**Microwave Engineering 2-3**

# **Chapter 2 Transmission Line**

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# **Learning Objectives**

- Learn how to realize microwave *L* and *C* with transmission line
- Learn what will be done in transmission lines connection
- Understanding decibel units
- Learn utilities of Smith chart

# **Learning contents**

- Microwave *L* and *C* using Transmission Line
- § Transmission Lines Connection
- § Decibel
- § Smith Chart

#### **Microwave** *L* **and** *C* **using Transmission Line 1**

**•** Transmission line circuit **terminated with short circuit**  $(Z_L = 0)$ 



- Repeated inductive & capacitive characteristics along transmission line
- Different reactance characteristics according to frequency (or wavelength)
- Repeated reactance characteristics according to harmonics  $(f_0, 3f_0,$  ⋯,  $(2n+1)f_0$ )

#### **Microwave** *L* **and** *C* **using Transmission Line 1**

- **•** Transmission line circuit **terminated with open circuit**  $(Z_L = \infty)$  $-$  At load,  $I = 0$  and  $V = \infty$ - Input impedance  $\rightarrow$  -*j* $\infty \le Z_{\text{in}} \le +j\infty$ 4 | \ | **EVOWAVE L and C using Transmission Line**<br>
ssion line circuit terminated with open circuit  $(Z_L = \infty)$ <br>  $\frac{-Z_0}{+Z_0}\Big|_{Z_t=\infty} = 1 = \frac{V^+}{V^-}$ <br>  $I = 0$  and  $V = \infty$ <br>
reedance<br>  $\frac{Z_L + jZ_0 \tan \beta \ell}{Z_0 + jZ_L \tan \beta \ell}\Big|_{Z_L=\infty} = -jZ_0 \cot \$ Figure 1.1 and capacitor (C) can be realized by using transmission line circuit terminated  $\frac{\lambda_L = 0}{\lambda_L}$ <br>  $Z_{\text{in}}$ ,  $\beta$   $Z_{\text{in}} - \infty$   $V_L$ <br>  $Z_{\text{in}} - \infty$ <br>  $Z$ **IMICYOWAVE L and C using Transmission Line**<br>
Transmission line circuit terminated with open circuit  $(Z_L = \infty)$ <br>  $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}\Big|_{Z_L = \infty} = 1 = \frac{V'}{V}$ <br>
At load,  $I = 0$  and  $V = \infty$ <br>
Imput impedance<br>  $Z_{\text{in}} = Z_0 \frac{Z_L$  $\theta = \beta l = \frac{2\pi}{3}l, \ 0 < \theta < \frac{\pi}{2} \Longleftrightarrow 0 < \frac{2\pi}{3}l < \frac{\pi}{2} \Longleftrightarrow 0 < l < \frac{\pi}{2} \frac{\lambda}{2\pi} = \frac{\lambda}{4}$  $\lambda$   $\lambda$   $\lambda$ simission line circuit terminated with open circuit  $(Z_L = \infty)$ <br>  $\frac{Z_L - Z_0}{Z_L + Z_0}\Big|_{Z_L - \infty} = 1 = \frac{V^+}{V}$ <br>  $\left[\begin{array}{ccc} \frac{Z_L - Z_0}{Z_L + Z_0}\Big|_{Z_L - \infty} & = -\frac{V^+}{V} \end{array}\right]$ <br>  $\left[\begin{array}{ccc} \frac{Z_L - Z_0}{Z_0} & \frac{Z_L - Z_0}{Z_0} & \frac{Z_L - Z_0}{Z_0} \\$ ansmission line circuit terminated with open circuit  $(Z_L = \infty)$ <br>  $\frac{Z_L - Z_u}{Z_L + Z_0}\Big|_{Z_L = \infty} = 1 = \frac{V^+}{V^-}$ <br>  $\frac{Z_0, \beta \qquad Z_L = \infty \qquad V_L = 0$ <br>
t load,  $I = 0$  and  $V = \infty$ <br>
put impedance<br>  $\left[\frac{Z_L - Z_u}{Z_0 + jZ_L \tan \beta t}\right]_{Z_L = \infty} = -jZ_$  $\frac{0}{-1}$   $\frac{1}{-1}$  $0|_{Z_t=\infty}$   $\blacksquare$  $1 = \frac{1}{1}$  $L^{\infty}$  $L \sim 0$   $-1-\frac{r}{r}$  $L + Z_0 \Big|_{Z_L = \infty}$   $V^-$ **Iicrowave L and C using Transmiss**<br>
smission line circuit terminated with open circuit  $(Z_L = \infty)$ <br>  $\left. \frac{Z_L - Z_0}{Z_L + Z_0} \right|_{Z_L = \infty} = 1 = \frac{V^+}{V^-}$ <br>
aad,  $I = 0$  and  $V = \infty$ <br>
t impedance **Iicrowave L and C using Transmission Line**<br>
imission line circuit terminated with open circuit  $(Z_L = \infty)$ <br>  $\frac{Z_L - Z_0}{Z_L + Z_0}\Big|_{Z_L = \infty} = 1 = \frac{V^+}{V^-}$ <br>
and,  $I = 0$  and  $V = \infty$ <br>  $Z + iZ$  tan  $B\ell$ + **-** *Company of the company of the company* Microwave L and C using Transmission Lin<br>
Fransmission line circuit terminated with open circuit  $(Z_L = \infty)$ <br>  $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}\Big|_{Z_L = \infty} = 1 = \frac{V^+}{V^-}$ <br>
At load,  $I = 0$  and  $V = \infty$ <br>
(nput impedance  $\begin{bmatrix} 0 & \tan P^{\chi} \end{bmatrix}$  \_\_ controlled the circuit **terminated with open circuit (Z<sub>L</sub>** =  $\infty$ )<br>  $\left[\frac{Z_L - Z_0}{Z_L + Z_0}\right]_{Z_L = \infty} = 1 = \frac{V^+}{V^-}$ <br>
t load,  $I = 0$  and  $V = \infty$ <br>
aput impedance<br>  $\left[\frac{Z_L + jZ_0 \tan \beta \ell}{Z_0 + jZ_L \tan \beta \ell}\right]_{Z_L = \infty} = -jZ_0 \cot \beta \ell$ <br>  $0 \perp J$   $\sim$   $L$  tan  $P$ <sup> $\sim$ </sup> $\mid$  $Z_{L}$  $\tan \beta \ell$   $\mathbb{Z}$  and  $\alpha \ell$  $\cot \beta \ell$  $\left.\tan \beta \ell \right|_{z=\infty}$   $\frac{1}{z}$  $L^{\infty}$  $L^{-1}$   $\int$   $L_0$  tail  $P^{\chi}$  $L$  *Lettar*  $P^{\mathcal{L}}|_{Z_L = \infty}$ **Microwave** *L* **and** *C* **using Transmission Line<br>
Fransmission line circuit terminated with open circuit**  $(Z_L = \infty)$ **<br>**  $\Gamma = \frac{Z_L - Z_u}{Z_L + Z_0}\Big|_{Z_L = \infty} = 1 = \frac{V^+}{V^-}$ **<br>
At load,**  $I = 0$  **and**  $V = \infty$ **<br>
Input impedance<br> Z\_m = Z\_0 \frac{Z\_L + j Prowave** *L* **and** *C* **using Transmission**<br>
sion line circuit terminated with open circuit  $(Z_L = \infty)$ <br>  $\frac{Z_0}{Z_0}\Big|_{Z_L = \infty} = 1 = \frac{V^+}{V^-}$ <br>  $I = 0$  and  $V = \infty$ <br>
pedance<br>  $\frac{Z_L + jZ_0 \tan \beta \ell}{Z_0 + jZ_L \tan \beta \ell}\Big|_{Z_L = \infty} = -jZ_0 \cot \beta \$  $\frac{+jZ_0 \tan \beta \ell}{+jZ_L \tan \beta \ell} \bigg|_{Z_L = \infty} = -jZ_0 \cot \beta \ell$  $\beta\ell$  , and the set of  $\beta$ Microwave L and C using Transmission Line<br>
ssmission line circuit terminated with open circuit  $(Z_L = \infty)$ <br>  $\frac{Z_L - Z_0}{Z_L + Z_0}\Big|_{Z_L = \infty} = 1 = \frac{V^+}{V^-}$ <br>  $\frac{Z_0}{Z_0}$ <br>  $\frac{Z_1}{Z_0} = 0$  and  $V = \infty$ <br>  $\frac{Z_{10}}{Z_0 + jZ_L \tan \beta \ell}\Big$ l  $\ell$   $\frac{1}{2}$  $\ell$ <sub>z</sub>  $\sim$   $\ell$ 
	- Same electrical characteristics as like transmission line terminated with short circuit
- § *Microwave inductor (L) and capacitor (C) can be realized by using transmission line circuit terminated with short or open circuits!!!*

# **2 Transmission Lines Connection**

**•** Input impedance in case of  $l = \lambda/2$  transmission line terminated with load  $(Z_L)$ 

**Transmission Lines Connection**  
put impedance in case of 
$$
l = \lambda/2
$$
 transmission line terminated with load  $(Z_L)$   

$$
Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} = Z_0 \frac{Z_L + jZ_0 \tan \frac{2\pi \lambda}{\lambda}}{Z_0 + jZ_L \tan \frac{2\pi \lambda}{\lambda} \frac{2}{2}} = Z_L \Leftrightarrow Z_{in} = Z_L
$$
put impedance in case of  $l = \lambda/4 + n\lambda/2$  transmission line terminated with load  $(Z_L)$ 

**•** Input impedance in case of  $l = \lambda/4 + n\lambda/2$  transmission line terminated with load  $(Z_L)$ 

**Transmission Lines Connection**  
put impedance in case of 
$$
I = \lambda/2
$$
 transmission line terminated with load  $(Z_L)$   

$$
Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} = Z_0 \frac{Z_L + jZ_0 \tan \frac{2\pi \lambda}{\lambda} \frac{\lambda}{2}}{Z_0 + jZ_L \tan \frac{2\pi \lambda}{\lambda} \frac{\lambda}{2}} = Z_L \Leftrightarrow Z_{in} = Z_L
$$
  
put impedance in case of  $I = \lambda/4 + n\lambda/2$  transmission line terminated with load  $(Z_L)$   

$$
Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta \ell}{Z_0 + jZ_L \tan \beta \ell} \Big|_{\ell = \lambda/4} = Z_0 \frac{Z_L + jZ_0 \tan \frac{2\pi \lambda}{\lambda} \frac{\lambda}{4}}{Z_0 + jZ_L \tan \frac{2\pi \lambda}{\lambda} \frac{\lambda}{4}} = \frac{Z_0^2}{Z_L} : \text{Quarter-wave (length) transcript}
$$

#### **Transmission Lines Connection 2**

- **Transmission line of characteristic impedance**  $Z_0$  **feeding different characteristic impedance**  $(Z_1)$  **transmission line** 
	- Reflection coefficient
		- $Z_1 + Z_0$  $\Gamma = \frac{Z_1 - Z_0}{Z_1 - Z_0}$
		- $\rightarrow$  Some portion of the incident wave is reflected and the remained is transmitted into the second line. of the incident wave is reflected and<br>is transmitted into the second line.<br>each transmission lines<br> $j^{\beta z} + \Gamma e^{j\beta z}$  (a)  $z < 0$ <br> $(j\beta z)$  (a)  $z > 0$ of the incident wave is reflected as<br> *is* transmitted into the second line<br>
		each transmission lines<br>  $\frac{d^2y}{dt^2} + \Gamma e^{j\beta z}$  (a)  $z < 0$ <br>
		(a)  $z > 0$ <br>
		smission coefficient at  $z = 0$ : of the incident wave is reflected and<br>s transmitted into the second line.<br>each transmission lines<br> $\beta^z + \Gamma e^{i\beta z}$  (a)  $z < 0$ <br> $\frac{\beta^z}{\beta^z}$  (a)  $z > 0$
	- Voltage waves on each transmission lines  $V(z) = V_0^+(e^{-j\beta z} + \Gamma e^{j\beta z})$ rtion of the incident wave is reflect<br>ined is transmitted into the second l<br>s on each transmission lines<br> $^{+}(e^{-j\beta z} + \Gamma e^{j\beta z})$  @  $z < 0$ <br> $^{+}Te^{-j\beta z}$  @  $z > 0$ rtion of the incident wave is reflected<br>
	ined is transmitted into the second l<br>
	s on each transmission lines<br>  ${}^{+}(e^{-j\beta z} + \Gamma e^{j\beta z})$  @ z < 0<br>  ${}^{+}Te^{-j\beta z}$  @ z > 0<br>  ${}^{+}Te^{-j\beta z}$  @ z > 0<br>  $\therefore$  transmission c
		- $V(z) = V_0^+ T e^{-j\beta z}$  $\omega$  z > 0
	- Continuity of transmission coefficient at *z* = 0:

$$
T = 1 + \Gamma = 1 + \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{2Z_1}{Z_1 + Z_0}
$$

- Insertion loss (IL):  $IL = -20 log |T| dB$ 



## **Decibel [dB] 3**

- § Decibel (dB): **relative unit** of measurement equal to one tenth of a **bel** (**B**)
- Power gain in decibel  $=10 \log \frac{I_1}{I_1}$  [dB] Ex.]  $P_1/P_2 = 0.5 = 1/2 \rightarrow -3$  dB,  $P_1/P_2 = 10 \rightarrow 10$  dB - Voltage gain in decibel: - Current gain in decibel: 10k where  $R_1, R_2$ : load resistances  $V_{1, 2}$ ,  $I_{1, 2}$ : voltages across specific ports and currents passing specific nodes Neper [Np]: ratio of voltages across equal load resistances 2 **a** it of measurement equal to one tenth of a **bel (B)**<br>
10log  $\frac{P_1}{P_2}$ [dB]<br>
→ -3 dB,  $P_1/P_2 = 10$  → 10 dB<br>
10log  $\frac{P_1}{P_2} = 10$ log  $\frac{V_1^2/R_1}{V_2^2/R_2} = 20$ log  $\frac{V_1}{V_2} \sqrt{\frac{R_2}{R_1}}$  [dB]  $P_2$  $=10 \log \frac{1}{D} [dB]$  $1 - \mathbf{R}_2$ 2 a set of  $\sim$  3 a set of  $\sim$  $\frac{1}{2}$  [dB]  $\leftarrow$  If  $R_1 = R_2$ 2  $[AB]$ 2  $V^{1}$  $1/\mathbf{R}$   $1$   $\mathbf{R}$  $1$   $2 \sqrt{11}$ 2  $\mathbf{p}$   $\sim$   $\sim$   $\mathbf{v}$   $\sim$   $\mathbf{v}$   $\sim$   $\sim$  $2 \frac{N_1}{N_2}$   $2 \frac{N_1}{N_1}$  $^{2} - 20 \cdot \frac{1}{1}$  $2\,$  D  $\qquad$   $\q$  $1^{12}$  – 2010<sup>'</sup> 2  $\binom{2}{1}$ 2  $\sqrt{D}$  1010 $\sigma$   $\sqrt{2}$ 2  $\binom{1}{2}$  $1 - 10 \log^{11} N_2$  $2/D$   $V^2$  $^{17}$   $^{10}$   $-10$   $^{10}$ 2  $V_2 / N_2$  $\frac{1}{V_2^2} = 10 \log \frac{V_1}{V_2^2/R_2} = 10 \log \frac{V_1}{V_2^2 R_1} = 20 \log \frac{V_1}{V_2} \sqrt{\frac{R_2}{R_1}}$  [dB]<br>= 20 log  $\frac{V_1}{V_1}$  [dB]  $\leftarrow$  If  $R_1 = R_2$  $/R_2$   $V_2^2$  $10\log\frac{P_1}{P_1} = 10\log\frac{V_1^2/R_1}{V_1^2/I} = 10\log\frac{V_1^2R_2}{V_1^2I} = 20$  $V_2$  $V_{1}$   $\sim$   $\Gamma$   $P$   $P$  $R_1$  $R_{2}$   $\epsilon$   $\Omega$  $V_2$   $\bigvee R_1$   $\bigvee$  $V_1$   $\left| R_2 \right|$   $\left| R_2 \right|$  $V_2^2 R_1$   $V_2 \sqrt{R_1}$  $V_1^2 R_2$  2012  $V_1$   $R_2$  5.401  $V_2^2/R_2$   $V_2^2R_1$   $V_2\sqrt[R_1]{R_1}$  $V_1^2/R_1$  **1012**  $V_1^2R_2$  **2012**  $V_1$   $|R_2|$  $P_2$   $V_2^2/R_2$   $V_2^2R_1$  $P_1$  101<sub>2</sub>,  $V_1^2/R_1$  101<sub>2</sub>,  $V_1^2R_2$  $= 20 \log \frac{1}{\epsilon} \left[ \text{dB} \right] \qquad \leftarrow \text{If} \quad R_1 = R_2$  $= 10 \log \frac{11 + 11}{2} = 10 \log \frac{11 + 11}{2} = 20 \log \frac{11}{2} \left( \frac{11}{2} \right)$  [dB] 0 → 10 dB<br>
<sup>2</sup>/R<sub>1</sub></sub> = 10 log  $\frac{V_1^2 R_2}{V_2^2 R_1}$  = 20 log  $\frac{V_1}{V_2} \sqrt{\frac{R_2}{R_1}}$  [dB]<br>
[dB] ← If  $R_1 = R_2$ <br>  $\frac{2R_1}{V_2}$  = 10 log  $\frac{I_1^2}{I_2^2}$  = 20 log  $\frac{I_1}{I_2}$  [dB] ← If  $R_1 = I_2$ <br>
ad resistances 3,  $P_1/P_2 = 10 \rightarrow 10 \text{ dB}$ <br>  $\frac{1}{2} = 10 \log \frac{V_1^2 / R_1}{V_2^2 / R_2} = 10 \log \frac{V_1^2 R_2}{V_2^2 R_1} = 20 \log \frac{V_1}{V_2} \sqrt{\frac{R_2}{R_1}}$  [dB]<br>  $= 20 \log \frac{V_1}{V_2}$  [dB]  $\leftarrow$  If  $R_1 = R_2$ <br>  $\frac{P_1}{P_2} = 10 \log \frac{I_1^2 R_1}{I_2^2 R_2} = 10 \log \frac$  $2^2/R_1 = 10 \log \frac{V_1^2 R_2}{V_2^2 R_1} = 20 \log \frac{V_1}{V_2} \sqrt{\frac{R_2}{R_1}}$  [dB]<br>
[dB]  $\leftarrow$  If  $R_1 = R_2$ <br>  $\frac{R_1}{V_2} = 10 \log \frac{I_1^2}{I_2^2} = 20 \log \frac{I_1}{I_2}$  [dB]  $\leftarrow$  If  $R_1 = R_2$ <br>
ad resistances<br>
voltages across specific ports 3,  $P_1/P_2 = 10 \rightarrow 10$  dB<br>  $\frac{1}{V_2^2 + R_1} = 10 \log \frac{V_1^2 R_2}{V_2^2 R_2} = 20 \log \frac{V_1}{V_2} \sqrt{\frac{R_2}{R_1}}$  [dB]<br>  $= 20 \log \frac{V_1}{V_2}$  [dB]  $\leftarrow$  If  $R_1 = R_2$ <br>  $\frac{P_1}{P_2} = 10 \log \frac{I_1^2 R_1}{I_2^2 R_2} = 10 \log \frac{I_1^2}{I_2^2} = 20 \log \frac{I$ **11** of measurement equal to one tenth of a **bel (B)**<br>
10log  $\frac{P_1}{P_2}$ [dB]<br>
→ 3 dB,  $P_1/P_2 = 10 \rightarrow 10$  dB<br>  $10 \log \frac{P_1}{P_2} = 10 \log \frac{V_1^2/R_1}{V_2^2/R_1} = 10 \log \frac{V_1^2 R_2}{V_2^2 R_1} = 20 \log \frac{V_1}{V_2} \sqrt{\frac{R_2}{R_1}}$  [dB]<br>
= reasurement equal to one tenth of a **bel (B)**<br> **P**<sub>2</sub> (dB)<br> **P**<sub>3</sub>  $P_1/P_2 = 10 \rightarrow 10$  dB<br> **P**<sub>1</sub> = 10log  $\frac{V_1^2 / R_1}{V_2^2 / R_2} = 10 \log \frac{V_1^2 R_2}{V_2^2 R_1} = 20 \log \frac{V_1}{V_2} \sqrt{\frac{R_2}{R_1}}$  [dB]<br>
= 20log  $\frac{V_1}{V_2}$  [dB] surement equal to one tenth of a bel (B)<br>
HB]<br>  $P_1/P_2 = 10 \rightarrow 10 \text{ dB}$ <br>  $= 10 \log \frac{V_1^2/R_1}{V_2^2/R_2} = 10 \log \frac{V_1^2 R_2}{V_2^2 R_1} = 20 \log \frac{V_1}{V_2} \sqrt{\frac{R_2}{R_1}}$  [dB]<br>  $= 20 \log \frac{V_1}{V_2}$  [dB]  $\leftarrow$  If  $R_1 = R_2$ <br>  $= 10 \log \frac{I_$ n in decibel:  $10 \log \frac{P_1}{P_2} = 10 \log \frac{I_1^2 R_1}{I_2^2 R_2} = 10 \log \frac{I_1^2}{I_2^2} = 20 \log \frac{I_1}{I_2}$  [dB]  $\leftarrow$  H<br>where  $R_1, R_2$ : load resistances<br> $V_{1,2}, I_{1,2}$ : voltages across specific ports and cur<br>ratio of voltages acro n in decibel:  $10 \log \frac{P_1}{P_2} = 10 \log \frac{I_1^2 R_1}{I_2^2 R_2} = 10 \log \frac{I_1^2}{I_2^2} = 20 \log \frac{I_1}{I_2}$  [dB] ← If<br>where  $R_1$ ,  $R_2$ : load resistances<br> $V_{1, 2}, I_{1, 2}$ : voltages across specific ports and cur<br>ratio of voltages a Ex.]  $P_1/P_2 = 0.5 = 1/2 \rightarrow 3$  dB,  $P_1/P_2 = 10 \rightarrow 10$  dB<br>  $\frac{P_1}{P_2} = 10 \log \frac{V_1^2 / R_1}{V_2 / R_2} = 10 \log \frac{V_1^2 R_2}{V_2 / R_1} = 20 \log \frac{V_1}{V_2} \sqrt{\frac{R_2}{R_1}}$  [dB]<br>  $= 20 \log \frac{V_1}{V_2}$  [dB]  $\leftarrow$  If  $R_1 = R_2$ <br>  $\therefore$  Current gain  $_{1}/P_{2} = 10 \rightarrow 10$  dB<br>  $_{1}/P_{2} = 10 \rightarrow 10$  dB<br>  $_{1}/P_{2} = 10 \log \frac{V_{1}^{2}/R_{2}}{V_{2}^{2}/R_{1}} = 20 \log \frac{V_{1}}{V_{2}} \sqrt{\frac{R_{2}}{R_{1}}}$  [dB]<br>  $_{2}/P_{2} = 20 \log \frac{V_{1}}{V_{2}}$  [dB]  $\leftarrow$  If  $R_{1} = R_{2}$ <br>  $_{3}/P_{2} = 10 \log \frac{I_{1}^{2}R_{1}}{I_{2}^{2}R_{2$  $V_2 = 0.5 = 1/2 \rightarrow -3$  dB,  $P_1/P_2 = 10 \rightarrow 10$  dB<br>
in in decibel:  $10 \log \frac{P_1}{P_2} = 10 \log \frac{V_1^2/R_1}{V_2^2/R_2} = 10 \log \frac{V_1^2 R_2}{V_2^2 R_1} = 20 \log \frac{V_1}{V_2} \sqrt{\frac{R_2}{R_1}}$  [dB]<br>  $= 20 \log \frac{V_1}{V_2}$  [dB]  $\leftarrow$  If  $R_1 = R_2$ <br>
in in d *V*<sub>2</sub> = 0.5 = 1/2 → 3 dB,  $P_1/P_2 = 10 \rightarrow 10$  dB<br>
in in decibel:  $10 \log \frac{P_1}{P_2} = 10 \log \frac{V_1^2 / R_1}{V_2^2 / R_2} = 10 \log \frac{V_1^2 R_2}{V_2^2 R_1} = 20 \log \frac{V_1}{V_2} \sqrt{\frac{R_2}{R_1}}$  [dB]<br>  $= 20 \log \frac{V_1}{V_2}$  [dB]  $\leftarrow$  1f  $R_1 = R_2$ <br>
in  $e^{i\theta}P_2$  = 0.5 = 1/2 → 3 dB,  $P_1/P_2$  = 10 → 10 dB<br>
e gain in decibel:  $10 \log \frac{P_1}{P_2}$  =  $10 \log \frac{V_1^2/R_1}{V_2^2/R_2}$  =  $10 \log \frac{V_1^2 R_2}{V_2^2/R_1}$  =  $20 \log \frac{V_1}{V_2} \sqrt{\frac{R_2}{R_1}}$  [dB]<br>  $= 20 \log \frac{V_1}{V_2}$  (dB]  $\leftarrow$

$$
\text{neper} = \ln \frac{V_1}{V_2} = \ln \left[ \left( \frac{V_1}{V_2} \right)^2 \right]^{1/2} = \ln \left[ \left( \frac{V_1^2 / R}{V_2^2 / R} \right) \right]^{1/2} = \frac{1}{2} \ln \frac{P_1}{P_2} \quad \text{[Np]} \quad \text{Ex.} \quad 1 \text{ Np} = 10 \log e^2 = 8.686 \text{ dB}
$$

## **Decibel [dB] 3**

- § Absolute decibel units: **absolute units** of measurement equal to one tenth to specific value
- If we let  $P_2 = 1$  mW,  $P_1$  [dBm] =  $10 \log \frac{T_1 \text{ [mW]}}{1 \text{ mW}}$ Ex.]  $P_1 = 1$  mW  $\rightarrow 0$  dBm,  $P_1 = 1$  W  $\rightarrow 30$  dBm - If we let  $V_2 = 1$  mV,  $V_1 = 20 \log \frac{V_1}{1 \text{ mV}}$  [dBmV] Ex.]  $V_1 = 1$  mV  $\rightarrow$  0 dBmV,  $V_1 = 1$  V  $\rightarrow$  60 dBmV - If we let  $I_2 = 1 \mu A$ ,  $I_1 = 20 \log \frac{I_1}{1 \mu A} [\text{dB} \mu \text{A}]$ Ex.]  $I_1=1 \mu A \rightarrow 0 \text{ dB} \mu A$ ,  $I_1=1 \text{ mA} \rightarrow 60 \text{ dB} \mu A$  $1 \frac{1}{1}$  [mW W<sub>1</sub>  $\int$ **bsolute units** of measurement equal to one tenth to specific value<br>  $[dBm] = 10 log \frac{P_1}{1} [mW]$ <br>
m,  $P_1 = 1 W \rightarrow 30 dBm$ <br>  $= 20 log \frac{V_1}{1} [dBmV]$  $1 \text{ mW}$ *P*<sub>1</sub> [mW]  $P_1$  [dBm] = 10 log  $\frac{P_1 + P_2}{P_1 + P_2}$ **absolute units** of measurement equal to one tenth to specific value<br>  $P_1$  [dBm] = 10log  $\frac{P_1$  [mW]<br>
1Bm,  $P_1 = 1 \text{ W} \rightarrow 30 \text{ dBm}$ <br>  $Y_1 = 20 \log \frac{V_1}{1 \text{ mV}}$  [dBmV]<br>
BmV,  $V_1 = 1 \text{ V} \rightarrow 60 \text{ dBmV}$ <br>
= 20log  $\frac{I_1}{1 \mu$  $V_1 = 20 \log \frac{1}{1 + V}$  [dBmV] 3<br>
2: absolute units of measurement equal to one tenth to specific value<br>  $P_1$  [dBm] = 10log  $\frac{P_1$  [mW]<br>
dBm,  $P_1 = 1 \text{ W} \rightarrow 30 \text{ dBm}$ <br>  $V_1 = 20 \log \frac{V_1}{1 \text{ mV}}$  [dBmV]<br>
dBmV,  $V_1 = 1 \text{ V} \rightarrow 60 \text{ dBmV}$ <br>  $V_1 = 20 \log \frac{I$  $I_1 = 20 \log \frac{I_1}{I_1}$  [d  $\mu$ A  $= 20 \log \frac{1}{1}$  [dB $\mu$ A]

- § **Reflection coefficient plane**
- $\Gamma = |\Gamma| e^{j\theta}$  $\theta$ Smith Chart<br>
Reflection coefficient plane<br>  $\Gamma = |\Gamma|e^{j\theta}$ <br>
where  $0 \le |\Gamma| \le 1$ ,  $0^{\circ} \le \theta \le 360^{\circ}$  (or  $0 \le \theta \le 2\pi$ )<br>
Developed by P. Smith<br>
at Rell Telenhone Laboratories in 1939
	- Developed by P. Smith at Bell Telephone Laboratories in 1939
	- Very useful when solving transmission line problems
		- $\rightarrow$  Visualizing transmission line phenomenon
		- $\rightarrow$  Intuition about transmission line and impedance-matching problems
	- **Normalized** impedance (or admittance):  $z = Z / Z_0$  (or  $y = Y / Y_0$ )
	- $-Z_0$  (or  $Y_0$ ): arbitrary value

#### Normalized Impedance **or** Admittance Coordinates



9



 $\overline{\mathbf{x}}$ 

 $\overline{ax}$ a



**If** a lossless transmission line of characteristic impedance  $Z_0$  is terminated with a load impedance  $Z_L$ , dance  $Z_0$  is terminated with a load impedance  $Z_L$ ,<br> $f(Z_L)$ <br> $f(Z_L)$ <br> $\frac{1+\Gamma e^{-j2\beta l}}{2}Z_0$ ,

**Smith Chart**  
a lossless transmission line of characteristic impedance 
$$
Z_0
$$
 is terminated with a load impedance  $Z_L$ ,  

$$
\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(Z_L/Z_0) - 1}{(Z_L/Z_0) + 1} = \frac{z_L - 1}{z_L + 1} = \Gamma |e^{j\theta}, \leftarrow \Gamma = f(Z_L)
$$
  
where  $z_L = Z_L/Z_0$ : normalized load impedance
$$
\text{since } Z_{in} = \frac{V(-l)}{Z_L + Z_0} = \frac{V_0^+ [e^{j\beta l} + \Gamma e^{-j\beta l}]}{Z_L + \Gamma e^{-j\beta l} + \Gamma e^{-j\beta l} + \Gamma e^{-j\beta l}} = \frac{e^{j\beta l} + \Gamma e^{-j\beta l}}{(j\beta l - \Gamma) + (l\beta l - \Gamma)^{j\beta l}} Z_0 = \frac{1 + \Gamma e^{-j2\beta l}}{(l\beta l - \Gamma) + (l\beta l - \Gamma)^{j\beta l}} Z_0,
$$

where  $z_L = Z_L/Z_0$ : normalized load impedance

- Since , , ( / ) 1 1 *Z Z Z z* - - - <sup>q</sup> G = = = = G ¬ G = + + + 2 0 in 0 0 <sup>2</sup> 0 0 ( ) 1 [ ] ( ) [ ] / 1 *j l j l j l j l j l j l j l j l j l j l V l e e e V e e Z Z Z I l V e e Z e e e* <sup>b</sup> + - - - + - - - - + G + G + G = = = = - - G - G - G 2 in 0 0 <sup>0</sup> <sup>2</sup> 0 1 1 1 1 *j l j <sup>L</sup> j l r i <sup>l</sup> l <sup>e</sup> Z Z Z Z e j e* b q b - = - <sup>=</sup> + G + G = = = ¬ G = G = G + G - G - G ( ) 1 1 *L L L <sup>j</sup> z z g Z Z e* + G + G = = = ¬ = - G - G

$$
z_L = \frac{Z_L}{Z_0} = \frac{1+\Gamma}{1-\Gamma} = \frac{1+\Gamma|e^{j\theta}}{1-|\Gamma|e^{j\theta}} \quad \leftarrow z_L = g(Z_L)
$$

It a tossess that  
substationation line of characteristic impedance Σ<sub>0</sub> is terminated with a load impedance  

$$
\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(Z_L / Z_0) - 1}{(Z_L / Z_0) + 1} = \frac{z_L - 1}{z_L + 1} = |\Gamma|e^{i\theta}, \quad \leftarrow \Gamma = f(Z_L)
$$
  
where  $z_L = Z_L / Z_0$ ; normalized load impedance  

$$
- Since Z_{in} = \frac{V(-l)}{I(-l)} = \frac{V_0^+ [e^{i\beta l} + \Gamma e^{-j\beta l}]}{V_0^+ [e^{i\beta l} - \Gamma e^{-j\beta l}]} / Z_0 = \frac{e^{i\beta l} + \Gamma e^{-j\beta l}}{e^{i\beta l} - \Gamma e^{-j\beta l}} Z_0 = \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}} Z_0,
$$
  

$$
Z_{in}|_{z=0} = Z_L = \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}} Z_0|_{z=0} = \frac{1 + \Gamma}{1 - \Gamma} Z_0 \quad \leftarrow \Gamma = |\Gamma|e^{i\theta} = \Gamma_r + j\Gamma_i
$$
  

$$
z_L = \frac{Z_L}{Z_0} = \frac{1 + \Gamma}{1 - \Gamma} = \frac{1 + |\Gamma|e^{i\theta}}{1 - |\Gamma|e^{i\theta}} \quad \leftarrow z_L = g(Z_L)
$$
  

$$
- Let \Gamma = \Gamma_r + j\Gamma_i
$$
 and  $z_L = r_L + jx_L$ .  

$$
z_L = r_L + jx_L = \frac{1 + \Gamma}{1 - \Gamma} = \frac{(1 + \Gamma_r) + j\Gamma_i}{(1 - \Gamma_r) - j\Gamma_i} = \frac{\{(1 + \Gamma_r) + j\Gamma_i\}\{(1 - \Gamma_r) + j\Gamma_i\}}{(1 - \Gamma_r)^2 + \Gamma_i^2}
$$
  

$$
= \frac{(1 - \Gamma_r^2) - \Gamma_i^2 + j\Gamma_i(1 + \Gamma_r) + j\Gamma_i(1 - \Gamma_r)}{(1 - \Gamma_r)^2 + \Gamma_i^2}
$$

11



**Smith Chart**  
\n
$$
r_{L} = \frac{1 - \Gamma_{r}^{2} - \Gamma_{i}^{2}}{(1 - \Gamma_{r})^{2} + \Gamma_{i}^{2}}, \qquad x_{L} = \frac{2\Gamma_{i}}{(1 - \Gamma_{r})^{2} + \Gamma_{i}^{2}}
$$
\n(2  
\n**Carrangement of real part (or resistance)**  
\n
$$
r_{L} = \frac{1 - \Gamma_{r}^{2} - \Gamma_{i}^{2}}{1 - \Gamma_{r}^{2} - \Gamma_{i}^{2}}
$$







The Smith chart can also be graphical solution of the transmission line impedance equation in terms of the generalized reflection coefficient as  $\longrightarrow$  *I*<sub>in</sub>  $\stackrel{L}{\longrightarrow}$   $I_L$ 

where  $\Gamma$ : reflection coefficient at load *l*: (positive) length of transmission line from load - The normalized input impedance seen looking into a length *l* of transmission line terminated with  $z_L$  can be found by rotating the point **clockwisely** an amount  $2\beta l$  (subtracting  $2\beta l$  from  $\theta$ ) around the center of the chart.  $\rightarrow$  The same radius is maintained, since the magnitude of  $\Gamma$ does not change with position along the transmission line.  $2j\beta l$ **Smith Chart**<br>
Smith chart can also be graphical solu<br>
ralized reflection coefficient as<br>  $\lim_{\text{in}} = Z_0 \frac{1 + \Gamma e^{-2j\beta l}}{1 - \Gamma e^{-2j\beta l}} \leftarrow \Gamma = |\Gamma| e^{j\theta}$ <br>
where  $\Gamma$ : reflection coefficient at loa  $1+\Gamma e^{-2j\beta l}$   $\Gamma$   $|\Gamma|$  $1 - \Gamma e^{-2j\beta l}$ art<br>also be graphical solution of the t<br>n coefficient as<br> $\frac{d^{j\beta l}}{d^{j\beta l}} \leftarrow \Gamma = |\Gamma| e^{j\theta} \quad V_g \leftarrow$  $j\theta$   $V_a$  ( **art**<br>also be graphical solution of the t<br>n coefficient as<br> $\frac{d}{d\beta l}$   $\leftarrow \Gamma = |\Gamma| e^{j\theta} V_g \bigotimes$ <br>tion coefficient at load **Smith Chart**<br>
e Smith chart can also be graphical solution of the transmission line impedance equation in<br>
eralized reflection coefficient as<br>  $Z_{\text{in}} = Z_0 \frac{1 + \Gamma e^{-2i\beta l}}{1 - \Gamma e^{-2i\beta l}}$   $\leftarrow \Gamma = |\Gamma| e^{j\theta} V_{\text{st}} \bigoplus_{Z_{\text{in}}$  $e^{-2j\beta l}$  1  $\beta l$  $\theta$  V (  $\lambda$  $\beta l$   $\left| \begin{array}{ccc} & & \\ & & \end{array} \right|$  $-2j\beta l$ **mith Chart**<br>
mith chart can also be graphical solution of the transmission line impedance equation<br>
lized reflection coefficient as<br>  $= Z_0 \frac{1 + \Gamma e^{-2/\beta l}}{1 - \Gamma e^{-2/\beta l}}$   $\leftarrow \Gamma = |\Gamma| e^{j\theta} V_R \oplus \frac{V_{\text{in}}}{Z_{\text{in}}} \oplus \frac{V_{\text{in}}}{Z$ **Chart**<br>
art can also be graphical solution of the transmission line<br>
flection coefficient as<br>  $+\Gamma e^{-2j\beta l}$ <br>  $-\Gamma e^{-2j\beta l}$ <br>  $\leftarrow \Gamma = |\Gamma| e^{j\theta} V_g \right\}$ <br>  $\frac{Z_g}{Z_{in} \bigoplus_{j}^{V_{in}} Y_{in}}$ <br>  $\therefore$  reflection coefficient at load<br>  $\therefore$  $V_g(\mathcal{S})$  $Z \left/ \begin{array}{c} \frac{\dot{x}}{2} \end{array} \right| \begin{array}{c} \frac{\dot{x}}{2} \\ \frac{\dot{y}}{2} \end{array}$ *-l*  $Z_{\text{g}} \longleftarrow$  $Z_{\text{in}} \overset{V \text{ in}}{\Longrightarrow} Z_0, \beta \qquad V_L \overset{V_L}{=} Z_L$ *+ - +*  $\overrightarrow{V}_{in}$   $Z_0$ ,  $\beta$   $V_L$   $\overrightarrow{Z_L}$  $V_L$   $Z_L$ 

(⸪ lossless transmission line)







- Microwave *L* and *C* using Transmission Line
	- Equivalent inductor
	- Equivalent capacitor
- Transmission lines connection
	- Reflection
	- Transmission
- § Smith chart
	- Reflection coefficient plane
	- Impedance and/or admittance chart
	- Transmission calculation tools