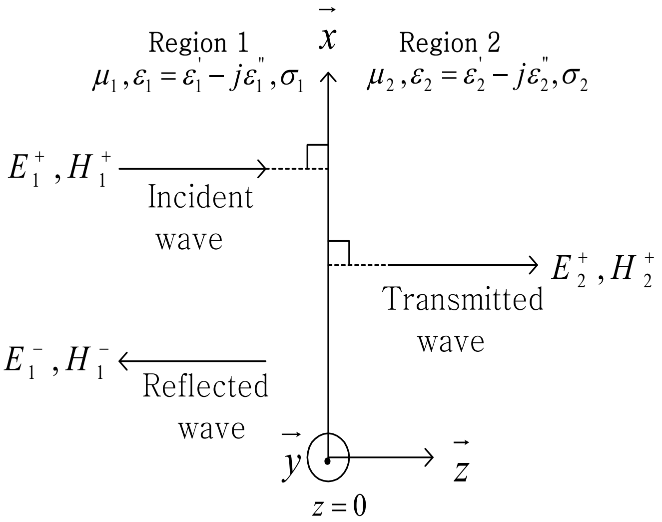
Chapter 12. Plane Wave Reflection and Dispersion

12.1 Reflection of Uniform Plane Wave at Normal Incidence

- A uniform plane wave is incident normally on the boundary between regions composed of two different materials.



- Wave traveling in the $+\vec{z}$ direction in region 1:

positive traveling direction

$$E_{x1}^+ = E_{x10}^+ e^{-\alpha_1 z} \cos(\omega t - \beta_1 z)$$

region

$$\xrightarrow{\text{phasor}} E_{xs1}^+ = E_{x10}^+ e^{-jk_1 z} \qquad (1)$$

where E_{x10}^+ : real, ($jk_1 = \alpha_1 + j\beta_1$)

$$\Rightarrow H_{ys1}^+ = \frac{1}{\eta_1} E_{x10}^+ e^{-jk_1 z} \qquad (2)$$

where $\eta_1 = \sqrt{\dfrac{j\omega\mu_1}{\sigma_1 + j\omega\varepsilon'_1}}$: intrinsic impedance

E_{x1}^+, H_{y1}^+: incident wave perpendicular to the boundary plane (경계면에 수직으로 입사되는 wave)

• Moving (or transmitted) wave in region 2 toward $(+\vec{z})$-direction:

$$\begin{cases} E_{xs2}^{+} = E_{x20}^{+} e^{-jk_2 z} & (3) \\ H_{ys2}^{+} = \dfrac{E_{x20}^{+}}{\eta_2} e^{-jk_2 z} & (4) \end{cases} \quad \leftarrow \eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\varepsilon_2^{'}}}$$

• (Special) Boundary condition at $z = 0$:

i) $\vec{E}$ -field in region 1, 2 must be equal.

Eq. (1) & (3) $\Rightarrow$ $E_{x10}^{+} = E_{x20}^{+}$ (tangential component) $z = 0$ 에서 고려하므로 ①, ③식의 진폭이 같게 됨.

ii) $\vec{H}$ -field in region 1, 2 must be equal.

Eq. (2) & (4) $\Rightarrow$ $\dfrac{E_{x10}^{+}}{\eta_1} = \dfrac{E_{x20}^{+}}{\eta_2}$ $\Rightarrow$ $\eta_1 = \eta_2$

$\Rightarrow$ This is very special condition $\Rightarrow$ Not in general.

• Let define reflected wave:

$$\begin{cases} E_{xs1}^{-} = E_{x10}^{-} e^{jk_1 z} & (5) \\ H_{ys1}^{-} = -\dfrac{E_{x10}^{-}}{\eta_1} e^{jk_1 z} & (6) \end{cases}$$

• Poynting Vector $\vec{E}_1^{-} \times \vec{H}_1^{-}$ 를 만족시키려면 $\vec{H}$ -field는 $-\vec{a}_y$ 이어야 함.

$$\vec{a}_x \times (-\vec{a}_y) = -\vec{a}_z$$

where E_{x10}^{-} : complex quantity (반사계수에 근거)

• Boundary condition:

i) Total electric field intensity at $z = 0$

$$E_{xs1} = E_{xs2}$$
$$\Rightarrow\ E_{xs1}^{+} + E_{xs1}^{-} = E_{xs2}^{+}$$
$$\Rightarrow\ E_{x10}^{+} + E_{x10}^{-} = E_{x20}^{+} \qquad (7)$$

ii) Total magnetic field intensity at $z = 0$

$$H_{ys1} = H_{ys2}$$
$$\Rightarrow\ H_{ys1}^{+} + H_{ys1}^{-} = H_{ys2}^{+}$$
$$\Rightarrow\ \frac{E_{x10}^{+}}{\eta_1} - \frac{E_{x10}^{-}}{\eta_1} = \frac{E_{x20}^{+}}{\eta_2} \qquad (8)$$

$$E_{x20}^{+} = E_{x10}^{+} + E_{x10}^{-} = \frac{\eta_2}{\eta_1} E_{x10}^{+} - \frac{\eta_2}{\eta_1} E_{x10}^{-}$$

$$\left(1 + \frac{\eta_2}{\eta_1}\right) E_{x10}^{-} = \frac{\eta_2 + \eta_1}{\eta_1} E_{x10}^{-} = \left(\frac{\eta_2}{\eta_1} - 1\right) E_{x10}^{+} = \frac{\eta_2 - \eta_1}{\eta_1} E_{x10}^{+}$$

$$\therefore\ E_{x10}^{-} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_{x10}^{+}$$

• Reflection coefficient (Γ): ratio of the amplitudes of the reflected and incident electric fields

$$\Gamma = \frac{E_{x10}^{-}}{E_{x10}^{+}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = |\Gamma| e^{j\phi} \qquad (9)$$

: complex (complex인 경우 위상지연 발생)
↓
phase shift

where $\eta_i = \sqrt{\dfrac{j\omega\mu_i}{j\omega\varepsilon_i' + \sigma_i}}$

$$E_{x20}^{+} = E_{x10}^{+} + E_{x10}^{-} = (1+\Gamma) E_{x10}^{+}$$

$$= \left(1 + \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) E_{x10}^{+} = \frac{2\eta_2}{\eta_2 + \eta_1} E_{x10}^{+}$$

$$= \tau E_{x10}^{+}$$

• Transmit coefficient (τ): ratio of the amplitudes of the transmitted and incident electric fields

$$\tau = 1 + \Gamma = \frac{E_{x20}^{+}}{E_{x10}^{+}} = \frac{2\eta_2}{\eta_2 + \eta_1} = |\tau| e^{j\phi_t} \qquad (10)$$

[Ex. 1] Region 1: Perfect dielectric

Region 2: Perfect conductor $\Rightarrow \sigma_2 = \infty$

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\varepsilon_2}} = 0$$

$$\tau = \frac{2\eta_2}{\eta_1 + \eta_2} = 0$$

$$\therefore \frac{E_{x20}^{+}}{E_{x10}^{+}} = 0 \qquad \Rightarrow \qquad E_{x20}^{+} = 0$$

region 2에서는 $\vec{E}$-field 가 존재하지 못함.
⇒ skin depth 의 영향으로 생각할 수도 있음.

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1$$

$$\therefore \quad E_{x10}^{-} = -E_{x10}^{+} \qquad \Rightarrow \quad \text{All wave is reflected}$$

$$E_{xs1} = E_{xs1}^{+} + E_{xs1}^{-}$$

$$= E_{x10}^{+} e^{-j\beta_1 z} - E_{x10}^{+} e^{j\beta_1 z}$$

perfect dielectric $jk_1 = \alpha_1 + j\beta_1 = j\beta_1$

$$= (e^{-j\beta_1 z} - e^{j\beta_1 z}) E_{x10}^{+} = -(j2) \frac{e^{j\beta_1 z} - e^{-j\beta_1 z}}{j2} E_{x10}^{+}$$

$$= -j2\sin\beta_1 z E_{x10}^+ \qquad (11)$$

$\xrightarrow[\text{form}]{\text{instantaneous}}$ $E_{x1}(z,t) = \text{Re}\left[(-j)2\sin(\beta_1 z)E_{x10}^+\{\cos\omega t + j\sin\omega t\}\right]$

$$= \underline{2E_{x10}^+\sin(\beta_1 z)\sin\omega t}$$

$$= E_{x10}^+\left[\cos(\omega t - \beta_1 z) - \cos(\omega t + \beta_1 z)\right] \rightarrow \sin A\sin B = \frac{1}{2}\{\cos(A-B) - \cos(A+B)\}$$

$$= E_{x10}^+\left[\cos\left\{\omega\left(t - \frac{z}{v_1}\right)\right\} - \cos\left\{\omega\left(t + \frac{z}{v_1}\right)\right\}\right]$$

→ Wave traveling in the ±z direction

For $\beta_1 z = \frac{2\pi}{\lambda_1}z = m\pi \quad (m = 0, \pm 1, \pm 2, \cdots), \quad E_{x1} = 0$

(Wave를 시간에 대한 term과 공간(z)에 대한 term으로 분리. $\beta_1 z = n\pi$ 이면 시간에 관계없이 $E_{x1} = 0$)

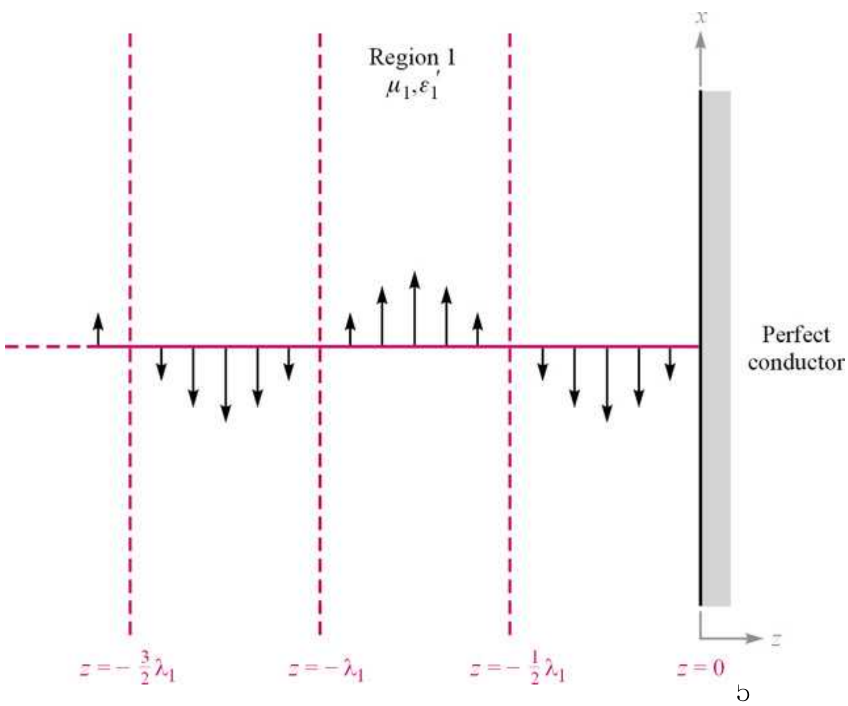
• $\omega t = m\pi$, $-\beta_1 z = m\pi$ 에 의해(서로 다른 두 조건) $E_{x1} = 0$ 이 될 수 있음

Null locations: $z = m\lambda_1/2$

⇒ Standing wave.

- For $E_{x1} = 0$,

$$\frac{2\pi}{\lambda_1}z = m\pi \qquad z = m\frac{\lambda_1}{2}$$



: Instantaneous values of the total field E_{x1} $\left(\text{at } \omega t = \frac{\pi}{2}\right)$

Since $E_{xs1}^{+} = H_{ys1}^{+}\eta_1$, $E_{xs1}^{-} = -H_{ys1}^{-}\eta_1$,

$$H_{ys1} = \frac{E_{x10}^{+}}{\eta_1}\left(e^{-j\beta_1 z} + e^{j\beta_1 z}\right)$$

$$\xrightarrow[\text{form}]{\text{instantaneous}} \quad H_{y1} = 2\frac{E_{x10}^{+}}{\eta_1}\cos\beta_1 z\cos\omega t$$

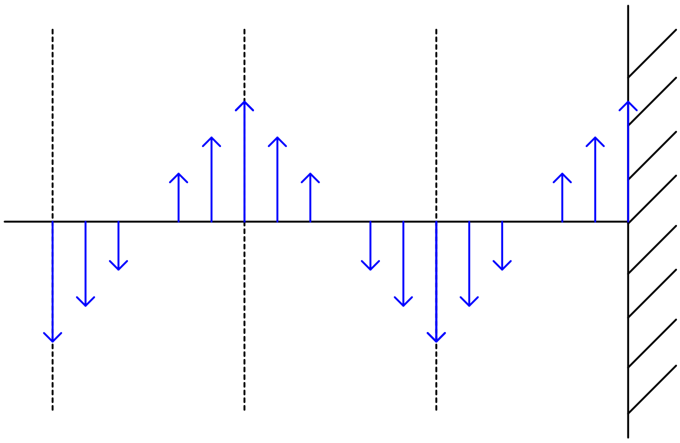
$$H_{ys1} = H_{ys1}^{+}e^{-j\beta_1 z} + H_{ys1}^{-}e^{j\beta_1 z} = \frac{E_{xs1}^{+}}{\eta_1}e^{-j\beta_1 z} - \frac{E_{xs1}^{-}}{\eta_1}e^{j\beta_1 z}$$

$$= \frac{E_{xs1}^{+}}{\eta_1}\left(e^{-j\beta_1 z} - \Gamma e^{j\beta_1 z}\right) \qquad \leftarrow \Gamma = -1$$

$$= \frac{E_{xs1}^{+}}{\eta_1}\left(e^{-j\beta_1 z} + e^{j\beta_1 z}\right)$$

$$= 2\frac{E_{xs1}^{+}}{\eta_1}\frac{e^{j\beta_1 z} + e^{-j\beta_1 z}}{2} = 2\frac{E_{xs1}^{+}}{\eta_1}\cos\beta_1 z$$

$\beta_1 z \rightarrow \frac{2\pi}{\lambda}\cdot z$



$\Rightarrow \vec{H}$ is also standing wave and 90° out of phase with $\vec{E}$.

[Ex. 12.1] Region 1, 2: perfect dielectrics.

$$\Rightarrow \alpha_1 = 0 = \alpha_2$$

$$\eta_1 = 100\ [\Omega], \qquad \eta_2 = 300\ [\Omega]$$

$$E_{x10}^{+} = 100 \quad [\text{V/m}]$$

$$\Rightarrow \qquad \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{300 - 100}{300 + 100} = 0.5$$

$$E_{x10}^{-} = 50\ [\text{V/m}]$$

$$H_{y10}^{+} = \frac{E_{x10}^{+}}{\eta_1} = \frac{100}{100} = 1 \quad [\mathrm{A/m}]$$

$$H_{y10}^{-} = -\frac{E_{x10}^{-}}{\eta_1} = -\frac{50}{100} = -0.5 \quad [\mathrm{A/m}]$$

$$\therefore < S_{1i} >= \left| \frac{1}{2} \mathrm{Re}\{\vec{E}_s \times \vec{H}_s^*\} \right| = \frac{1}{2} \frac{E_{x10}^2}{\eta_1} = \frac{1}{2} E_{x10}^{+} H_{y10}^{+} = \frac{1}{2} \times 100 \times 1 = 50 \ [\mathrm{W/m^2}] : \text{incident average power}$$

$$< S_{1r} >= -\frac{1}{2} E_{x10}^{-} H_{y10}^{-} = -\frac{1}{2} \times 50 \times (-0.5) = 12.5 \ [\mathrm{W/m^2}] : \text{ reflected average power}$$

$\to -\vec{z}$ 축으로 진행하므로

• Region 2.

$$E_{x20}^{+} = \frac{2\eta_2}{\eta_1 + \eta_2} E_{x10}^{+} = \frac{600}{100 + 300} \times 100 = 150 \quad [\mathrm{V/m}]$$

$$H_{y20}^{+} = \frac{E_{x20}^{+}}{\eta_2} = \frac{150}{300} = 0.5 \quad [\mathrm{A/m}]$$

$$\therefore < S_2 >= \frac{1}{2} E_{x20}^{+} H_{y20}^{+} = \frac{1}{2} \times 150 \times 0.5 = 37.5 \, [\mathrm{W/m^2}] : \text{ transmit average power.}$$

$\therefore \quad < S_{1i} >=< S_{1r} > + < S_2 >$: 입사전력=반사전력+ 전달전력

$\Rightarrow \quad E_{x10}^{+} + E_{x10}^{-} = E_{x20}^{+}$: 경계면에서 region 1 에서의 $\vec{E}$-field의 합.
= region 2 에서의 $\vec{E}$-field의 합.

• General rule on the transfer of power through reflection and transmission.

- Incident power density,

$$< S_{1i} >= \frac{1}{2} \mathrm{Re}\left\{ E_{xs1}^{+} H_{ys1}^{+\,*} \right\} = \frac{1}{2} \mathrm{Re}\left\{ E_{x10}^{+} \frac{1}{\eta_1^*} E_{x10}^{+*} \right\} = \frac{1}{2} \mathrm{Re}\left\{ \frac{1}{\eta_1^*} \right\} \left| E_{x10}^{+} \right|^2$$

- Reflected power density:

$$< S_{1r} > = -\frac{1}{2}\mathrm{Re}\left\{ E_{x10}^{-} H_{y10}^{-\ *} \right\} = \frac{1}{2}\mathrm{Re}\left\{ \Gamma E_{x10}^{+} \frac{1}{\eta_1^*} \Gamma^* E_{x10}^{+\ *} \right\}$$

$$= \frac{1}{2}\mathrm{Re}\left\{ \frac{1}{\eta_1^*} \right\} \left| E_{x10}^{+} \right|^2 \left| \Gamma \right|^2$$

- General relation between the reflected and incident power

$$< S_{1r} > = \left| \Gamma \right|^2 < S_{1i} > \qquad (15)$$

- Transmitted power:

$$< S_2 > = \frac{1}{2}\mathrm{Re}\{ E_{x20}^{+} H_{y20}^{+\ *} \} = \frac{1}{2}\mathrm{Re}\left\{ \tau E_{x10}^{+} \frac{1}{\eta_2^*} \tau^* E_{x10}^{+\ *} \right\} = \frac{1}{2}\mathrm{Re}\left\{ \frac{1}{\eta_2^*} \right\} \left| E_{x10}^{+} \right|^2 \left| \tau \right|^2$$

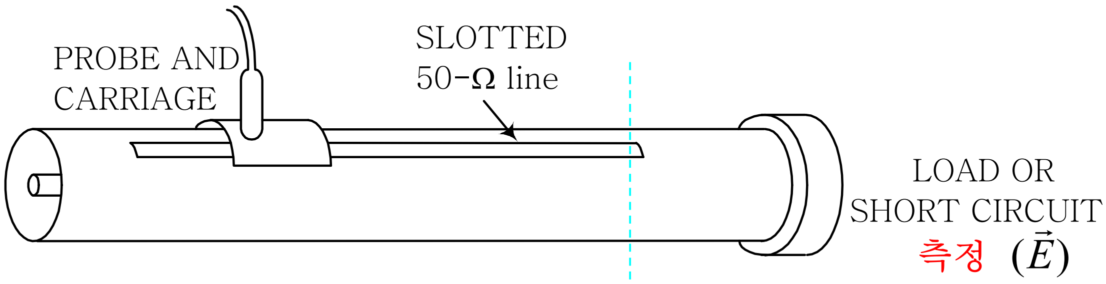
$$= \frac{\mathrm{Re}\{1/\eta_2^*\}}{\mathrm{Re}\{1/\eta_1^*\}} \left| \tau \right|^2 \cdot \frac{1}{2}\mathrm{Re}\left\{ \frac{1}{\eta_1^*} \right\} \left| E_{x10}^{+} \right|^2 = \frac{\mathrm{Re}\{1/\eta_2^*\}}{\mathrm{Re}\{1/\eta_1^*\}} \left| \tau \right|^2 < S_{1i} >$$

$$= \frac{\frac{1}{2}\left(\frac{1}{\eta_2} + \frac{1}{\eta_2^*} \right)}{\frac{1}{2}\left(\frac{1}{\eta_1} + \frac{1}{\eta_1^*} \right)} \left| \tau \right|^2 < S_{1i} > = \frac{\left(\eta_2 + \eta_2^* \right) / \left(\eta_2 \eta_2^* \right)}{\left(\eta_1 + \eta_1^* \right) / \left(\eta_1 \eta_1^* \right)} \left| \tau \right|^2 < S_{1i} >$$

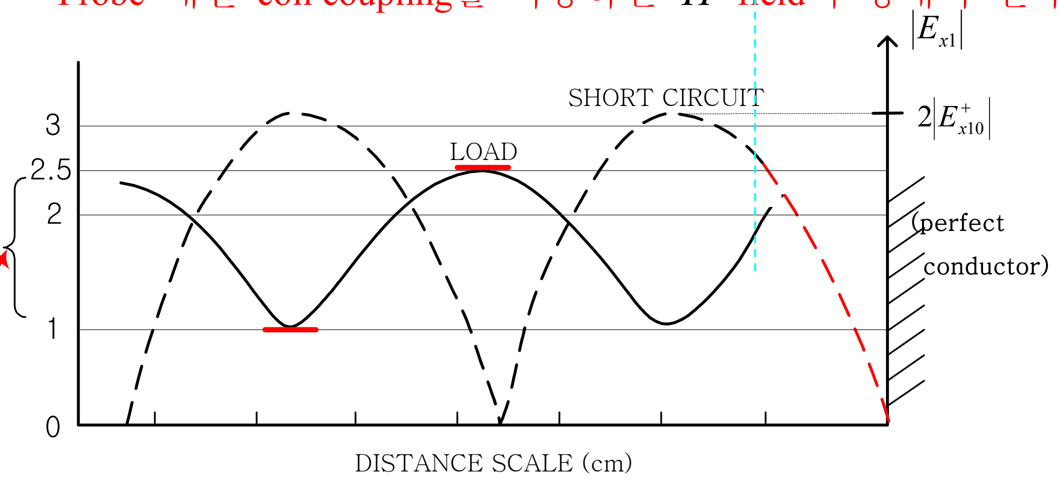
$$= \left| \frac{\eta_1}{\eta_2} \right|^2 \frac{\eta_2 + \eta_2^*}{\eta_1 + \eta_1^*} \left| \tau \right|^2 < S_{1i} > \qquad (16)$$: Too difficult to calculate

$$\Leftrightarrow \quad P_{2,\mathrm{av}}^{+} = \left(1 - \left| \Gamma \right|^2 \right) P_{1,av}^{+} \qquad (17)$$: Alternate method.

12.2 Standing Wave Ratio



- Probe를 통해 relative amplitude of electric field intensity
- Probe 대신 coil coupling을 사용하면 $\vec{H}$ -field의 상대적 진폭 측정 가능.



- Perfect short이면 $\Gamma = -1$이 되어 standing wave 가 됨.

  $0 \le |E_{x1}| \le 2|E_{x10}^{+}|$

- 그러나 임의의 load가 달린다면 임의의 신호들은 전달되고 임의의

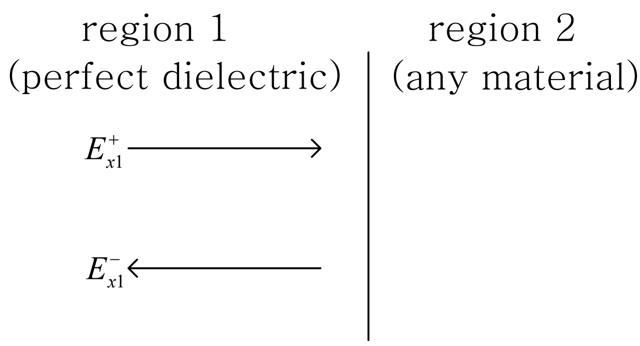
  신호는 반사가 됨.

  그래서 region 1에서는 traveling wave + standing wave 형태가 됨.

  ➔ $|E_{x1}| > 0$

- Standing wave ratio: ratio of the maximum and minimum amplitudes of electric field intensity

• Total electric field phasor in region 1



$$E_{x1T} = E_{x1}^{+} + E_{x1}^{-}$$
$$= E_{x10}^{+} e^{-j\beta_1 z} + \Gamma E_{x10}^{+} e^{j\beta_1 z} \quad (18)$$

Reflection coefficient:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = |\Gamma| e^{j\phi}$$

where η_1: positive real, η_2 : arbitrary complex

• If region 2 is a perfect conductor,

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\varepsilon_2}} \cong 0 \qquad \Rightarrow \Gamma = -1 = 1\angle\pi$$

• If η_2 is real and less than η_1,

$$\Gamma = |\Gamma| \angle\pi \qquad \leftarrow \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -\Delta = |\Delta| \angle\pi$$

• If η_2 is real and greater than η_1,

$$\Gamma = |\Gamma| \angle 0$$

• Revised total electric field phasor in region 1:

$$E_{x1T} = \left(e^{-j\beta_1 z} + |\Gamma| e^{j(\beta_1 z + \phi)}\right) E_{x10}^{+} \quad (19)$$

$$|E_{x1T}|_{\max} = \left(1 + |\Gamma|\right) E_{x10}^{+} \quad (20)$$

$$\Rightarrow -\beta_1 z_{\max} = \beta_1 z_{\max} + \phi + 2m\pi \qquad (m = 0, \pm 1, \pm 2 \cdots) \quad (21)$$

$$-\beta_1 z_{\max} = \frac{\phi}{2} + m\pi$$

$$z_{\max} = -\frac{1}{2\beta_1}(\phi + 2m\pi) \qquad (22)$$

← $z_{\max}$: E_{x1T} 를 일으키는 z 좌표값.

- If $\phi = 0 \Rightarrow \Gamma$: real positive $\Rightarrow \eta_2 > \eta_1$ & (η_1 , η_2 : real)

$\Rightarrow$ Voltage maximum is located at the boundary plane (z = 0)

($\because$ @ m = 0)

$$\Delta z_{\max} = -\frac{m\pi}{\beta_1} = -\frac{m\pi}{2\pi/\lambda_1} = -\frac{m\lambda_1}{2}$$ ($z_{\max}$ point들의 간격)

- For the perfect conductor ($\phi = \pi$) ← $\Gamma = -1 = 1\angle\pi$

$$-\beta_1 z_{\max} = \frac{\phi}{2} + m\pi \Big|_{\phi=\pi} = \frac{\pi}{2} + m\pi$$

(경계면에서 $-\pi/2$만큼 떨어진 곳에서 maximum point 발생.

$$-\beta_1 z_{\max} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \cdots)$$ (@ $m = 0$, $z_{\max} = -\frac{\pi}{2\beta_1}$)

- Voltage minimum:

$$\left|E_{x1T}\right|_{\min} = \left(1 - |\Gamma|\right) E_{x10}^{+} \qquad (23)$$

$$\Rightarrow -\beta_1 z_{\min} = \beta_1 z_{\min} + \phi + \pi + 2m\pi \quad (m = 0, \pm 1, \pm 2, \cdots) \qquad (24)$$

$$-\beta_1 z_{\min} = \frac{\phi}{2} + m\pi + \frac{\pi}{2}$$

$$z_{\min} = -\frac{1}{2\beta_1}\left[\phi + (2m+1)\pi\right] \qquad (25)$$

← $z_{\min}$: $E_{xs1,\min}$ 를 일으키는 z 좌표값.

- For the perfect conductor,

$$\Gamma = -1 = 1\angle -\pi \qquad \Rightarrow \quad \phi = -\pi$$

$\Rightarrow$ Voltage minimum is found at $z = 0$ $\quad z_{\min} = -\frac{1}{2\beta_1}[-\pi + \pi] = 0$

$\Rightarrow$ Γ: real negative $\quad \Rightarrow \eta_2 < \eta_1$ & (η_1, η_2: real)

• Revised total electric field:

$$E_{xs1} = \left(e^{-j\beta_1 z} + |\Gamma| e^{j(\beta_1 z + \phi)}\right) E_{x10}^{+}$$

$$= E_{x10}^{+}\left(1 - |\Gamma|\right) e^{-j\beta_1 z} + E_{x10}^{+}|\Gamma|\left(e^{-j\frac{\phi}{2}} e^{-j\beta_1 z} + e^{j\frac{\phi}{2}} e^{j\beta_1 z}\right) e^{j\frac{\phi}{2}}$$

$$= \left(1 - |\Gamma|\right) E_{x10}^{+} e^{-j\beta_1 z} + 2|\Gamma| E_{x10}^{+} e^{j\frac{\phi}{2}} \frac{e^{j\frac{\phi}{2}} e^{j\beta_1 z} + e^{-j\frac{\phi}{2}} e^{-j\beta_1 z}}{2}$$

$$= \left(1 - |\Gamma|\right) E_{x10}^{+} e^{-j\beta_1 z} + 2|\Gamma| E_{x10}^{+} e^{j\frac{\phi}{2}} \cos\left(\beta_1 z + \frac{\phi}{2}\right)$$

instantaneous form →

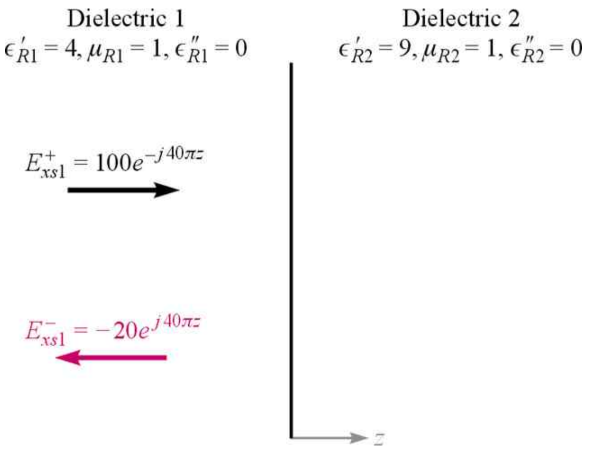
$$E_{x1T}(x,t) = \underbrace{\left(1 - |\Gamma|\right) E_{x10}^{+} \cos(\omega t - \beta_1 z)}_{\text{traveling wave}}$$

계속 진행되는 traveling wave

$$+ \underbrace{2|\Gamma| E_{x10}^{+} \cos(\beta_1 z + \phi/2)\cos(\omega t + \phi/2)}_{\text{standing wave}} \qquad (26)$$

reflected 되어 standing wave를 일으키는 성분.

[Ex. 12.2] Propagating wave in material 1: 100 [V/m], 3 [GHz]



$$\omega = 2\pi f = 6\pi \times 10^9 \quad \text{[rad/sec]}$$

$$\beta_1 = \omega\sqrt{\mu_1\varepsilon_1} = 6\pi \times 10^9 \frac{\sqrt{4\times 1}}{3\times 10^8} = 40\pi \quad \text{[rad/m]}$$

$$\beta_2 = \omega\sqrt{\mu_2\varepsilon_2} = 60\pi \quad \text{[rad/m]}$$

$$\lambda_1 = \frac{2\pi}{\beta_1} = \frac{2\pi}{40\pi} = 0.05 \quad \text{[m]}$$

$$\lambda_2 = \frac{2\pi}{\beta_2} = \frac{2\pi}{60\pi} = 0.0333 \quad \text{[m]}$$

$$\eta_1 = \sqrt{\frac{j\omega\mu_1}{\sigma + j\omega\varepsilon_1}} = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \frac{1}{\sqrt{\varepsilon_{R1}}} \times 120\pi = 60\pi \quad [\Omega] \leftarrow \sigma = \omega\varepsilon''$$

$$\eta_2 = \frac{1}{\sqrt{\varepsilon_{R2}}} \times 120\pi = 40\pi \quad [\Omega]$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{40\pi - 60\pi}{40\pi + 60\pi} = -0.2 = 0.2\angle\pi$$

$\Rightarrow$ $\vec{E}_{\min}$ will be at the boundary and repeated at half-waelength (0.025m) intervals in dielectric 1.

$$E_{xs1,\min} = \left(1 - |\Gamma|\right) E^+_{x10} = 80 \quad \text{[V/m]}$$

$\Rightarrow$ $E_{xs1,\max} = \left(1 + |\Gamma|\right) E^+_{x10} = 120 \quad \text{[V/m]}$ at ($z = -0.0125, -0.0375, -0.0625, \cdots$ [m]

(Region 2에서는 반사파가 없으므로 max, min점들이 없음)

• Standing wave ratio (SWR): ratio of maximum to minimum electric field amplitudes

$$s = \frac{|E_{x1T}|_{\max}}{|E_{x1T}|_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|} \qquad (27)$$

Since $|\Gamma| \le 1$, $\quad s \ge 1$

[Ex.] ① $\Gamma = -0.2 \quad \Rightarrow s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1.2}{0.8} = 1.5 \qquad \Rightarrow 1.5:1$

② $|\Gamma| = 1 \quad \Rightarrow$ The reflected and incident wave amplitude are equal.

$\Rightarrow$ All the incident wave energy is reflected.

$\Rightarrow$ E_{x1} is zero which plane is separated $\lambda_1/2$.

E_{x1} is $2E_{xs10}$ which plane is separated $\lambda_1/2$.

③ $\eta_2 = \eta_1 \quad \Rightarrow \Gamma = 0 \quad \Rightarrow$ No energy reflectded.

$\Rightarrow \quad s = 1$

④ If 1/2 of incident power is reflected. $\Rightarrow |\Gamma|^2 = 0.5 \quad \Rightarrow |\Gamma| = 0.707$

$$\Rightarrow \quad s = \frac{1+0.707}{1-0.707} = \frac{1.707}{0.293} = 5.83$$

(SWR은 반사신호의 진폭 변화 비를 의미하므로 반사신호의 진폭 측정에 의해 반사계수를 알 수 있다. $s \rightarrow |\Gamma|$)

Ex.] An uniform plane wave is reflected from an unknown material to the air. The measured electric field is 1.5 m between maxima.

z_{max} = -0.75 [m], s = 5. η_u = ?

➔ $\lambda = 1.5 \times 2 = 3$ [m] → $f = c / \lambda = (3\times10^8) / 3 = 10^8$ [Hz]

or 100 [MHz]

z_{max} = -0.75 [m] → The 1st minimum occurs at the boundary.

$$|\Gamma| = \frac{s-1}{s+1} = \frac{5-1}{5+1} = \frac{2}{3}$$

$$\Gamma = -\frac{2}{3} = \frac{\eta_u - \eta_0}{\eta_u + \eta_0}$$

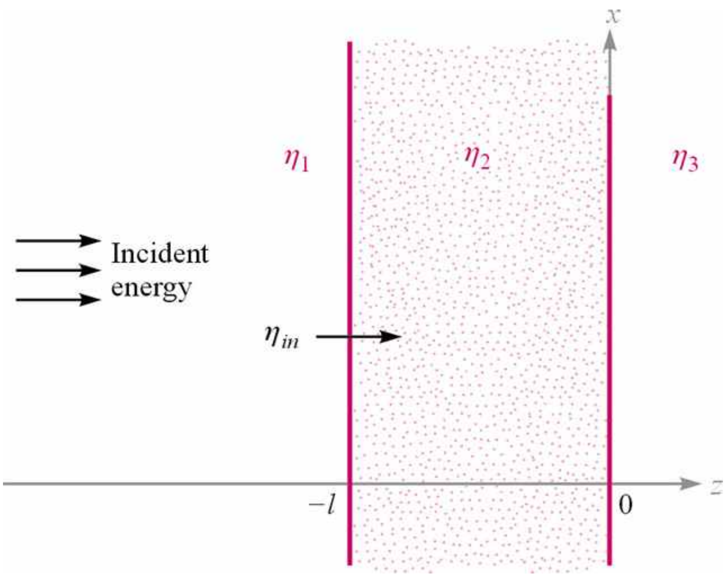
$$\eta_u = \eta_0 / 5 = 377 / 5 = 75.4 \ [\Omega]$$

12.3 Wave Reflection from Multiple Interfaces

• Consider wave reflection from materials that are finite in extent.

➔ Effects of the front and back surfaces.

[Ex.] Light incident on a flat piece of glass. (coated glasses, etc.)



• Let us assume all regions are composed of lossless media.

• Electric fields in region 2:

$$E_{xs2} = E_{x20}^{+} e^{-j\beta_2 z} + E_{x20}^{-} e^{j\beta_2 z} \quad (28a)$$

where $\beta_2 = \omega\sqrt{\mu_2\varepsilon_2} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_{r2}}$

$$= \frac{\omega\sqrt{\varepsilon_{r2}}}{c}$$

E_{x20}^{+}, E_{x20}^{-} : complex

• Magnetic fields in region 2:

$$H_{ys2} = H_{y20}^{+} e^{-j\beta_2 z} + H_{y20}^{-} e^{j\beta_2 z} \qquad (28b)$$

Reflection coefficient at the second interface of region 2:

$$\Gamma_{23} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} \qquad (29)$$

$$\Rightarrow \; E_{x20}^{-} = \Gamma_{23} E_{x20}^{+} \qquad (30)$$

$$H_{y20}^{+} = \frac{E_{x20}^{+}}{\eta_2} \qquad (31a)$$

$$H_{y20}^{-} = -\frac{\mathrm{E}_{x20}^{-}}{\eta_2} = -\frac{\Gamma_{23}}{\eta_2} E_{x20}^{+} \qquad (31b)$$

• Wave impedance (η_w): ratio of total electric field to total magnetic field at arbitrary position z in region 2

$$\eta_w(z) = \frac{E_{xs2}}{H_{ys2}} = \frac{E_{x20}^{+} e^{-j\beta_2 z} + E_{x20}^{-} e^{j\beta_2 z}}{H_{y20}^{+} e^{-j\beta_2 z} + H_{y20}^{-} e^{j\beta_2 z}}$$

$$= \frac{E_{x20}^{+} e^{-j\beta_2 z} + \Gamma_{23} E_{x20}^{+} e^{j\beta_2 z}}{\frac{E_{x20}^{+}}{\eta_2} e^{-j\beta_2 z} - \frac{\Gamma_{23}}{\eta_2} E_{x20}^{+} e^{j\beta_2 z}} = \eta_2 \frac{e^{-j\beta_2 z} + \Gamma_{23} e^{j\beta_2 z}}{e^{-j\beta_2 z} - \Gamma_{23} e^{j\beta_2 z}}$$

$$= \eta_2 \frac{(\cos\beta_2 z - j\sin\beta_2 z) + \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2}(\cos\beta_2 z + j\sin\beta_2 z)}{(\cos\beta_2 z - j\sin\beta_2 z) - \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2}(\cos\beta_2 z + j\sin\beta_2 z)}$$

$$= \eta_2 \frac{(\eta_3+\eta_2)(\cos\beta_2 z - j\sin\beta_2 z)+(\eta_3-\eta_2)(\cos\beta_2 z + j\sin\beta_2 z)}{(\eta_3+\eta_2)(\cos\beta_2 z - j\sin\beta_2 z)-(\eta_3-\eta_2)(\cos\beta_2 z + j\sin\beta_2 z)}$$

$$= \eta_2 \frac{2\eta_3 \cos\beta_2 z - j2\eta_2 \sin\beta_2 z}{2\eta_2 \cos\beta_2 z - j2\eta_3 \sin\beta_2 z}$$

$$= \eta_2 \frac{\eta_3 \cos\beta_2 z - j\eta_2 \sin\beta_2 z}{\eta_2 \cos\beta_2 z - j\eta_3 \sin\beta_2 z} \qquad (32)$$

$$= \eta_2 \frac{\eta_3 - j\eta_2 \tan\beta_2 z}{\eta_2 - j\eta_3 \tan\beta_2 z}$$

- Boundary condition at $z = -l$,

$$\begin{cases} E_{xs1}^{+} + E_{xs1}^{-} = E_{xs2} & (33a) \\ H_{ys1}^{+} + H_{ys1}^{-} = H_{ys2} & (33b) \end{cases}$$

$\Rightarrow \; E_{x10}^{+} + E_{x10}^{-} = E_{xs2}(z=-l) \;\leftarrow$ from eq. (7), (8) in text

region 2 에도 E_{x20}^{+}, E_{x20}^{-} 성분이 있으므로 E_{xs2} 로 표시.

$H_{y10}^{+} + H_{y10}^{-} = H_{ys2}(z=-l) \;\rightarrow\; \dfrac{E_{x10}^{+}}{\eta_1} - \dfrac{E_{x10}^{-}}{\eta_1} = \dfrac{E_{xs2}(z=-l)}{\eta_w(-l)}$

where E_{x10}^{+}, E_{x10}^{-}: amplitudes of incident and reflected fields in region 1.

- Input impedance $(\eta_{in}) = \eta_w(-l)$ to the two-interface combination

$$\frac{E_{x10}^{+}}{\eta_1} - \frac{E_{x10}^{-}}{\eta_1} = \frac{E_{xs2}(z=-l)}{\eta_{\text{in}}} = \frac{E_{x10}^{+} + E_{x10}^{-}}{\eta_{\text{in}}}$$

$$E_{x10}^{+}\left(\frac{1}{\eta_1} - \frac{1}{\eta_{\text{in}}}\right) = E_{x10}^{-}\left(\frac{1}{\eta_{\text{in}}} + \frac{1}{\eta_1}\right)$$

$$E_{x10}^{+}\frac{\eta_{in}-\eta_1}{\eta_1\eta_{in}} = E_{x10}^{-}\frac{\eta_{in}+\eta_1}{\eta_1\eta_{in}}$$

$$\frac{E_{x10}^{-}}{E_{x10}^{+}} = \Gamma = \frac{\eta_{\text{in}}-\eta_1}{\eta_{\text{in}}+\eta_1} \qquad (35)$$

where $$\eta_{\text{in}} = \eta_w(z=-l) = \eta_2\frac{\eta_3\cos\beta_2 l + j\eta_2\sin\beta_2 l}{\eta_2\cos\beta_2 l + j\eta_3\sin\beta_2 l} \qquad (36)$$

• In case of $\eta_{\text{in}} = \eta_1$, $\Gamma = 0$

$\Rightarrow$ No reflected power $\left(\because\ P_r = \left(1-|\Gamma|^2\right)P_{in}\right) \Rightarrow$ Matched

• Suppose that $\eta_3 = \eta_1$ and $\beta_2 l = m\pi$ (m : integer)

$$\beta_2 l = \frac{2\pi}{\lambda_2}l = m\pi \qquad \left(\text{or } l = \frac{m\lambda_2}{2} \qquad (37)\right)$$

$$\eta_{\text{in}} = \eta_2\left.\frac{\eta_3\cos\beta_2 l + j\eta_2\sin\beta_2 l}{\eta_2\cos\beta_2 l + j\eta_3\sin\beta_2 l}\right|_{\beta_2 l = m\pi}$$

$$= \eta_2\frac{\eta_3}{\eta_2} = \eta_3$$

$$= \eta_1 \qquad \Rightarrow \quad \text{Matched.}$$

Ex.] Antenna inside of the aircraft.

• (Refractive) Index:

$$n = \sqrt{\varepsilon_r} \qquad (38)$$

For lossless media (ε_r" = 0 and μ_r = 1),

$$\beta = k = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_r} = \frac{n\omega}{c} \qquad (39)$$

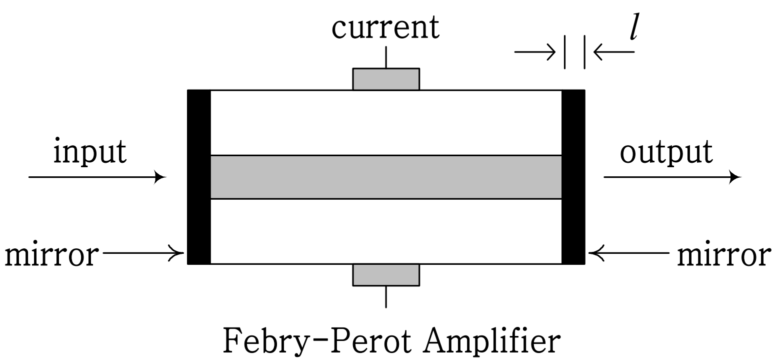
$$\eta = \frac{1}{\sqrt{\varepsilon_r}}\sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{\eta_0}{n} \qquad (40)$$

Phase velocity and wavelength in a material of index n:

$$v_p = \frac{1}{\sqrt{\mu_0\varepsilon_0\varepsilon_r}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_r}} = \frac{c}{n} \qquad (41)$$

$$\lambda = \frac{v_p}{f} = \frac{c}{nf} = \frac{\lambda_0}{n} \qquad (42)$$

Ex.] Fabry-Perot interferometer: optic filter which transmit spcific



wavelength satisfying

the condition. $\lambda = \lambda_0 / n = 2l / m$

$$\lambda_{m-1} - \lambda_m = \Delta\lambda_f = \frac{2l}{m-1} - \frac{2l}{m} = \frac{2l}{m(m-1)} \approx \frac{2l}{m^2}$$

where m: number of half-wavelength in region 2 or

$$m = 2l/\lambda = 2nl/\lambda_0$$

λ_0: desired free-space wavelength for transmission.

Free-space wavelength in region 2:

$$\Delta\lambda_f = \frac{2l}{m^2} = 2l\frac{\lambda^2}{4l^2} = \frac{\lambda_2^2}{2l} \qquad (43a)$$

Free spectral range: wavelength in free space

$$\Delta\lambda_{f0} = n\Delta\lambda_f = n\frac{\lambda_2^2}{2l} = n\frac{\lambda_0^2/n^2}{2l} = \frac{\lambda_0^2}{2nl} \qquad (43b)$$

[Ex. 12.4] $\Delta\lambda_s = 50\,\text{nm}$, $\underline{\lambda_0 = 600\,\text{nm}}$, $(\lambda = 600 \pm 50\,\text{nm})$

→ Red part of the visible spectrum.

Febry-Perot filter: $n = 1.45$

Required range of glass thickness?

$\Rightarrow$ Optical spectral width $\Delta\lambda_{f0} > \Delta\lambda_s$ (∵ 원하는 대역폭의 신호가 모두 통과 되어야 하므로)

$$\frac{\lambda_2^2}{2l} = \Delta\lambda_f > \Delta\lambda_s$$

$$\therefore l < \frac{\lambda_2^2}{2\Delta\lambda_s} = \frac{\left(600/\sqrt{1.45}\right)^2 \ \text{nm}^2}{2\times 50 \ \ \text{nm}} = \frac{600^2}{2\times 1.45\times 50} = 2.5\times 10^3\ \text{nm} = 2.5\,\mu\text{m}$$

///

• Another matching condition:

For $\beta_2 l = (2m-1)\frac{\pi}{2}$ (m : integer)

$$\beta_2 l = \frac{2\pi}{\lambda_2} l = (2m-1)\frac{\pi}{2} \qquad (44)$$

$$\Rightarrow \quad l = (2m-1)\frac{\lambda_2}{4} \quad \text{: odd multiple of a quarter wavelength}$$

$$\eta_{\text{in}} = \eta_2 \frac{\eta_3 \cos\beta_2 l + j\eta_2 \sin\beta_2 l}{\eta_2 \cos\beta_2 l + j\eta_3 \sin\beta_2 l}\bigg|_{l=(2m-1)\frac{\lambda_2}{4}}$$

$$= \eta_2 \frac{j\eta_2}{j\eta_3} = \frac{\eta_2^2}{\eta_3} \qquad (45) \qquad \leftarrow \quad \eta_{in} = \eta_1 \quad (\text{ for matching })$$

$$\eta_2 = \sqrt{\eta_{in}\eta_3} = \sqrt{\eta_1\eta_3} \quad (46) : \text{ quarter wave matching.}$$

[Ex. 12.5] Coating glass. Transmission wavelength from air to glass: 570 [nm]

$n_3 = 1.45$

Required index for the coating and its minimum thickness?

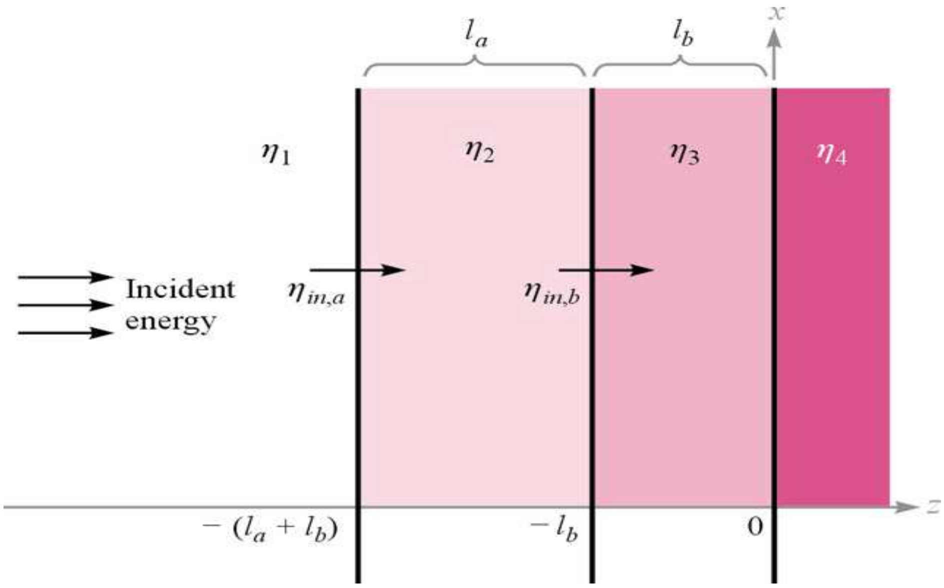
$$\Rightarrow \quad \eta_1 = 377\,\Omega \quad (\text{free space}), \quad \eta_3 = \frac{377}{1.45} = 260\,\Omega \quad (\text{glass})$$

$$\eta_2 = \sqrt{\eta_1\eta_3} = \sqrt{377 \times 260} = 313\,\Omega$$

$$\therefore \quad \lambda_2 = \frac{570}{1.45} = 475 \text{ nm}$$

$$l = \frac{\lambda_2}{4} = 119 \text{ nm} = 0.119\ \mu\text{m} \quad : \text{ coating width}$$

• Multiple-interface situation



Step 1) $$\eta_{\text{in},b} = \eta_3 \frac{\eta_4 \cos \beta_3 l_b + j\eta_3 \sin \beta_3 l_b}{\eta_3 \cos \beta_3 l_b + j\eta_4 \sin \beta_3 l_b} \qquad (47)$$

Step 2) $$\eta_{\text{in},a} = \eta_2 \frac{\eta_{\text{in},b} \cos \beta_2 l_a + j\eta_2 \sin \beta_2 l_a}{\eta_2 \cos \beta_2 l_a + j\eta_{\text{in},b} \sin \beta_2 l_a} \qquad (48)$$

Step 3) $$\Gamma = \frac{\eta_{\text{in},a} - \eta_1}{\eta_{\text{in},a} + \eta_1}$$

Step 4) Reflected power: $P_r = |\Gamma|^2 P_{in}$

Transmitted power at layer 4: $P_t = \left(1 - |\Gamma|^2\right) P_{in}$