Chapter 11. The Uniform Plane Wave

11.1 Wave Propagation in (source) Free Space

• Consider wave motion in free space first. $(\rho = 0 \, , J = 0)$

$$\begin{cases} \nabla \times \vec{H} = \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} & (1) \\ \nabla \times \vec{E} = -\mu_0 \dfrac{\partial \vec{H}}{\partial t} & (2) \\ \nabla \cdot \vec{E} = 0 & (3) \\ \nabla \cdot \vec{H} = 0 & (4) \end{cases}$$

(1) : $\vec{E}$가 한 점에서 시간적인 변화가 있으면 그 주위에 $\vec{H}$ curl 발생.
→ closed loop linking → $\vec{H}$도 시간적인 변화를 유도

(2) : $\vec{H}$가 한 점에서 시간적인 변화가 있으면 그 주위에 $\vec{E}$ curl 발생.
→ closed loop linking → $\vec{E}$도 시간적인 변화를 유도

⇒ 원래 source point로부터 약간 이격된 곳에 새로운 field를 유기시킴.

• Given the electric vector field:

$\vec{E} = E_x \vec{a}_x$: 진폭 방향은 $\vec{a}_x$이되 진행방향은 $\vec{a}_z$ ➔ $\vec{E} = E_x(z) e^{\pm \gamma z} \vec{a}_x$

Using Eq. (2),

$$\nabla \times \vec{E} = \frac{\partial E_x}{\partial z} \vec{a}_y = -\mu_0 \frac{\partial \vec{H}}{\partial t} = -\mu_0 \frac{\partial H_y}{\partial t} \vec{a}_y \qquad (5)$$

$\Rightarrow \vec{E}$ and $\nabla \times \vec{E}$ are mutually orthogonal.

$$\nabla \times \vec{H} = -\frac{\partial H_y}{\partial z} \vec{a}_x = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \varepsilon_0 \frac{\partial E_x}{\partial t} \vec{a}_x \qquad (6)$$

➔ $\vec{H} = H_y(z) e^{\pm \gamma z} \vec{a}_y$

With Eqs. (5) and (6),

$$\frac{\partial E_x}{\partial z} = -\mu_0 \frac{\partial H_y}{\partial t} \qquad (7)$$

$$\frac{\partial H_y}{\partial z} = -\varepsilon_0 \frac{\partial E_x}{\partial t} \qquad (8)$$

↔Telegraphist's equations for lossless line (Eq. (20) and (21) in Ch.10)

Differentiate Eq. (7) with respect to z and Eq. (8) with respect to t:

$$\frac{\partial^2 E_x}{\partial z^2} = -\mu_0 \frac{\partial^2 H_y}{\partial t \partial z} \qquad (9)$$

$$\frac{\partial^2 H_y}{\partial z \partial t} = -\varepsilon_0 \frac{\partial^2 E_x}{\partial t^2} \qquad (10)$$

Eq. (9) ← Eq. (10):

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2} \qquad (11)$$

: **<u>Wave equation</u>** for x-polarized TEM electric field

↔ Analogy to Eq. (13) in Ch. 10

Propagation velocity:

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times 10^8 \text{ m/sec} = c \qquad (12)$$

where c: velocity of light in free space

• Differentiate Eq. (7) with respect to t and Eq. (8) with respect to z:

$$\frac{\partial^2 E_x}{\partial z \partial t} = -\mu_0 \frac{\partial^2 H_y}{\partial t^2}$$

$$\frac{\partial^2 H_y}{\partial z^2} = -\varepsilon_0 \frac{\partial^2 E_x}{\partial t \partial z} = \mu_0 \varepsilon_0 \frac{\partial^2 H_y}{\partial t^2} \qquad (13)$$

: **<u>Wave equation</u>** for y-polarized TEM magnetic field

• General electric field solution of Eq. (11):

= forward-propagating waves + backward-propagating waves

$$E_x(z,t) = f_1(t - z/v) + f_2(t + z/v) \qquad (14)$$

Since the waves are sinusoidal, electric field can be expressed with a phase velocity (v_p) as below.

$$\begin{aligned} E_x(z,t) &= \mathrm{E}_x(z,t) + \mathrm{E}_x^{'}(z,t) \\ &= |E_{x0}|\cos[\omega(t - z/v_p) + \phi_1] + |E_{x0}^{'}|\cos[\omega(t + z/v_p) + \phi_2] \\ &= \underbrace{|E_{x0}|\cos[\omega t - k_0 z + \phi_1]}_{\text{forward z travel}} + \underbrace{|E_{x0}^{'}|\cos[\omega t + k_0 z + \phi_2]}_{\text{backward z travel}} \qquad (15) \end{aligned}$$

where $k_0 \equiv \dfrac{\omega}{c}$ [rad/m] (16)

: wavenumber in free space

ω: radian time frequency (or angular frequency) [rad/s]

k_0: spatial frequency [rad/m] ↔ phase constant

Wavelengh: distance in free space over which the special phase shifts by 2π

$$k_0 z = k_0 \lambda = 2\pi \quad \rightarrow \quad \lambda = \frac{2\pi}{k_0} \quad \text{(free space)} \quad (17)$$

• Condition for considering the m^{th} crest of the wave:

$k_0 z = 2m\pi$

For periodic cosine waveform, the entire cosine argument be the same multiple of 2π for all time.

$$\omega t - k_0 z = \omega t - (\omega / c)z = \omega(t - z/c) = 2m\pi \qquad (18)$$

If t ↑, then z ↑. ➔ moving positive z direction with phase velocity c (in free space)

$$\omega t + k_0 z = \omega(t + z/c) = 2m\pi$$

If t ↑, then z ↓. ➔ moving negative z direction with phase velocity c (in free space)

• Real instantaneous fields in terms of their phasor forms:

$$\mathrm{E}_x(z,t) = \frac{1}{2}\underbrace{|E_{x0}|e^{j\phi_1}}_{E_{x0}} e^{-jk_0 z}e^{j\omega t} + c.c. = \frac{1}{2}E_{xs}e^{j\omega t} + c.c. = \mathrm{Re}\left[E_{xs}e^{j\omega t}\right] \qquad (19)$$

where $E_{xs} = E_{x0}e^{-jk_0 z}$: phasor electric field

$E_{x0} = |E_{x0}|e^{j\phi_1}$: complex amplitude

$c.c.$: complex conjugate

[Ex. 11.1] $E_y(z,t) = 100\cos(10^8 t - 0.5z + 30^\circ)$ [V/m]

$$\xrightarrow{phasor} E_{ys}(z) = 100e^{-j(0.5z - 30^\circ)}$$

[Ex. 11.2] Magnitude $\vec{E}_s = 100\vec{a}_x + 20\angle 30^\circ \vec{a}_y$ [V/m]

$$= 100e^{j0^\circ}\vec{a}_x + 20e^{j30^\circ}\vec{a}_y, \quad f = 10^7 \text{ Hz}$$

$k_0 = \omega / c = (2\pi \times 10^7) / (3 \times 10^8) = 0.21$ rad/m

$$\vec{E}(z,t) = \mathrm{Re}[100 e^{-j0.21z} e^{j2\pi\times10^7 t}\vec{a}_x + 20 e^{j30^o} e^{-j0.21z} e^{j2\pi\times10^7 t}\vec{a}_y]$$
$$= \mathrm{Re}[100 e^{j(2\pi\times10^7 t - 0.21z)}\vec{a}_x + 20 e^{j(2\pi\times10^7 t - 0.21z + 30^o)}\vec{a}_y]$$
$$= 100\cos(2\pi\times10^7 t - 0.21z)\vec{a}_x$$
$$+ 20\cos(2\pi\times10^7 t - 0.21z + 30^\circ)\vec{a}_y$$

• Taking the partial derivative of any field quantity with respect to time is equivalent to multiplying the corresponding phasor by ($j\omega$):

$$\frac{\partial H_y}{\partial z} = -\varepsilon_0 \frac{\partial E_x}{\partial t} \qquad (20)$$

where $E_x(z,t) = \frac{1}{2}E_{xs}(z)e^{j\omega t} + c.c.$ and $H_y(z,t) = \frac{1}{2}H_{ys}(z)e^{j\omega t} + c.c.$ (21)

$$\frac{dH_{ys}(z)}{dz} = -j\omega\varepsilon_0 E_{xs}(z) \qquad (22)$$

• Maxwell's phasor form equations (in free space).

$$\begin{cases} \nabla\times\vec{H}_s = j\omega\varepsilon_0\vec{E}_s & (23) \\ \nabla\times\vec{E}_s = -j\omega\mu_0\vec{H}_s & (24) \\ \nabla\cdot\vec{E}_s = 0 & (25) \\ \nabla\cdot\vec{H}_s = 0 & (26) \end{cases}$$

- Wave equation in free space:

$$\nabla \times \vec{E}_s = -j\omega\mu_0 \vec{H}_s$$

$$\nabla \times \nabla \times \vec{E}_s = (-j\omega\mu_0)\nabla \times \vec{H}_s$$

$$\nabla(\nabla \cdot \vec{E}_s) - \nabla^2 \vec{E}_s = (-j\omega\mu_0)(j\omega\varepsilon_0)\vec{E}_s \quad (27)$$

$$-\nabla^2 \vec{E}_s = \omega^2 \mu_0 \varepsilon_0 \vec{E}_s \qquad (\because \quad \nabla \cdot \vec{E}_s = 0)$$

$$\therefore \quad \nabla^2 \vec{E}_s = -\omega^2 \mu_0 \varepsilon_0 \vec{E}_s$$

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \nabla \times \vec{E} = -\mu_0 \nabla \times \frac{\partial \vec{H}}{\partial t}$$

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \frac{\partial}{\partial t}(\nabla \times \vec{H})$$

$$= -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\therefore \quad \nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$\vec{E}$ – *field* 를 공간적 성격과 시간적인 성격으로 분리시킨 식.

$$\nabla^2 \vec{E}_s = -k_0^2 \vec{E}_s \quad \text{:(vector) Helmholtz equation} \qquad (28)$$

where $k_0 = \omega\sqrt{\mu_0 \varepsilon_0}$: free space wave number

- One-dimensional form: $\nabla^2 \vec{E}_{xs} = -k_0^2 \vec{E}_{xs}$ (29)

($\vec{E}$ – field 가 $\vec{x}$ 방향성분만 가지고 있다고 가정한 것.)

- Expansion of the operator leads to the second-order partial differential equation:

$$\frac{\partial^2 E_{xs}}{\partial x^2} + \frac{\partial^2 E_{xs}}{\partial y^2} + \frac{\partial^2 E_{xs}}{\partial z^2} = -k_0^2 E_{xs} \qquad : \vec{E} = E_x(z) e^{\pm \gamma z} \vec{a}_x$$

If E_{xs} does not vary with *x* or *y*,

$$\frac{d^2 E_{xs}}{dz^2} = -k_0^2 E_{xs} \qquad (30) \text{ (편미분 → 상미분)}$$

$$\Rightarrow \quad E_{xs} = E_{x0} e^{-jk_0 z} + E'_{x0} e^{jk_0 z} \qquad (31)$$

where E_{x0} and E_{x0}' : real

• Determine the form of the $\vec{H}-$field

$$\nabla\times\vec{E}_s=-j\omega\mu_0\vec{H}_s \qquad (24)$$

Simplify for a simple E_{xs} component varying only with z

$$\frac{\partial E_{xs}}{\partial z}=-j\omega\mu_0 H_{ys} \quad \leftarrow (31)$$

$$\nabla\times\vec{E}=\left(\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}\right)\vec{a}_x+\left(\frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}\right)\vec{a}_y+\left(\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}\right)\vec{a}_z=\frac{\partial E_x}{\partial z}\vec{a}_y$$

$$\vec{E}=E_x(z)\vec{a}_x$$

$$\nabla\times\vec{E}=\frac{\partial E_x}{\partial z}\vec{a}_y$$

$$H_{ys}=\frac{1}{-j\omega\mu_0}[(-jk_0)E_{x0}e^{-jk_0z}+(jk_0)E'_{x0}e^{jk_0z}] \leftarrow k_0=\omega\sqrt{\mu_0\varepsilon_0}$$

$$=E_{x0}\sqrt{\frac{\varepsilon_0}{\mu_0}}e^{-jk_0z}-E'_{x0}\sqrt{\frac{\varepsilon_0}{\mu_0}}e^{jk_0z}=H_{y0}e^{-jk_0z}+H'_{y0}e^{jk_0z} \quad (32)$$

Instantaneous form of magnetic field:

$$H_y(z,t)=E_{x0}\sqrt{\frac{\varepsilon_0}{\mu_0}}\cos(\omega t-k_0z)-E'_{x0}\sqrt{\frac{\varepsilon_0}{\mu_0}}\cos(\omega t+k_0z) \qquad (33)$$

• By comparing the electric and magnetic field amplitudes of the forward-propagating wave in free space,

$$E_{x0}=\sqrt{\frac{\mu_0}{\varepsilon_0}}H_{y0}=\eta_0H_{y0} \qquad (34a)$$

Backward propagating wave:

$$E'_{x0}=-\sqrt{\frac{\mu_0}{\varepsilon_0}}H'_{y0}=-\eta_0H'_{y0} \qquad (34b)$$

Intrinsic impedance of free space (η[eta]):

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377 \cong 120\pi \quad [\Omega] \qquad (35)$$

$$= E_{xs} / H_{ys}$$

$\Rightarrow$ in-phase in space as well as time

$\Rightarrow$ constant

$\Rightarrow$ analogy to the characteristic impedance, Z_0

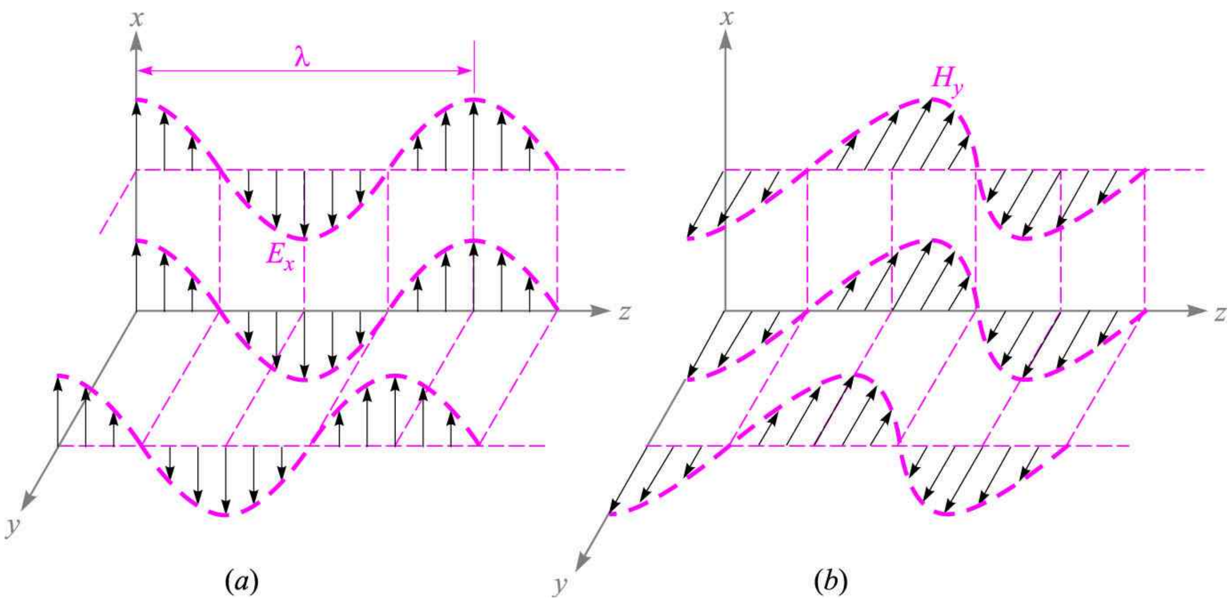
• Poynting vector:

$$\vec{S} = \vec{E} \times \vec{H}$$

• Uniform plane wave: The magnetic field intensity $\vec{H}$ lies in a plane normal to propagation direction $(\vec{z})$, and is perpendicular to $\vec{E}$

$\Rightarrow$ TEM-wave (Transverse Electromagnetic wave)

$\Rightarrow \quad (\vec{z} \perp \vec{E}, \vec{z} \perp \vec{H}, \vec{E} \perp \vec{H})$



11.2 Wave Propagation in Dielectrics

- Propagation of uniform plane wave in a perfect (lossless) dielectric of permittivity ε and permeability μ which medium is isotropic and homogeneous

- Helmholtz equation:

$$\nabla^2 \vec{E}_s = -k^2 \vec{E}_s \qquad (36)$$

$$= -\omega^2 \mu\varepsilon \vec{E}_s = -\omega^2 \mu_r \mu_0 \varepsilon_r \varepsilon_0 \vec{E}_s = -\omega^2 \mu_0 \varepsilon_0 \mu_r \varepsilon_r \vec{E}_s$$

$$= -k_0^2 \mu_r \varepsilon_r \vec{E}_s \quad \leftarrow k_0^2 = \omega^2 \mu_0 \varepsilon_0$$

$$k = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r} = k_0\sqrt{\mu_r\varepsilon_r} \qquad (37)$$

- For E_{xs},

$$\frac{d^2 E_{xs}}{dz^2} = -k^2 E_{xs} \qquad (38)$$

- General solution:

$$E_{xs} = E_{x0} e^{-jkz} = E_{x0} e^{-\alpha z} e^{-j\beta z} \qquad (40)$$

z 방향으로 $\vec{E}$ – field가 전달되면서 감쇄 성분도 함께 표현.

$$\xrightarrow{\text{instantaneous}} \quad E_x = E_{x0} e^{-\alpha z} \cos(\omega t - \beta z) \qquad (41)$$

lossy propagation model

where $\begin{cases} \alpha \ : \ \text{attenuation constant.} \\ \beta \ : \ \text{phase constant} \end{cases}$ $\qquad \alpha = 0$ for lossless.

➔ $jk = \alpha + j\beta$

$e^{-\alpha z}$:

where α : positive $\rightarrow$ attenuation

negative $\rightarrow$ gain

- Physical processes in material can affect the wave electric field and be described complex permittivity as.

$$\varepsilon = \varepsilon' - j\varepsilon'' = \varepsilon_0(\varepsilon_r' - j\varepsilon_r'') \qquad (42)$$

- Losses arising from the response of the medium (e.g. ferromagnetic material or ferrite) to the magnetic field are modeled as

$$\mu = \mu' - j\mu'' = \mu_0(\mu_r' - j\mu_r'')$$

Ex.] Ferrimagnetic material (or Ferrites)

➔ Very weaker than that of dielectric material $\mu \approx \mu_0$.

- Eq. (37) ← Eq. (42)

$$k = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu(\varepsilon' - j\varepsilon'')} = \omega\sqrt{\mu\varepsilon'}\sqrt{1 - j\frac{\varepsilon''}{\varepsilon'}} \qquad (43)$$

$$\alpha = \mathrm{Re}\{jk\} = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\left(\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right)^{1/2} \qquad (44)$$

$$\beta = \mathrm{Im}\{jk\} = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\left(\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} + 1\right)^{1/2} \qquad (45)$$

where $\varepsilon''/\varepsilon'$: loss tangent (if $\varepsilon''/\varepsilon' = 0$, then $\alpha = 0$)

• From electric field intensity

$$E_x = E_{x0} e^{-\alpha z} \cos(\omega t - \beta z)$$

$$= E_{x0} e^{-\alpha z} \cos\left\{\omega\left(t - \frac{\beta}{\omega} z\right)\right\} = E_{x0} e^{-\alpha z} \cos\left\{\omega\left(t - \frac{z}{(\omega/\beta)}\right)\right\}$$

$$= E_{x0} e^{-\alpha z} \cos\left\{\omega\left(t - \frac{z}{v_p}\right)\right\}$$

time dimension

$$\therefore \quad v_p = \frac{\omega}{\beta} \qquad (46) \qquad : \text{phase velocity}$$

• Wavelength: distance required to effect a phase change of 2π

$$\beta\lambda = 2\pi$$

$$\lambda = \frac{2\pi}{\beta} \qquad (47)$$

• By uniform plane wave property,

$$H_{ys} = \frac{E_{x0}}{\eta} e^{-\alpha z} e^{-j\beta z}$$

where $\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon' - j\varepsilon''}} = \sqrt{\frac{\mu}{\varepsilon'}} \frac{1}{\sqrt{1 - j(\varepsilon''/\varepsilon')}}$ (48)

: intrinsic impedance in dielectric

➔ Electric and magnetic fields are no longer in phase.

• Lossless medium (or perfect dielectric): $\varepsilon'' = 0$ and $\varepsilon = \varepsilon'$

➔ From Eq. (44), $\alpha = 0$

From Eq. (45), $\beta = \omega\sqrt{\mu\varepsilon'}$ (lossless medium) (49)

• With $\alpha = 0$,

$$E_x = E_{xo}\cos(\omega t - \beta z)$$

• Phase velocity and wavelength in medium except free space:

speed of light

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}} < c$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu\varepsilon'}} = \frac{1}{f\sqrt{\mu\varepsilon'}} = \frac{c}{f\sqrt{\mu_r\varepsilon_r}} = \frac{\lambda_0}{\sqrt{\mu_r\varepsilon_r}} < \lambda_0$$

• Magnetic field associated with E_x:

$$H_y = \frac{E_{x0}}{\eta}\cos(\omega t - \beta z)$$

Where $\eta = \sqrt{\frac{\mu}{\varepsilon}}$ (52)

[Ex. 11.3] 1 MHz plane wave propagating in fresh water.

Neglect losses in fresh water (ε" = 0)

$\mu_r = 1$, $\varepsilon_r = \varepsilon_r' = 81$

$$\Rightarrow \beta = \omega\sqrt{\mu\varepsilon'} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_r} = \frac{\omega\sqrt{\varepsilon_r}}{c} = \frac{2\pi \times 10^6 \times \sqrt{81}}{3 \times 10^8} = 0.19 \quad \text{[rad/m]}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.19} = 33\,\text{[m]} \quad < \quad 300\,\text{[m]} \; = \lambda_0$$

$$v_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^6}{0.19} = 3.3 \times 10^7 \text{ [m/sec]} \;\; < \;\; 3 \times 10^8 \text{ [m/sec]} = v_{p0}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon'}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \frac{1}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{81}} = 42 \; [\Omega]$$

If $E_{x0} = 0.1 \quad \text{[V/m]}$,

$$E_x = 0.1\cos\left(2\pi 10^6 t - 0.19z\right) \;\; \text{[V/m]}$$

$$H_y = \frac{E_x}{\eta} = \frac{0.1}{42}\cos\left(2\pi \times 10^6 t - 0.19z\right) = 2.4 \times 10^{-3}\cos\left(2\pi \times 10^6 t - 0.19z\right) \text{ [A/m]}$$

[Ex. 11.4] In previous example, f = 2.5 GHz, ε_r' = 78, ε_r" = 7

(Dielectric loss can't be ignored.)

$$\Rightarrow \alpha = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\left(\sqrt{1 + \left(\frac{\varepsilon_r''}{\varepsilon_r'}\right)^2} - 1\right)^{1/2} = \omega\frac{\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_r'}}{\sqrt{2}}\left(\sqrt{1 + \left(\frac{\varepsilon_r''}{\varepsilon_r'}\right)^2} - 1\right)^{1/2}$$

$$= \frac{\omega\sqrt{\varepsilon_r'}}{\sqrt{2}c}\left(\sqrt{1 + \left(\frac{\varepsilon_r''}{\varepsilon_r'}\right)^2} - 1\right)^{1/2}$$

$$= \frac{2\pi \times 2.5\times 10^9 \times \sqrt{78}}{\sqrt{2}\times 3\times 10^8}\left(\sqrt{1+\left(\frac{7}{78}\right)^2}-1\right)^{1/2} = 21 \quad \text{[Np/m]}$$

- Frequency of microwave oven: 2.5 GHz.

$e^{-\alpha z} = e^{-1}$ $\qquad z = \frac{1}{\alpha} = \frac{1}{21} = 0.048$ m : penetration depth.

$=0.3679$

- If $f > 2.5\,\text{GHz}$, $\varepsilon'' \uparrow$. But $z = \frac{1}{\alpha}\downarrow\downarrow$ $\Rightarrow$ Too much power is needed.

$f < 2.5$ GHz, $\quad z = \frac{1}{\alpha}\uparrow \qquad \varepsilon''\downarrow\downarrow$.

$$\beta = \frac{2\pi \times 2.5\times 10^9 \times \sqrt{78}}{\sqrt{2}\times 3\times 10^8}\left(\sqrt{1+\left(\frac{7}{78}\right)^2}+1\right)^{1/2} = 464 \quad \text{rad/m}$$

$$\lambda = \frac{2\pi}{\beta} = 0.014 \quad < \quad 0.12 = \lambda_0$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\cdot\frac{1}{\sqrt{\varepsilon_r' - j\varepsilon_r''}} = \frac{377}{\sqrt{78}\sqrt{1-j\left(\frac{7}{78}\right)}}$$

$$= 43 + j1.9 = 43.04\angle 2.6^\circ\,[\Omega]$$

• Real conductor does not have infinite conductivity. ($\sigma \neq \infty$)

$\Rightarrow$ The wave loses power through resistive heating of the material.

These phenomena can be explained with the complex permittivity.

$$\nabla \times \vec{H}_s = j\omega(\varepsilon' - j\varepsilon'')\vec{E}_s = \omega\varepsilon''\vec{E}_s + j\omega\varepsilon'\vec{E}_s \quad (53)$$

$$\nabla \times \vec{H}_s = \vec{J}_s + j\omega\varepsilon\vec{E}_s = \sigma\vec{E}_s + j\omega\varepsilon\vec{E}_s \qquad \leftarrow \quad \varepsilon' \gg \varepsilon''$$

conduction current

$$= (\sigma + j\omega\varepsilon')\vec{E}_s = \vec{J}_{\sigma s} + \vec{J}_{ds} \quad (55)$$

displacement current

displacement current와
비교되는 측면에서 "σ"를 붙임.

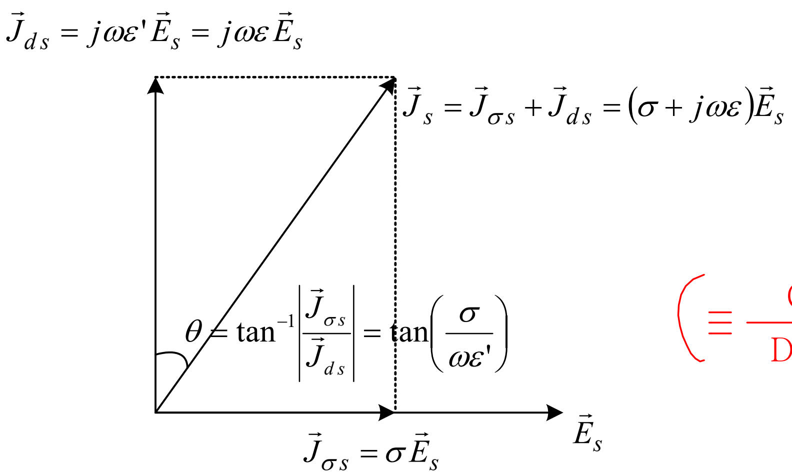
$$\vec{J}_{\sigma s} = \omega\varepsilon''\vec{E}_s = \sigma\vec{E}_s$$

$$\therefore \quad \varepsilon'' = \frac{\sigma}{\omega} \qquad (56)$$

• Loss tangent (ε”/ε’) is a barometer to judge whether the loss is small or not.

$$\because \quad \alpha = \mathrm{Re}\{jk\} = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\left(\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right)^{1/2} \qquad (44)$$

$$\frac{\vec{J}_{\sigma s}}{\vec{J}_{ds}} = \frac{\omega\varepsilon''\vec{E}_s}{j\omega\varepsilon'\vec{E}_s} = \frac{\varepsilon''}{j\varepsilon'} = \frac{\sigma/\omega}{j\varepsilon'} = \frac{\sigma}{j\omega\varepsilon'} \qquad (57)$$



$$\therefore \quad \tan\theta = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon'}$$

$$\left(\equiv \frac{\text{Conduction current density}}{\text{Displacement current density}} \right)$$

• If the loss tangent is small,

$$jk = j\omega\sqrt{\mu\varepsilon'}\sqrt{1 - j\frac{\varepsilon''}{\varepsilon'}}$$

$$= j\omega\sqrt{\mu\varepsilon'}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon'}} \qquad (59) \qquad \leftarrow (1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots\cdots$$

where $|x| << 1$

$$= j\omega\sqrt{\mu\varepsilon'}\left[1 - j\frac{\sigma}{2\omega\varepsilon'} + \frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon'}\right)^2 - \cdots\cdots\right]$$

$$= \alpha + j\beta$$

$$\therefore \quad \alpha = \mathrm{Re}(jk) = j\omega\sqrt{\mu\varepsilon'}\left(-j\frac{\sigma}{2\omega\varepsilon'}\right) = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon'}} \qquad (60a) \leftarrow (43) \sim (45)$$

$$\beta = \mathrm{Im}(jk) = \omega\sqrt{\mu\varepsilon'}\left[1 + \frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon'}\right)^2\right] \qquad (60b) \leftarrow \quad |\sigma| << |\omega\varepsilon'|$$

$$\approx \omega\sqrt{\mu\varepsilon'} \qquad (61)$$

$$\leftarrow\ (1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots\cdots$$

where $|x| << 1$

$$\eta = \sqrt{\frac{\mu}{\varepsilon'}}\frac{1}{\sqrt{1-j\left(\frac{\varepsilon''}{\varepsilon'}\right)}} \qquad \leftarrow \qquad x = -j\frac{\sigma}{\omega\varepsilon'} \qquad n = -\frac{1}{2}$$

$$= \sqrt{\frac{\mu}{\varepsilon'}}\left[1 + j\frac{\sigma}{2\omega\varepsilon'} + \frac{(-1/2)(-3/2)}{2}\left(-j\frac{\sigma}{\omega\varepsilon'}\right)^2 + \cdots\cdot\right]$$

$$\approx \sqrt{\frac{\mu}{\varepsilon'}}\left[1 - \frac{3}{8}\left(\frac{\sigma}{\omega\varepsilon'}\right)^2 + j\frac{\sigma}{2\omega\varepsilon'}\right] \qquad (62a)$$

$$\approx \sqrt{\frac{\mu}{\varepsilon'}}\left(1 + j\frac{\sigma}{2\omega\varepsilon'}\right) \qquad (62b)$$

[Ex. 11.5] Repeat the computation of [Ex. 11.4].

$$\frac{\varepsilon''}{\varepsilon'} = \frac{7}{78} = 0.09 \qquad \varepsilon'' = \frac{\sigma}{\omega} \qquad \sigma = \varepsilon''\omega$$

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon'}} = \frac{\varepsilon''\omega}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{1}{\sqrt{78}}$$

$$= \frac{7\times 8.854\times 10^{-12}\times 2\pi\times 2.5\times 10^9}{2}\frac{377}{\sqrt{78}} = 20.78\ (\approx 21) \quad [\text{Np/m}]$$

$$\beta = \omega\sqrt{\mu\varepsilon'} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{78} = \frac{2\pi\times 2.5\times 10^9\times\sqrt{78}}{3\times 10^8} = 462.4\ (\approx 464)\ [\text{rad/m}]$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon'}}\left(1 + j\frac{\sigma}{2\omega\varepsilon'}\right) = \sqrt{\frac{\mu_0}{\varepsilon_0}}\cdot\frac{1}{\sqrt{78}}\left(1 + j\frac{\varepsilon''}{2\varepsilon'}\right)$$

$$= \frac{377}{\sqrt{78}}\left(1 + j\frac{0.09}{2}\right) = 42.7 + j1.92\ (\approx 43 + j1.9)\ \Omega \quad \Rightarrow \text{very similar results.}$$

11.3 Poynting's Theorem and Wave Power

• Consider Maxwell's equation

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (63)$$

$$\vec{E} \cdot \nabla \times \vec{H} = \vec{J} \cdot \vec{E} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t} \quad (64)$$

Using vector identity,

$$\nabla \cdot \left(\vec{E} \times \vec{H}\right) = -\vec{E} \cdot \nabla \times \vec{H} + \vec{H} \cdot \nabla \times \vec{E} \quad (65)$$

$$\vec{H} \cdot \nabla \times \vec{E} - \nabla \cdot \left(\vec{E} \times \vec{H}\right) = \vec{E} \cdot \nabla \times \vec{H} = \vec{J} \cdot \vec{E} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t} \quad (66)$$

$$-H \cdot \frac{\partial \vec{B}}{\partial t} - \nabla \cdot \left(\vec{E} \times \vec{H}\right) = \vec{J} \cdot \vec{E} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t}$$

$$-\nabla \cdot \left(\vec{E} \times \vec{H}\right) = \vec{J} \cdot \vec{E} + \varepsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} + \mu \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} \quad (67)$$

Since $\varepsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{\varepsilon}{2} \frac{\partial E^2}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\varepsilon E^2}{2}\right) = \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{D} \cdot \vec{E}\right) \leftarrow \frac{\partial}{\partial t} F(x) = F'(x) \cdot \frac{\partial x}{\partial t}$

and $\mu \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} = \frac{\mu}{2} \frac{\partial H^2}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\mu H^2}{2}\right) = \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{B} \cdot \vec{H}\right)$,

$$-\nabla \cdot \left(\vec{E} \times \vec{H}\right) = \vec{J} \cdot \vec{E} + \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{D} \cdot \vec{E}\right) + \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{B} \cdot \vec{H}\right) \quad (69)$$

Integrate throughout a volume,

$$-\int_{vol} \nabla \cdot (\vec{E} \times \vec{H}) dv = \int_{vol} \vec{J} \cdot \vec{E} dv + \int_{vol} \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{D} \cdot \vec{E}\right) dv + \int_{vol} \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{B} \cdot \vec{H}\right) dv$$

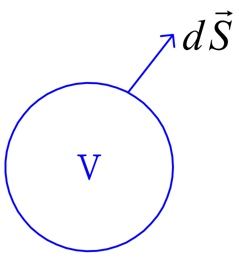
Apply divergence theorem

$$-\oint_S (\vec{E} \times \vec{H}) \cdot d\vec{S} = \int_{vol} \vec{J} \cdot \vec{E} dv + \frac{d}{dt}\int_{vol} \frac{1}{2}\vec{D} \cdot \vec{E} dv + \frac{d}{dt}\int_{vol} \frac{1}{2}\vec{B} \cdot \vec{H} dv \quad (70)$$

(-)부호이므로 구(체적) 안으로 들어오는 total transmitting power

volume 안에서 소모되는 (ohmic) power

체적 안에서 electric 또는 megnetic field 형태로 저장되는 total energy. (in Ch. 8)



$$\vec{J} \cdot \vec{E} = \frac{|\vec{J}|^2}{\sigma} = R|\vec{J}|^2$$

: Poynting Theorem

• Poynting vector:

$\vec{S} = \vec{E} \times \vec{H}$ [W/m^2] (72): Instantaneous power density, measured in watts per square meter [W/m^2]

$\Rightarrow$ $(\vec{S} \perp \vec{E})$ and $(\vec{S} \perp \vec{H})$ and $(\vec{E} \perp \vec{H})$

• In the uniform plane wave,

$$E_x\vec{a}_x \times H_y\vec{a}_y = S_z\vec{a}_z$$

In a perfect dielectric

$$E_x = E_{x0}\cos(\omega t - \beta z)$$

$$H_y = \frac{E_{x0}}{\eta}\cos(\omega t - \beta z)$$

$$\Rightarrow \quad S_z = \frac{E_{x0}^2}{\eta}\cos^2(\omega t - \beta z) \qquad (73)$$

To find the time-average power density,

$$< S_z >= \frac{1}{T}\int_0^T \frac{E_{x0}^2}{\eta}\cos^2(\omega t - \beta z)\,dt$$

$$= \frac{1}{2T}\frac{E_{x0}^2}{\eta}\int_0^T \left[1 + \cos(2\omega t - 2\beta z)\right]dt$$

$$= \frac{1}{2T}\frac{E_{x0}^2}{\eta}\left[t + \frac{1}{2\omega}\sin(2\omega t - 2\beta z)\right]_0^T$$

$$= \frac{E_{x0}^2}{2\eta}$$

$$\therefore \quad <S_z>= \frac{E_{x0}^2}{2\eta} = \frac{\left(E_{x0}/\sqrt{2}\right)^2}{\eta} = \frac{E_{x0.rms}^2}{\eta}$$

E_{x0} : peak amplitude

$E_{x0.rms}$: rms amplitude

Average power flowing through any area S normal to the propagation direction:

$$S_{z.av} = \frac{1}{2}\frac{E_{x0}^2}{\eta}S \quad [\mathrm{W}]$$

• In the lossy dielectric, E_x and H_y are not in-phased.

$$E_x = E_{xo} e^{-\alpha z} \cos(\omega t - \beta z)$$

Let $\eta = |\eta| \angle \theta_\eta$

$$\Rightarrow \quad H_y = \frac{E_{x0}}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)$$

$$S_z = E_x H_y = \frac{E_{x0}^2}{|\eta|} e^{-2\alpha z} \cos(\omega t - \beta z) \cos(\omega t - \beta z - \theta_\eta) \qquad (74)$$

$$= \frac{E_{x0}^2}{2|\eta|} e^{-2\alpha z} \left[\cos(2\omega t - 2\beta z - \theta_\eta) + \cos\theta_\eta \right]$$

$$\leftarrow \cos A \cos B = [\cos(A+B) + \cos(A-B)]/2$$

Time-average power density:

$$< S_z >= \frac{1}{T} \int_0^T \frac{E_{x0}^2}{2|\eta|} e^{-2\alpha z} [\cos(2\omega t - 2\beta z - 2\theta_\eta) + \cos\theta_\eta] dt \qquad (75)$$

$$= \frac{E_{x0}^2}{2|\eta|} e^{-2\alpha z} \cos\theta_\eta \qquad (76)$$

$$\therefore \quad < S_z >= \frac{1}{2} \mathrm{Re}\left[\vec{E}_s \times \vec{H}_s^* \right] \quad \mathrm{W/m^2} \qquad (77)$$

phsor form

E_x, H_y 는 $e^{-\alpha z}$ 로 감쇠하나
$P_{z.av}$ 는 $e^{-2\alpha z}$ 로 감쇠. 더 빠른 감쇠

$$\Rightarrow < \vec{S} >= \frac{1}{2} \mathrm{Re}[E_{x0} e^{-\alpha z} e^{-j\beta z} \vec{a}_x \times \frac{E_{x0}}{|\eta|} e^{-\alpha z} e^{j(\beta z + \theta_\eta)} \vec{a}_y] = \frac{1}{2} \mathrm{Re}[\frac{E_{x0}^2}{|\eta|} e^{-2\alpha z} e^{j\theta_\eta} \vec{a}_z]$$

$$= \frac{1}{2} \frac{E_{x0}^2}{|\eta|} e^{-2\alpha z} \cos\theta_\eta \vec{a}_z$$

11.4 Propagation in Good Conductors: Skin Effect

- Investigate the behavior of a good conductor

  For example, consider nichrome (Ni) $\sigma \cong 10^6$, $f = 100$ MHz, $\varepsilon'_r \cong 90$

$$\frac{\sigma}{\omega\varepsilon'} = \frac{10^6}{2\pi \times 100 \times 10^6 \times 8.854 \times 10^{-12} \times 90} = 199.7 \times 10^4 \;>>\; 1$$

$$\Rightarrow \vec{J}_{\sigma S} >> \vec{J}_{dS}$$

- Propagation constant:

$$jk = j\omega\sqrt{\mu\varepsilon'}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon'}} \approx j\omega\sqrt{\mu\varepsilon}\sqrt{-j\frac{\sigma}{\omega\varepsilon}} \quad \leftarrow (59)$$

$$= j\sqrt{-j\omega\mu\sigma} = j\sqrt{-2j}\sqrt{\pi f \mu\sigma}$$

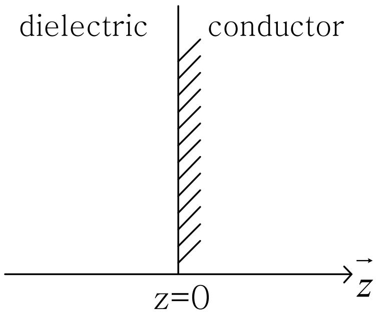
$$= 1\angle 90^\circ \times \sqrt{2}\angle -45^\circ \sqrt{\pi f \mu\sigma} = \sqrt{2}\angle 45^\circ \sqrt{\pi f \mu\sigma}$$

$$= (1 + j)\sqrt{\pi f \mu\sigma} = \alpha + j\beta \qquad (78)$$

$$\therefore \qquad \alpha = \beta = \sqrt{\pi f \mu\sigma} \qquad (79)$$

- E_x-component traveling in the $(+z)$-direction,

$$E_x = E_{x0} e^{-\sqrt{\pi f \mu\sigma}\, z} \cos\left(\omega t - \sqrt{\pi f \mu\sigma}\, z\right) \qquad (80)$$

- 

At $z = 0$, $E_x = E_{x0}\cos\omega t$.
Since the displacement current is negligible in the conductor,

$$\vec{J} = \sigma\vec{E}$$

$$J_x = \sigma E_x = \sigma E_{x0} e^{-\sqrt{\pi f \mu\sigma}\, z} \cos\left(\omega t - \sqrt{\pi f \mu\sigma}\, z\right)$$

$\Rightarrow$ Conduction current decreases exponentially in the conductor.

• Skin depth (or the depth of penetration):

$$z = \delta = \frac{1}{\sqrt{\pi f \mu\sigma}} = \frac{1}{\alpha} = \frac{1}{\beta} \quad (82) \quad \left(\begin{array}{l} J_x = \sigma E_{x0} e^{-\sqrt{\pi f \mu\sigma}\, z} \cos\left(\omega t - \sqrt{\pi f \mu\sigma}\, z\right) \text{에서} \\ e^{-\sqrt{\pi f \mu\sigma}\, z} = e^{-1} \qquad \therefore\ z = \frac{1}{\sqrt{\pi f \mu\sigma}} = \frac{1}{\alpha} \end{array} \right)$$

[Ex.] Copper $\sigma = 5.8\times10^7$ [S/m]

$$\delta_{\mathrm{Cu}} = \frac{1}{\sqrt{\pi\mu\sigma}\sqrt{f}} = \frac{1}{\sqrt{\pi\times4\pi\times10^{-7}\times5.8\times10^{7}}\sqrt{f}} = \frac{0.0661}{\sqrt{f}}$$

At $f = 60\text{[Hz]}$, $\delta_{\mathrm{Cu}} = 8.532\ \text{[mm]} = \frac{1}{3}\ \text{[inch]}$ (→ 대전력선은 2~4” 정도로 하는 것이 좋음)

$$f = 10\text{[GHz]},\quad \delta_{\mathrm{Cu}} = \frac{0.0661}{\sqrt{10^{10}}} = 6.61\times10^{-7}\ \text{[m]}$$

$\Rightarrow$ All field intensities in a good conductor are zero at distance greater than a few skin depths from the surfaces.

$\Rightarrow$ Electromagnetic energy is not transmitted in interior of a conductor and travels surrounding conductor.

• Velocity and wavelength within a good conductor:

$$\alpha = \beta = \frac{1}{\delta} = \sqrt{\pi f \mu\sigma} = \frac{2\pi}{\lambda}$$

$$\therefore\quad \lambda = 2\pi\delta \qquad (83)$$

$$v = \frac{\omega}{\beta} = \omega\delta \qquad (84)$$

$$\leftarrow \beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{v_p},\quad v_p = \frac{\omega}{\beta} = \omega\delta$$

[Ex.] For copper at 60[Hz]

$$\lambda = 2\pi\delta = 2\pi\sqrt{\frac{1}{\pi\mu\sigma f}} = 2\pi \times 8.532 \times 10^{-3} = 0.0536\,[\mathrm{m}] \quad \boxed{< 0.5 \times 10^{7}\,[\mathrm{m}] = \lambda_0}$$

$$v = \omega\delta = 2\pi \times 60 \times 8.532 \times 10^{-3} = 3.216\,[\mathrm{m/s}] \quad \boxed{< 3 \times 10^{8}\,[\mathrm{m/s}] = v_0}$$

[Ex. 11.6] Sea water $f = 1$ MHz, $\sigma = 4$ S/m, $\varepsilon_r' = 81$

→ $H_2O + NaCl \Leftrightarrow H_2O + Na^{+} + Cl^{-}$

→ charged ions

$$\left(\frac{\varepsilon''}{\varepsilon'}\right) = \frac{\sigma}{\omega\varepsilon'} = \frac{4}{2\pi \times 10^{6} \times 81 \times 8.854 \times 10^{-12}} = 887.7 \quad >> 1$$

→ Good conductor

$$\delta = \frac{1}{\sqrt{\pi f \mu\sigma}} = \frac{1}{\sqrt{\pi \times 10^{6} \times 4\pi \times 10^{-7} \times 4}} = 0.25\ \mathrm{m}$$

$$\lambda = 2\pi\delta = 1.58\ \mathrm{m} \quad << 300\ \mathrm{m} = \lambda_0$$

$$v_p = \omega\delta = 2\pi f\delta = 1.58 \times 10^{6}\ \mathrm{m/s} \ << 3 \times 10^{8}\ \mathrm{m/s} = v_0$$

→RF communication in seawater is quite impractical.

$\left(\because \delta \propto \frac{1}{\sqrt{f}}\right)$

→ $f = 10$ Hz, $\delta = 79.6$ m, $\lambda = 500$ m $<<< \lambda_0 = 3 \times 10^{7}$ m, $v_p = 5000$ m/s

- A suspended wire antenna of length shorter than 500m is sufficient to receive signal.
- Drawback: slow signal data rate. ($\because v_p$=5 km/s)

• To find H_y associated with E_x in good conductor,

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon'(1 - j\frac{\varepsilon''}{\varepsilon'})}} = \sqrt{\frac{\mu}{\varepsilon'(1 - j\frac{\sigma}{\omega\varepsilon'})}} = \sqrt{\frac{\mu}{\varepsilon' - j\frac{\sigma}{\omega}}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon'}}$$

$$\cong \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{j2\pi f\mu\sigma}{\sigma^2}} = \frac{\sqrt{j2}}{\sigma\delta} = \frac{\sqrt{2}\angle 45^o}{\sigma\delta} = \frac{1}{\sigma\delta} + j\frac{1}{\sigma\delta} \qquad (85)$$

(intrinsic impedance)

$$E_x = E_{x0} e^{-\sqrt{\pi f \mu\sigma}\, z} \cos\left(\omega t - \sqrt{\pi f \mu\sigma}\, z\right)$$

$$= E_{x0} e^{-\frac{z}{\delta}} \cos\left(\omega t - \frac{z}{\delta}\right) \qquad (86)$$

$$H_{ys} = \frac{E_{xs}}{\eta} = \frac{E_{x0} e^{-\frac{z}{\delta}} e^{j(-z/\delta)}}{\frac{\sqrt{2}}{\sigma\delta} e^{j\pi/4}}$$

$$= \frac{\sigma\delta}{\sqrt{2}} E_{x0} e^{-\frac{z}{\delta}} e^{j(-z/\delta - \pi/4)}$$

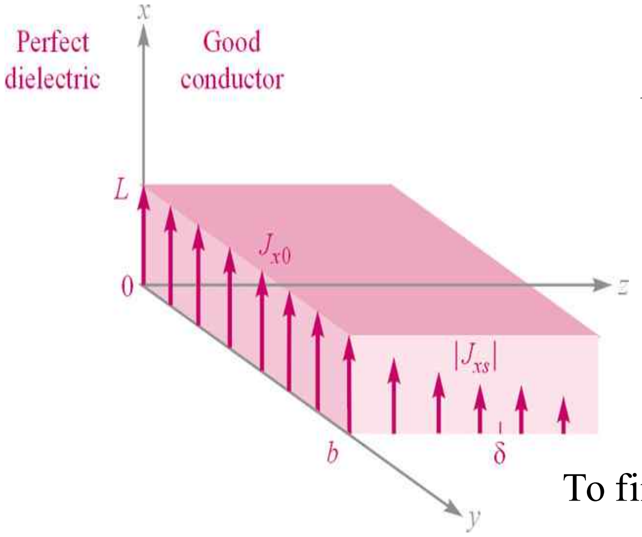
$$H_y = \frac{E_x}{\eta} = \frac{\sigma\delta}{\sqrt{2}} E_{xo} e^{-\frac{z}{\delta}} \cos\left(\omega t - \frac{z}{\delta} - \frac{\pi}{4}\right) \qquad (87)$$

$$< S_z > = \frac{1}{T}\int_0^T \frac{\sigma\delta}{\sqrt{2}} E_{x0}^2 e^{-\frac{2z}{\delta}} \cos\left(\omega t - \frac{z}{\delta}\right)\cos\left(\omega t - \frac{z}{\delta} - \frac{\pi}{4}\right) dt$$

$$= \frac{\sigma\delta}{2\sqrt{2}} E_{x0}^2 e^{-\frac{2z}{\delta}} \cos\frac{\pi}{4}$$

$$= \frac{1}{4}\sigma\delta E_{x0}^2 e^{-\frac{2z}{\delta}}$$

• Total average power loss in $0 < y < b$, $0 < x < L$:



$$P_{L,av} = \int_S < S_z > dS = \int_0^b \int_0^L \frac{1}{4}\sigma\delta E_{x0}^2 e^{-2z/\delta}\Big|_{z=0} dxdy$$

$$= \frac{1}{4}\sigma\delta bL E_{x0}^2$$

$$= \frac{1}{4\sigma}\delta bL J_{x0}^2$$

$$(88)\ J_x = \sigma E_x \quad (\because \text{ good conductor})$$

$$= \sigma E_{x0} e^{-z/\delta}\cos\left(\omega t - \frac{z}{\delta}\right)$$

$$= J_{x0} e^{-z/\delta}\cos\left(\omega t - \frac{z}{\delta}\right)$$

To find total current

$$I = \int_0^\infty \int_0^b J_x dydz$$

where $J_x = J_{x0} e^{-z/\delta}\cos\left(\omega t - \frac{z}{\delta}\right)$

$$J_{xs} = J_{x0} e^{-z/\delta} e^{-jz/\delta} = J_{x0} e^{-(1+j)z/\delta}$$

$$\therefore \quad I_s = \int_0^\infty \int_0^b J_{xs} dydz = \int_0^\infty \int_0^b J_{x0} e^{-(1+j)z/\delta} dydz$$

$$= J_{x0} b \frac{(-\delta)}{1+j} e^{-(1+j)z/\delta}\Big|_0^\infty = \frac{J_{x0} b\delta}{1+j} = \frac{J_{x0} b\delta}{\sqrt{2}} e^{-j\frac{\pi}{4}}$$

$$\therefore \quad I = \frac{J_{x0} b\delta}{\sqrt{2}}\cos\left(\omega t - \frac{\pi}{4}\right)$$

• If the above current is distributed with a uniformly throughout

$(0 < y < b, \ 0 < z < \delta)$

($\bar{z}$ 축 방향으로는 $\alpha = \frac{1}{\sqrt{\pi f \mu \sigma}}$ 에
의해 감쇠하지만, total 전류를 구해서
$0 < y < b, \ 0 < z < \delta$ 의 체적에
고르게 분포되었다고 가정)

$$I = J'S = J'b\delta = \frac{J_{x0} b\delta}{\sqrt{2}} \cos\left(\omega t - \frac{\pi}{4}\right)$$

$$J' = \frac{J_{x0}}{\sqrt{2}} \cos\left(\omega t - \frac{\pi}{4}\right)$$

Ohmic power loss per unit volume: $\vec{J} \cdot \vec{E}$ ←poynting vector 에서 ohmic loss로 이미 언급됨.

Total instantaneous power dissipated in the volume:

$$P_{Li}(t) = \left(J' \cdot E\right) bL\delta = J' \cdot \frac{J'}{\sigma} bL\delta = \frac{(J')^2}{\sigma} bL\delta$$

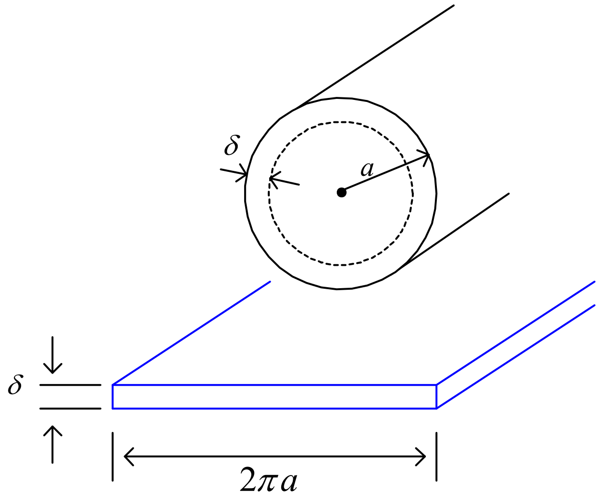
$$= \frac{J_{x0}^2}{2\sigma} bL\delta \cos^2\left(\omega t - \frac{\pi}{4}\right)$$

$$P_{L,av} = \frac{J_{x0}^2}{4\sigma} bL\delta \quad \text{(same result with Eq.(88))}$$

⇒ The average power loss in a conductor with skin effect may be calculated by assuming that the total current is distributed uniformly in one skin depth.

(저항적인 측면에서는 폭이 b이고 길이가 L인 무한두께의 slab이 skin effect를 가질때의 저항은 skin effect가 없이 폭이 b이고 길이가 L이면서 두께가 δ인 slab의 저항과 같다.)

[Ex.] Circular cross section copper wire.



$$R = \frac{L}{\sigma S} \qquad f = 1 \text{ [MHz]}$$

$$a = 1 \text{ [mm]},\ L = 1 \text{ [km]},\ \sigma = 5.8 \times 10^{7}$$

$$R_{dc} = \frac{10^{3}}{5.8 \times 10^{7} \times \pi \times 10^{-6}} = 5.488 \quad [\Omega]$$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi \times 10^{6} \times 4\pi \times 10^{-7} \times 5.8 \times 10^{7}}} = 66.09 \times 10^{-6}$$

$$\therefore\ R_{RF} = \frac{L}{\sigma 2\pi a \delta} = \frac{10^{3}}{5.8 \times 10^{7} \times 2\pi \times 10^{-3} \times 66.09 \times 10^{-6}} = 41.52 \ [\Omega]$$