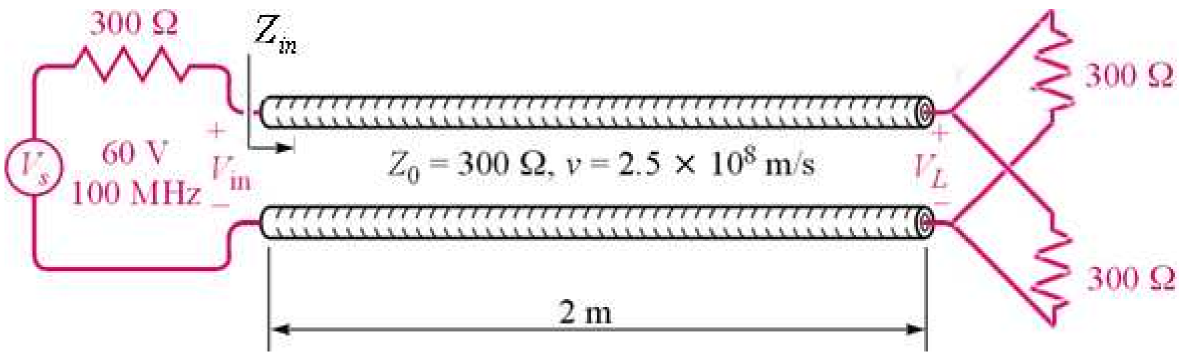
$$V_L = 30\cos(\omega t - \beta l) = 30\cos(2\pi 10^8 t - 1.6\pi) \quad [\mathrm{V}]$$

$$I_{\text{in}} = \frac{V_{\text{in}}}{Z_{\text{in}}} = \frac{30\cos 2\pi 10^8 t}{300} = 0.1\cos 2\pi 10^8 t \quad [\mathrm{A}]$$

$$I_L = \frac{V_L}{Z_L} = 0.1\cos(2\pi 10^8 t - 1.6\pi) \quad [\mathrm{A}]$$

$$P_{\text{in}} = P_L = \frac{1}{2}\times 30 \times 0.1 = 1.5 \quad [\mathrm{W}] \leftarrow \quad P_L = 30\cos(2\pi 10^8 t)\times 0.1\cos(2\pi 10^8 t) = 3\cos^2(2\pi 10^8 t)$$

[Ex. 2]



$$Z_{L1} = Z_{L2} = 300\,[\Omega], \qquad Z_L = Z_{L1} // Z_{L2} = 150\,[\Omega]$$

$$\Gamma = \frac{150-300}{150+300} = -\frac{1}{3} = \frac{1}{3}\angle\pi, \qquad s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{4/3}{2/3} = 2$$

$$Z_{\text{in}} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = 300\frac{150 + j300\tan 288^\circ}{300 + j150\tan 288^\circ} = 300\frac{150 - j923.3}{300 - j461.7}$$

$$= 300\frac{935.1\angle -80.77^\circ}{550.6\angle -56.99^\circ} = 509.5\angle -23.78^\circ$$

$$= 466.2 - j205.4$$ → capacitive ⇒ store electric field energy

$$I_{s,\text{in}} = \frac{V_{s,\text{in}}}{Z_s + Z_{\text{in}}} = \frac{60\angle 0^\circ}{300 + (466.2 - j205.4)} = \frac{60\angle 0^\circ}{766.2 - j205.4}$$

$$= \frac{60\angle 0^\circ}{793.3\angle -15^\circ} = 0.0756\angle 15^\circ \quad [\text{A}]$$

$$P_{\text{in}} = \frac{1}{2}|I_{s,\text{in}}|^2 \cdot \text{Re}(Z_{\text{in}}) = \frac{1}{2} \times 0.0756^2 \times 466.2 = 1.332 \quad [\text{W}]$$

$$= P_L \qquad \because \quad \text{lossless transmission line}$$

$$< 1.5\ [\text{W}] \quad (\text{in previous example result})$$

P_L is divided equally between two receivers. $\Rightarrow 0.666 \quad [\text{W / receiver}]$

Voltage across the receiver:

$$\frac{1}{2}P_L = \frac{1}{2}\left(|V_{s,L}|^2 / R_L\right)$$

$$0.666 = \frac{1}{2}\frac{|V_{s,L}|^2}{300}$$

$$|V_{s,L}| = \sqrt{0.666 \times 600} = 19.99 \quad [\text{V}]$$

$$< 30\ [\text{V}]\ (\text{in previous example result})$$

•Voltage maxima:

$$-\beta z_{\max} = \frac{\phi}{2} + n\pi \quad (n = 0, \pm 1, \pm 2, \cdots) \quad \text{cf.)} \ -\beta z_{\min} = \frac{\phi}{2} + n\pi + \frac{\pi}{2}$$

$$\Gamma = |\Gamma| e^{j\phi}$$

$\because \ \Gamma = -\frac{1}{3} = \frac{1}{3}\angle \pi$

When $\beta = 0.8\pi$, $\phi = \pi$, $\leftarrow$ region 2가 conductor이거나 $Z_L < Z_0$인 경우이므로

$$z_{\max} = -\frac{1}{0.8\pi}\left[\frac{\pi}{2} + n\pi\right] = -0.625 - 1.25n$$

$$= -0.625 \quad \text{or} \quad -1.875 \quad \text{m}$$

$$z_{\min} = z_{\max} \pm \frac{\lambda}{4} \quad \leftarrow \quad \text{Fig 11.6}$$

=0 or -1.25 [m]

General conclusion: voltage minimum at the load if $Z_L < Z_0$
in case Z_L and Z_0 are real
voltage maximum at the load if $Z_L > Z_0$
in case Z_L and Z_0 are real

• $V_{s,\text{in}} = I_{s,\text{in}} Z_{\text{in}} = (0.0756\angle 15^\circ) \times (509.5\angle -23.78^\circ)$

$$= 38.52\angle -8.78^\circ$$

cf.)
$\left|V_{s,L}\right| = 19.99 \text{ V} \approx 20 \text{ V} \quad \because \; Z_L < Z_0$ 이므로.
$V_{s,\max} = S \times V_{s,\min} = 2 \times 20 = 40 \quad \text{V}$
$V_{s,\text{in}}$ 은 40 V 에 근접함 $\because L = 2 \text{ m}$ 이고 $\lambda = 2.5 \text{ m}$ 이므로
$(3/4)\lambda = 1.875 \text{ m}$ 로 L 은 $(3/4)\lambda$ 근처이므로

Near voltage maximum location

•Voltage at an arbitrary point on the line:

$$V_s = \left(e^{-j\beta z} + \Gamma e^{j\beta z}\right) V_0^+ \quad (104)$$

At $z = -l$, $(z = -2 \text{ m})$

$$V_{s,\text{in}} = (e^{j\beta l} + \Gamma e^{-j\beta l}) V_0^+ \quad (105)$$

$$V_0^+ = \frac{V_{s,\text{in}}}{e^{j\beta l} + \Gamma e^{-j\beta l}} = \frac{38.52\angle -8.78^\circ}{e^{j1.6\pi} - (1/3)e^{-j1.6\pi}} = \frac{38.52\angle -8.78^\circ}{(0.309 - j0.957) - (0.103 + j0.317)}$$

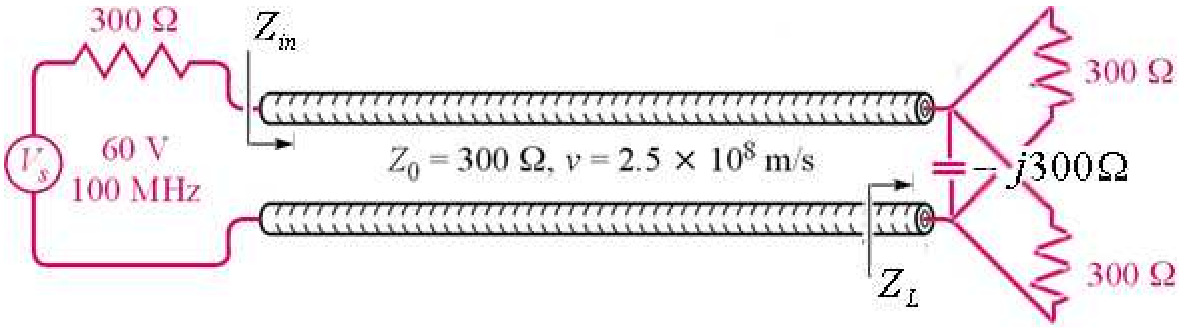
$$= \frac{38.52\angle -8.78^\circ}{0.206 - j1.268} = \frac{38.52\angle -8.78^\circ}{1.285\angle -80.77^\circ} = 29.98\angle 71.99^\circ$$

At $z = 0$,

$$V_{s,L} = (1+\Gamma)V_0^+ = \frac{2}{3} \times 29.98\angle 71.99^\circ = 19.99\angle 71.99^\circ = \underline{19.99\angle -288.1^\circ}$$

: Same result with the previous

[Ex. 10.8]



$$Z_L = \frac{150(-j300)}{150 - j300} = \frac{-j300}{1-j2} = -j\frac{300}{5}(1+j2) = 120 - j60 \quad \Omega$$

$$\Gamma = \frac{(120-j60)-300}{(120-j60)+300} = \frac{-180-j60}{420-j60} = \frac{189.7\angle -161.6^\circ}{424.3\angle -8.13^\circ} = 0.447\angle -153.47^\circ$$

$$s = \frac{1+0.447}{1-0.447} = 2.62$$

$\Rightarrow$ Standing wave ratio is higher than example 2.

$$Z_{\text{in}} = 300\frac{(120-j60)+j300\tan 288^\circ}{300+j(120-j60)\tan 288^\circ} = 300\frac{(120-j60)+j300\times(-3.08)}{300+j(120-j60)(-3.08)}$$

$$= 300\frac{120-j984}{115.2-j369.6} = 300\frac{991.3\angle -83^\circ}{387.1\angle -72.7^\circ}$$

$$= 768.3\angle -10.3^\circ = 755.9 - j137.4$$

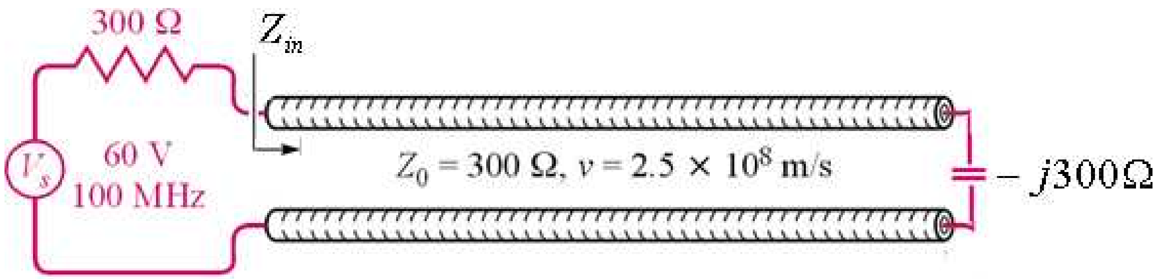
$$I_{s,\text{in}} = \frac{V_{Th}}{Z_{Th} + Z_{\text{in}}} = \frac{60}{300 + 755.9 - j137.4} = \frac{60}{1065\angle -7.41^\circ} = 0.0563\angle 7.41^\circ \quad \text{A}$$

$$P_{\text{in}} = \frac{1}{2} \times 0.0563^2 \times 755.9 = 1.198 \quad \text{W}$$

$$= P_L \qquad (\because \text{ Transmission line is lossless.})$$

➔ Each load dissipates about 0.6 W and less than the previous Ex.

[Ex. 10.9]



$$Z_L = -j300$$

$$\Gamma = \frac{-j300 - 300}{-j300 + 300} = \frac{\sqrt{2}\angle -135^\circ}{\sqrt{2}\angle -45^\circ} = 1\angle -90^\circ$$

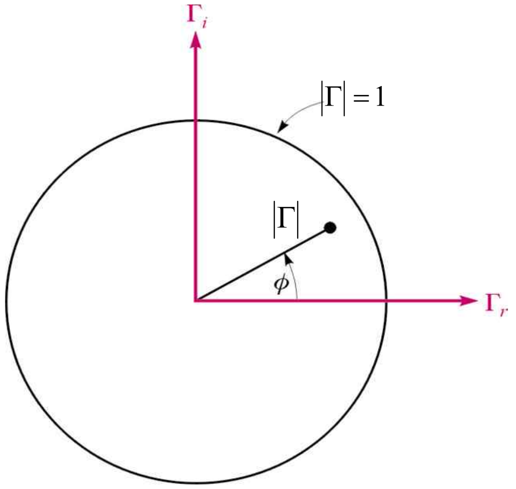
$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \infty$$

$$Z_{\text{in}} = 300\frac{(-j300) + j300\tan 288^\circ}{300 + j(-j300)\tan 288^\circ} = 300\frac{-j1224}{300 - 924} = -j588.5$$

$\Rightarrow$ No average power can be delivered. $\left(\because \frac{1}{2} I_{s,\text{in}}^2 \cdot \underset{\uparrow\,0}{\underline{R_{\text{in}}}}\right)$

10.13 Graphic Methods: The Smith Chart

• Polar coordinates of the Smith chart



$$\Gamma = |\Gamma| e^{j\phi}$$

where $0 \le |\Gamma| \le 1$

• Basic relationship

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{106}$$

Normalized load impedance

$$z_L = r + jx = \frac{Z_L}{Z_0} = \frac{R_L + jX_L}{Z_0}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(Z_L/Z_0) - 1}{(Z_L/Z_0) + 1} = \frac{z_L - 1}{z_L + 1} = \Gamma_r + j\Gamma_i \tag{108}$$

$$z_L = \frac{1+\Gamma}{1-\Gamma} \tag{107}$$

$$= \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i} = \frac{\left[(1+\Gamma_r) + j\Gamma_i\right]\left[(1-\Gamma_r) + j\Gamma_i\right]}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$= \frac{\left[(1-\Gamma_r^2) - \Gamma_i^2\right] + j\left[\Gamma_i(1-\Gamma_r) + \Gamma_i(1+\Gamma_r)\right]}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$= \frac{1-\Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2} + j\frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2} = r + jx \tag{109}$$

$$\therefore \quad r = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2} \qquad (110)$$

$$x = \frac{2\Gamma_i}{(1-\Gamma_r)^2+\Gamma_i^2} \qquad (111)$$

1) Real part

$$\frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2} = r$$

$$(1-\Gamma_r^2)-\Gamma_i^2 = r(1-\Gamma_r)^2 + r\Gamma_i^2$$

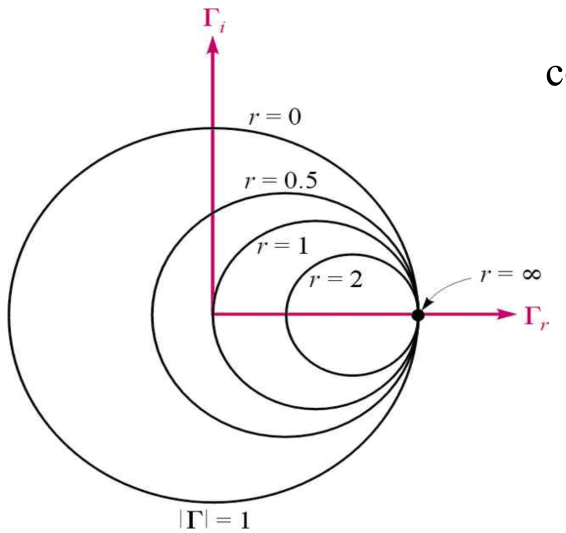
$$\Gamma_r^2(r+1) - 2r\Gamma_r + \Gamma_i^2(r+1) = 1-r$$

$$\Gamma_r^2 - \frac{2r}{1+r}\Gamma_r + \Gamma_i^2 = \frac{1-r}{1+r}, \quad \Gamma_r^2 - \frac{2r}{1+r}\Gamma_r + \left(\frac{r}{r+1}\right)^2 + \Gamma_i^2 = \frac{1-r}{1+r} + \left(\frac{r}{r+1}\right)^2$$

$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \frac{1-r}{1+r} + \left(\frac{r}{r+1}\right)^2 = \frac{1}{(r+1)^2} \qquad (112)$$

Constant r - circles: radius - $\dfrac{1}{r+1}$

center - $\left(\dfrac{r}{r+1}, 0\right)$



2) Imaginary part

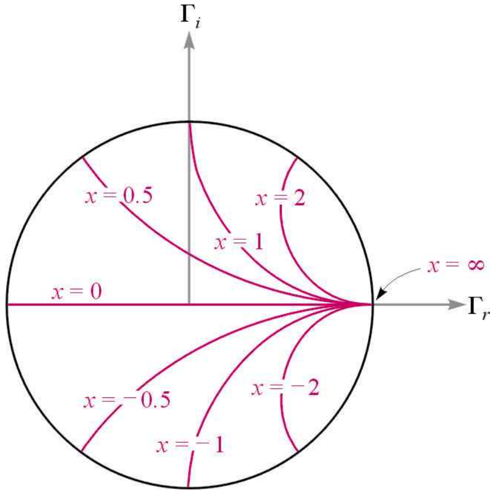
$$\frac{2\Gamma_i}{(1-\Gamma_r)^2+\Gamma_i^2}=x$$

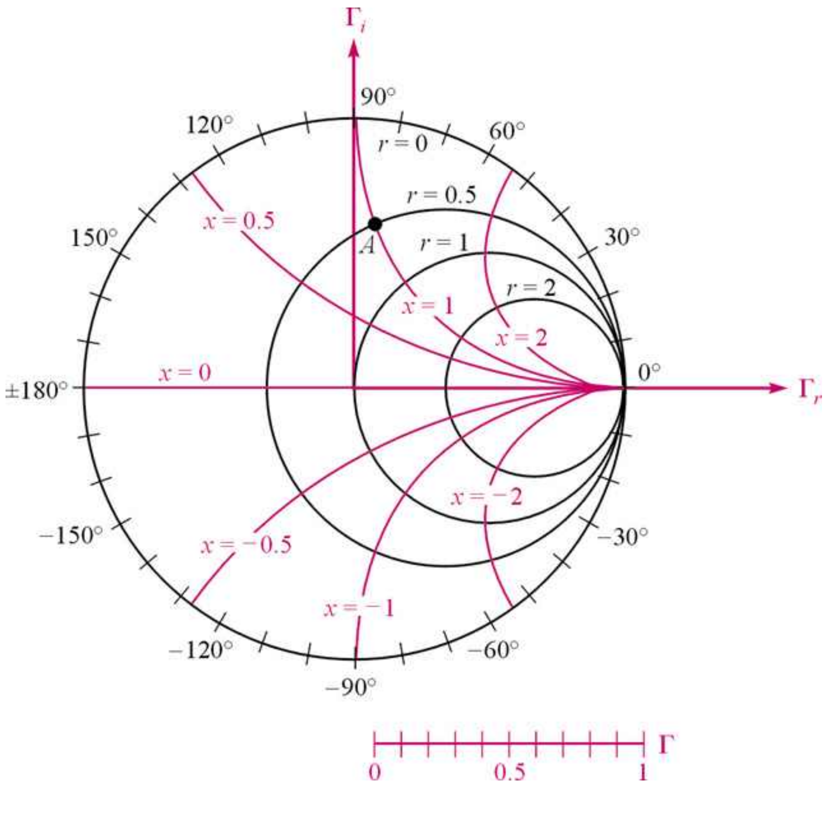
$$(\Gamma_r-1)^2+\Gamma_i^2-\frac{2\Gamma_i}{x}=0$$

$$(\Gamma_r-1)^2+\left(\Gamma_i-\frac{1}{x}\right)^2=\left(\frac{1}{x}\right)^2 \qquad (113)$$

constant x - circles: radius - $\left|\frac{1}{x}\right|$

center - $\left(1,\frac{1}{x}\right)$





[Ex.] $Z_L = 25 + j50 \quad \rightarrow \quad z_L = 0.5 + j1$

From the Smith chart, $|\Gamma| = 0.62, \ \phi = 83^\circ$

• Second scale on the circumference

$$V_s = V_0^+(e^{-j\beta z} + \Gamma e^{j\beta z})$$

$$I_s = \frac{V_0^+}{Z_0}(e^{-j\beta z} - \Gamma e^{j\beta z})$$

$$z_{\text{in}} = \frac{Z_{\text{in}}}{Z_0} = \frac{1}{Z_0} \cdot \frac{V_0^+(e^{-j\beta z} + \Gamma e^{j\beta z})}{\left(V_0^+ / Z_0\right)(e^{-j\beta z} - \Gamma e^{j\beta z})} = \frac{e^{-j\beta z} + \Gamma e^{j\beta z}}{e^{-j\beta z} - \Gamma e^{j\beta z}} \quad \leftarrow z = -l$$

$$= \frac{e^{j\beta l} + \Gamma e^{-j\beta l}}{e^{j\beta l} - \Gamma e^{-j\beta l}} = \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}} = \frac{1 + |\Gamma| e^{j(\phi - 2\beta l)}}{1 - |\Gamma| e^{j(\phi - 2\beta l)}} \quad (114)$$

If $l = 0$,

$$z_{\text{in}} = \frac{1 + \Gamma}{1 - \Gamma} = z_L$$

If $z = -l$

$$z_{\text{in}} = \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}} \quad : \quad \leftarrow \quad (\Gamma e^{-j2\beta l} \leftarrow \Gamma)$$

(위 두 식을 비교하면, Γ 대신에 $\Gamma e^{-j2\beta l}$을 대입한 것과 동일. 부하에서 l만큼 떨어진 지점에서의 입력 임피던스는 반사계수 평면에서 부하에서의 반사계수 Γ를 전기각 $-2\beta l$ 만큼 신호원 방향으로 회전시켜 측정하는 것과 같음)

∴ As we proceed from the load z_L to the input impedance z_{in}, we move toward the generator a distance l and move through a clockwise angle of $2\beta l$ on the Smith chart.

- Since the magnitude of Γ stays constant, the movement toward the source is made along a constant-radius circle

  (∵ 전송선로가 lossless 이므로)

(l 만큼의 물리적 위치 변화에 $2\beta l$ 만큼의 위상 변화를 가져옴. 따라서 부하에서 $l = \lambda/2$ 떨어진 지점은 $2\beta l = 2\pi$ 가 되어 부하와 동일한 임피던스를 갖게 됨. 동일지점. ⇒ 외곽 테두리에 반파장을 명시.)

[Ex. 10.10]

$Z_L = 25 + j50 \quad \Omega$

$z_L = 0.5 + j1 \ \rightarrow$ point A

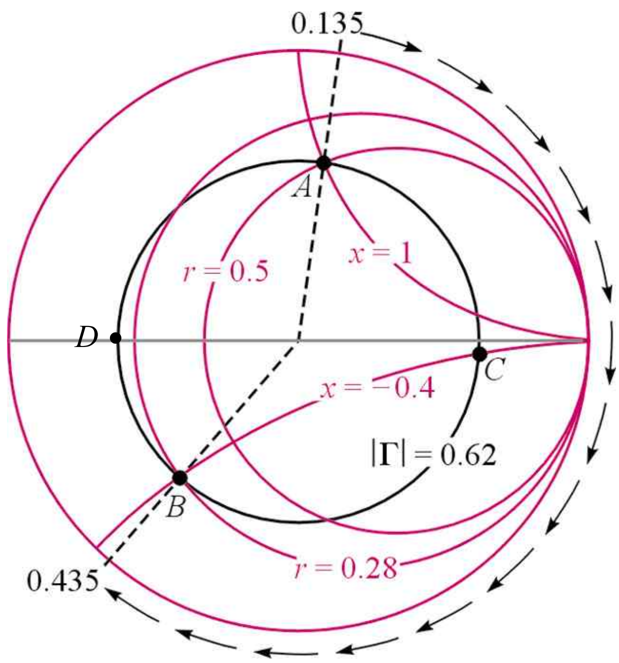
$\lambda = 2$ m 일 때 load로부터 60 cm 떨어진 곳의 입력 임피던스:

$\frac{l}{\lambda} = \frac{0.6}{2} = 0.3\lambda$

$0.135\lambda + 0.3\lambda = 0.435\lambda \ \rightarrow$ point B

∴ $z_{in} = 0.28 - j0.4$,

$Z_{in} = 14 - j20$



- Voltage maxima: @wtg = 0.25   *wtg* : wavelength toward generator

∴ point C (∵ $R_L > Z_0$, pure resistance)

• Voltage minima: @ wtg = 0 or 0.5

$\therefore$ point D ($\because$ $R_L < Z_0$, pure resistance)

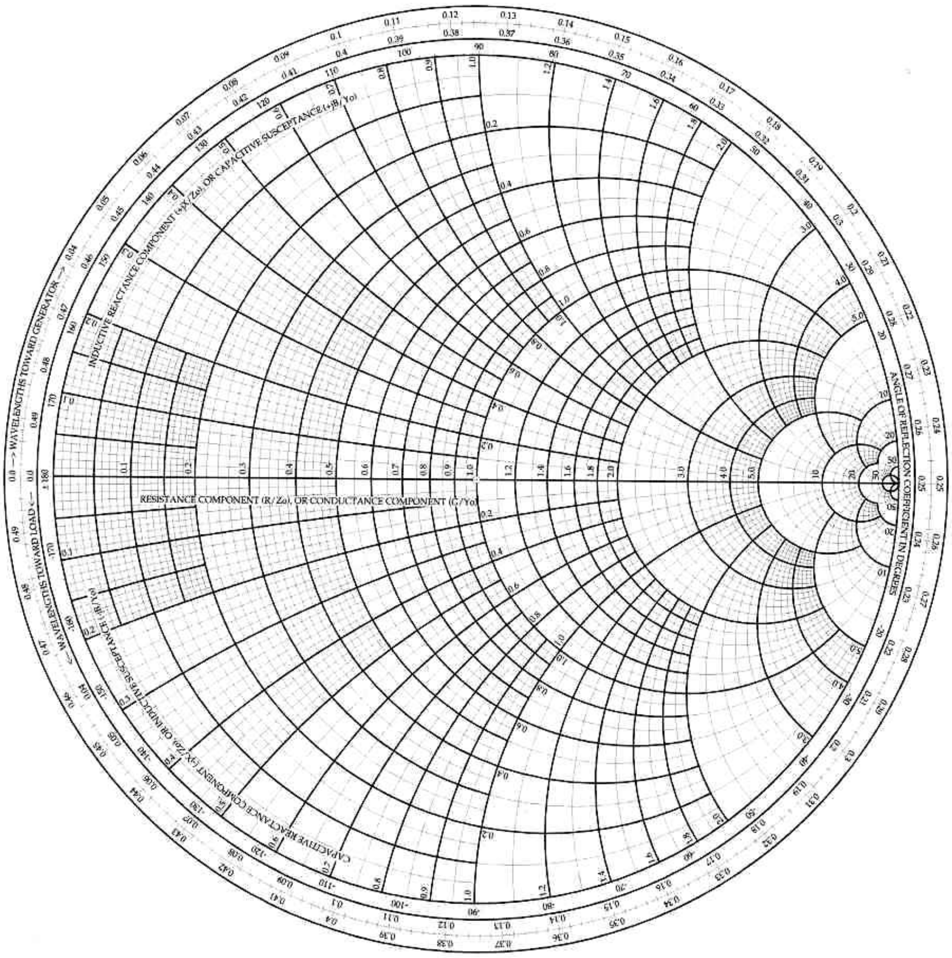
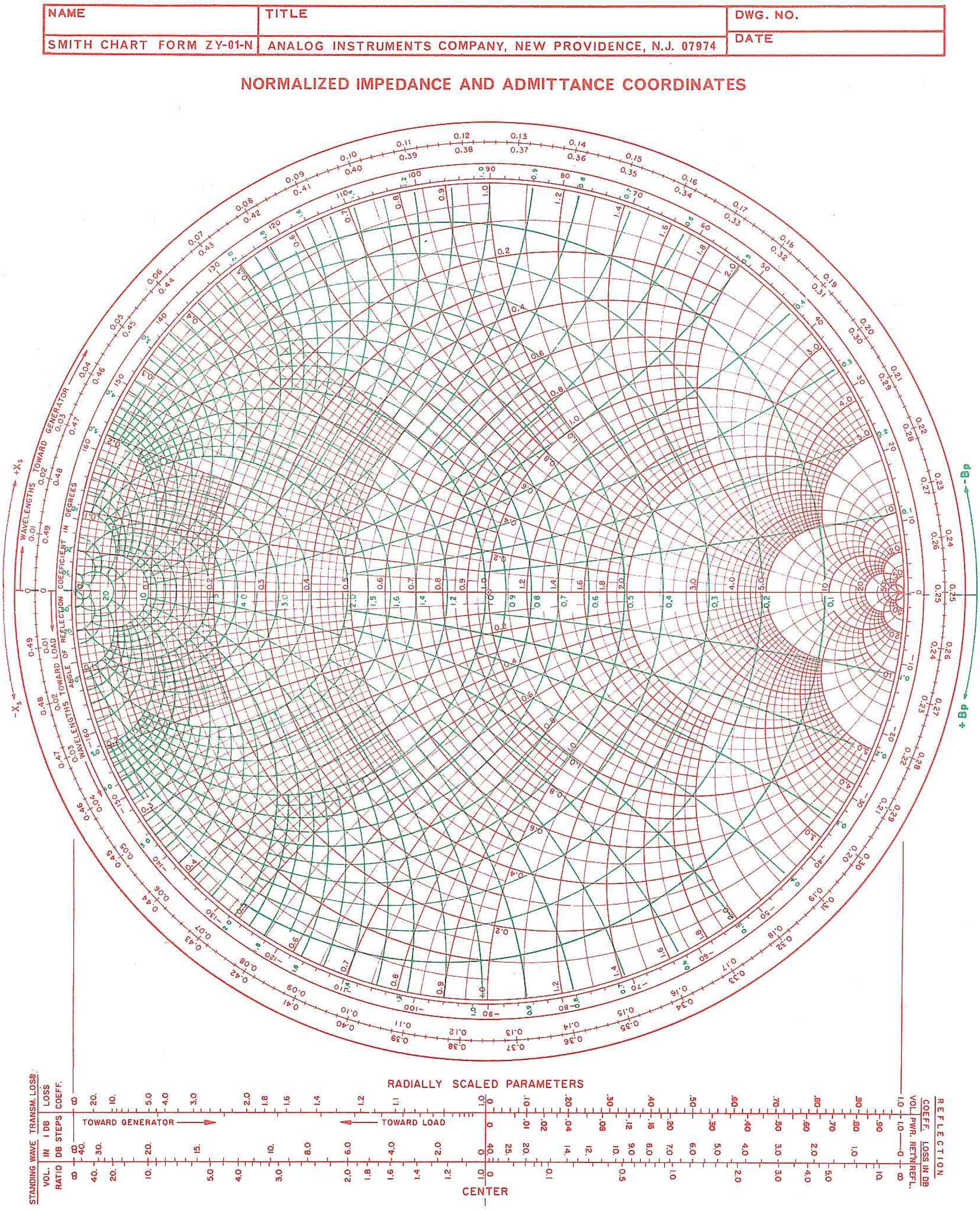


Figure 10.13 A photographic reduction of one version of a useful Smith chart.

• Normalized Impedance **and** Admittance Coordinates



[Ex.] The determination of load impedance

① $s = 2.5$, operating freq. = 400 MHz $\Rightarrow \lambda = 75$ cm

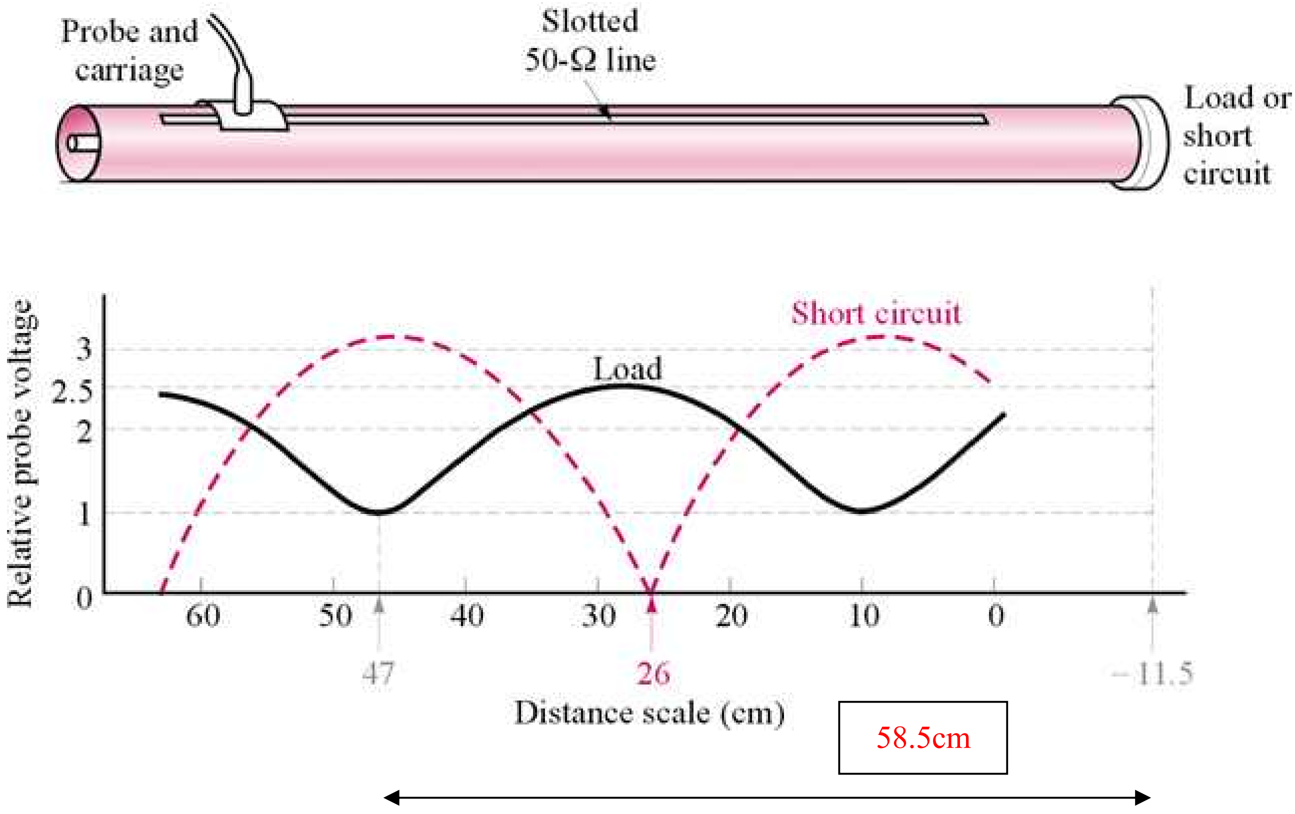
② The carriage moves indicates that a minimum occurs at 47 cm

③ Replace load with a short circuit

④ Repeat step ②.   Minimum at 26cm.

⑤ Short circuit locates one half-wavelength away and this is

load location : 26 - 37.5 = -11.5 cm



⑥ The minimum is 47.0 – (-11.5) = 58.5 cm from the load.
(load를 달고 시험할 때 minimum point 가 load로부터 58.5 cm 이격)

⑦ The maximum is $58.5-\left(\frac{3}{4}\times 75\right)=58.5-56.25=2.25\,\text{cm}$ from the load.

(47 cm 지점(= load로부터 58.5 cm지점)에서 $\lambda/2$ 이격되면 또 minimum. 다시 $\lambda/4$ 이격하면 maximum.)

⑧ So 2.25 cm $\rightarrow \dfrac{2.25}{75} = 0.03\lambda$

⑨ $|\Gamma| = \dfrac{s-1}{s+1} = \dfrac{1.5}{3.5} = 0.4258$

⑩ $z_L = 2.1 + j0.8 \quad \rightarrow \quad Z_L = 105 + j40$

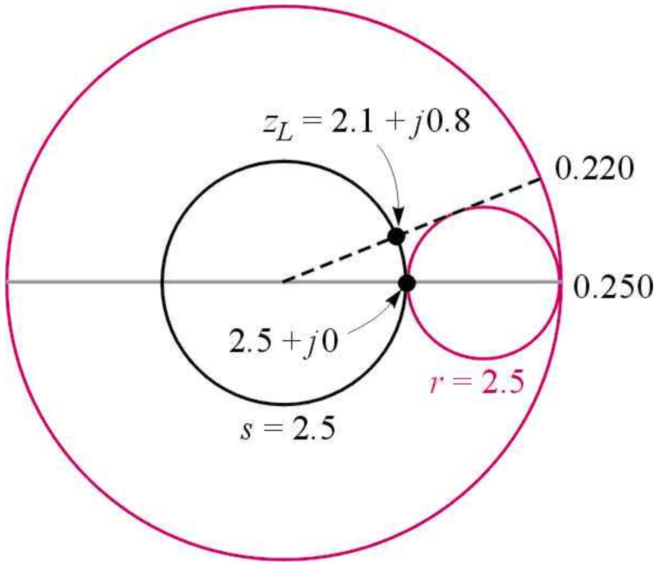
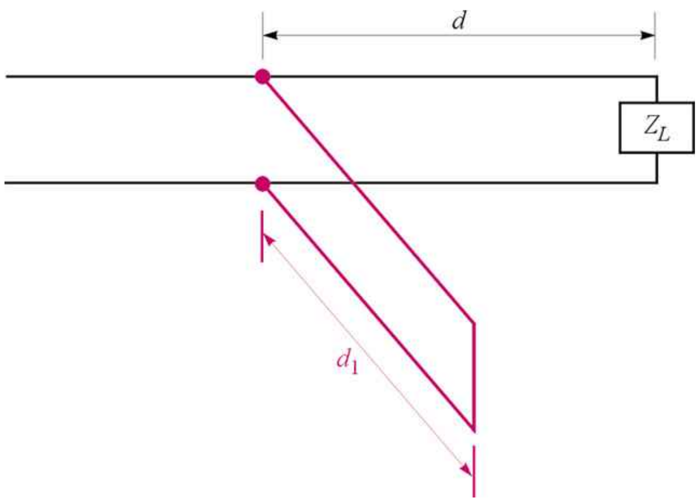


Figure 10.16

If $z_{in} = 2.5 + j0$ on a line 0.03 wavelength long, then $z_L = 2.1 + j0.8$

[Ex.] Match the above load to the 50 Ω by placing a short –circuited stub of length d_1 at a distance d from the load.



•Adding one-quarter wavelength ($\lambda/4$) to Z_{in}. (z_{in})

$$z_{in}' = \frac{z_{in} + j\tan\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}}{1 + jz_{in}\tan\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}} = \frac{1}{z_{in}} = y_{in}$$

(load 임피던스를 180° rotation시키면 load admittance 와 동일)

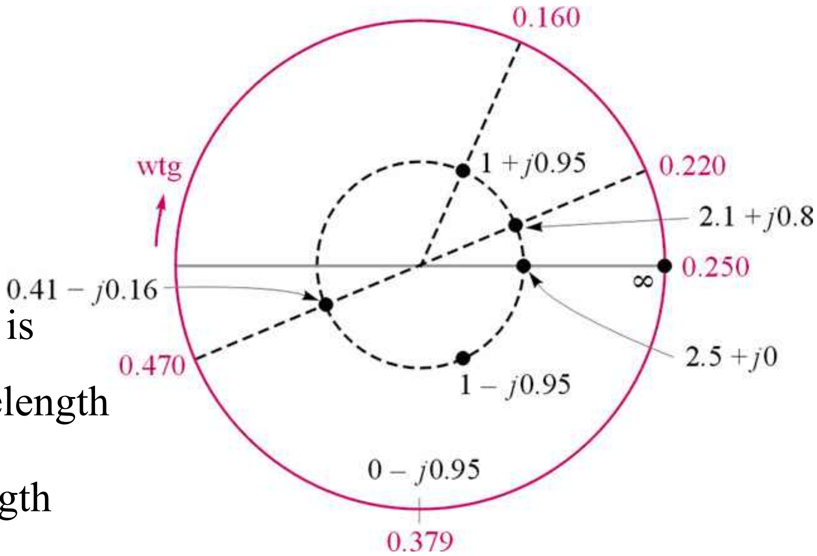


Figure 10.18

A normalized load $z_L = 2.1 + j0.8$ is matched by placing a 0.129-wavelength short-circuited stub 0.19 wavelength from the load.

•Target: $y_{in} = 1 + j0$ ➔ $z_{in} = 1$

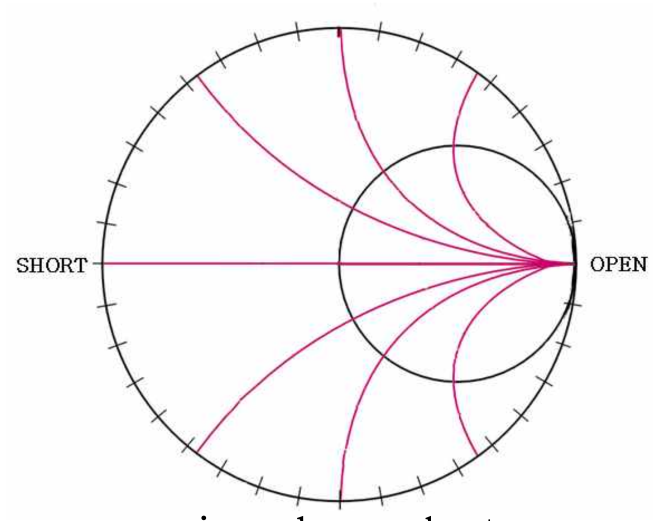
• $y_L = 0.41 - j0.16$

•Moving toward generator by $0.16\lambda + (0.5 - 0.47)\lambda = 0.19\lambda$

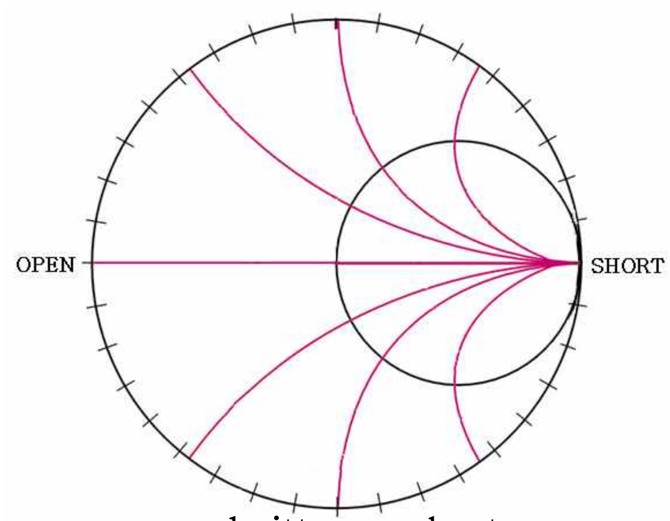
$$\Rightarrow \quad d = 0.19 \times 75\,\text{cm} = 14.25\,\text{cm}$$

$$\Rightarrow \quad y_{in}' = 1 + j0.95$$

• $y_{in}'' = 0 - j0.95 \quad \rightarrow \quad (0.379 - 0.25)\lambda = 0.129\lambda$ short stub.



impedance chart



admittance chart

$$0.129\lambda \quad \rightarrow \quad d_1 = 0.129 \times 75\ \text{cm} = 9.675\ \text{cm}$$

$$\therefore \quad y_{in} = y'_{in} + y''_{in} = (1 + j0.95) + (-0.95\lambda) = 1 \qquad \text{(Matching)}$$