Chapter 2
Transmission Line

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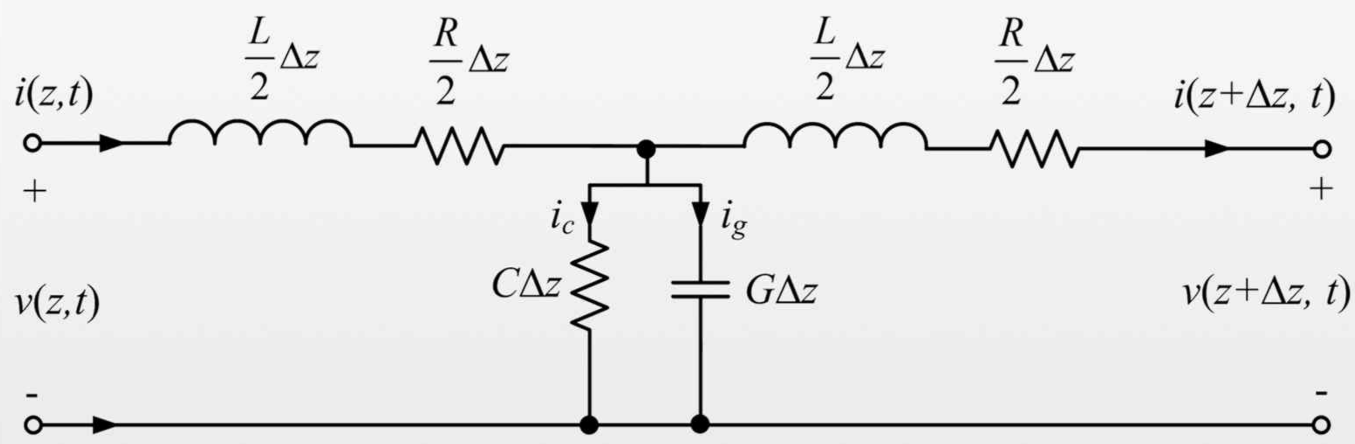


Learning Objectives

- Learn how to realize microwave *L* and *C* with transmission line
- Learn what will be done in transmission lines connection
- Understanding decibel units
- Learn utilities of Smith chart

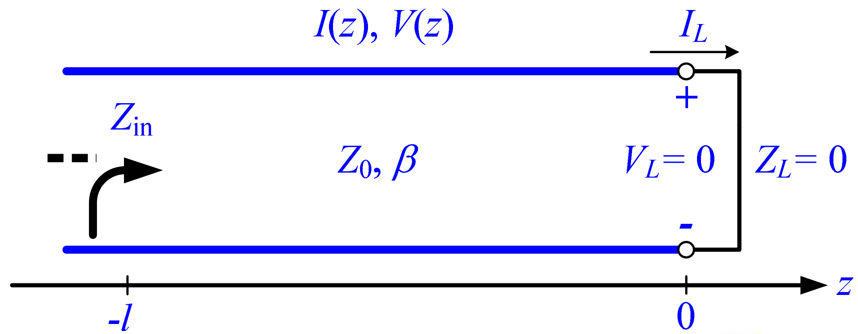
Learning contents

- Microwave *L* and *C* using Transmission Line
- Transmission Lines Connection
- Decibel
- Smith Chart

1 Microwave L and C using Transmission Line

- Transmission line circuit **terminated with** **short circuit ($Z_L = 0$)**

$$\Gamma = \left.\frac{Z_L - Z_0}{Z_L + Z_0}\right|_{Z_L=0} = -1 = \frac{V^+}{V^-}$$



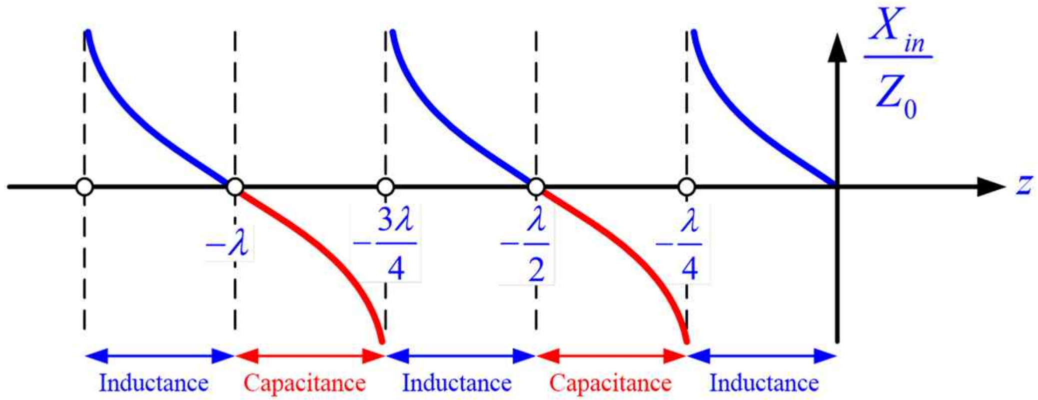
- At load, $V = 0$ and $I = \infty$

- Input impedance

$$Z_{\text{in}} = Z_0 \left.\frac{Z_L + jZ_0 \tan \beta\ell}{Z_0 + jZ_L \tan \beta\ell}\right|_{Z_L=0} = jZ_0 \tan \beta\ell = jX_{\text{in}}$$

➔ $-j\infty \le Z_{\text{in}} \le +j\infty$

$$\theta = \beta l = \frac{2\pi}{\lambda} l,\ 0 < \theta < \frac{\pi}{2} \Leftrightarrow 0 < \frac{2\pi}{\lambda} l < \frac{\pi}{2} \Leftrightarrow 0 < l < \frac{\pi}{2}\frac{\lambda}{2\pi} = \frac{\lambda}{4}$$



- Repeated inductive & capacitive characteristics along transmission line

- Different reactance characteristics according to frequency (or wavelength)

- Repeated reactance characteristics according to harmonics (f_0, $3f_0$, ⋯, $(2n+1) f_0$)

1 Microwave *L* and *C* using Transmission Line

- Transmission line circuit **terminated with** **open circuit ($Z_L = \infty$)**

$$\Gamma = \left. \frac{Z_L - Z_0}{Z_L + Z_0} \right|_{Z_L=\infty} = 1 = \frac{V^+}{V^-}$$

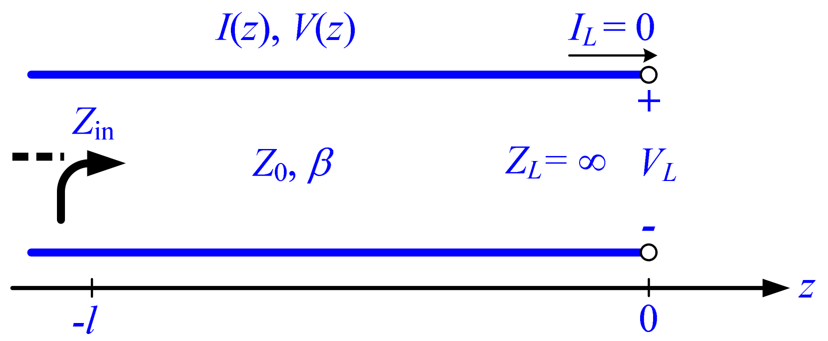
- At load, $I = 0$ and $V = \infty$

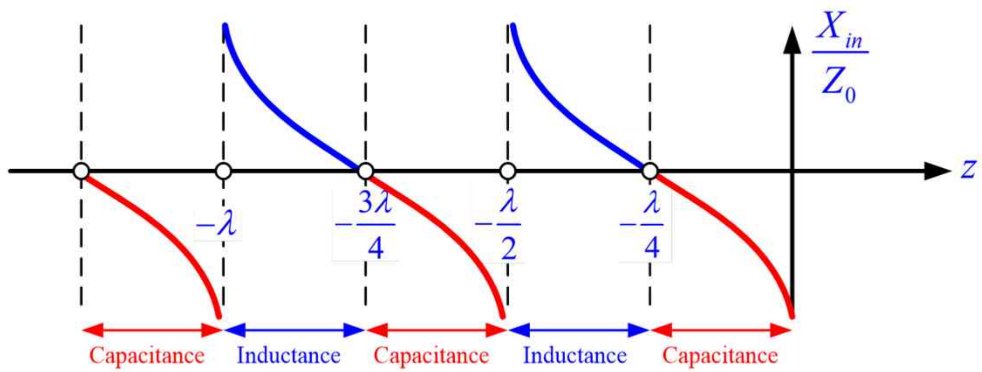
- Input impedance

$$Z_{\text{in}} = Z_0 \left. \frac{Z_L + jZ_0 \tan \beta\ell}{Z_0 + jZ_L \tan \beta\ell} \right|_{Z_L=\infty} = -jZ_0 \cot \beta\ell$$

➔ $-j\infty \le Z_{\text{in}} \le +j\infty$

$$\theta = \beta l = \frac{2\pi}{\lambda} l,\ 0 < \theta < \frac{\pi}{2} \Leftrightarrow 0 < \frac{2\pi}{\lambda} l < \frac{\pi}{2} \Leftrightarrow 0 < l < \frac{\pi}{2}\frac{\lambda}{2\pi} = \frac{\lambda}{4}$$





- Same electrical characteristics as like transmission line terminated with short circuit

- ***Microwave inductor (L) and capacitor (C) can be realized by using transmission line circuit terminated with short or open circuits!!!***

2 Transmission Lines Connection

- Input impedance in case of $\boldsymbol{l} = \lambda/2$ **transmission line terminated with load** $(\boldsymbol{Z_L})$

$$Z_{\text{in}} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}\bigg|_{l=\lambda/2} = Z_0 \frac{Z_L + jZ_0 \tan \dfrac{2\pi}{\lambda}\dfrac{\lambda}{2}}{Z_0 + jZ_L \tan \dfrac{2\pi}{\lambda}\dfrac{\lambda}{2}} = Z_L \quad \Leftrightarrow \quad Z_{\text{in}} = Z_L$$

- Input impedance in case of $\boldsymbol{l} = \lambda/4 + n\lambda/2$ **transmission line terminated with load** $(\boldsymbol{Z_L})$

$$Z_{\text{in}} = Z_0 \frac{Z_L + jZ_0 \tan \beta \ell}{Z_0 + jZ_L \tan \beta \ell}\bigg|_{\ell=\lambda/4} = Z_0 \frac{Z_L + jZ_0 \tan \dfrac{2\pi}{\lambda}\dfrac{\lambda}{4}}{Z_0 + jZ_L \tan \dfrac{2\pi}{\lambda}\dfrac{\lambda}{4}} = \frac{Z_0^2}{Z_L}$$

: **Quarter-wave (length) transformer**

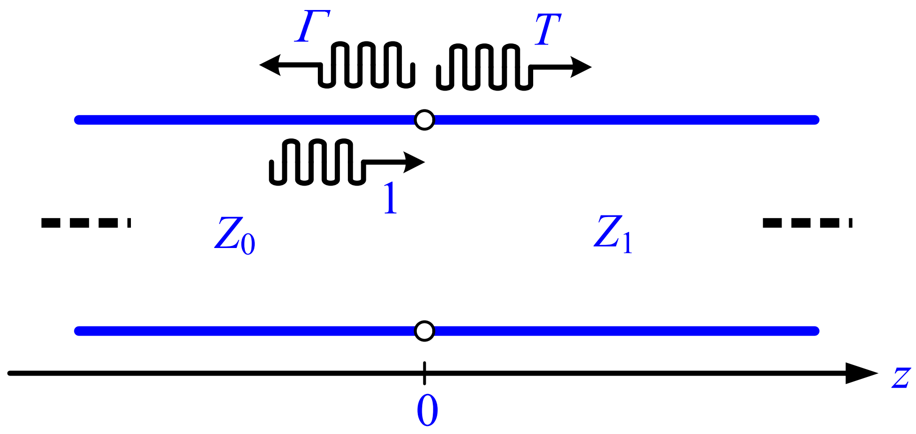
2 Transmission Lines Connection

- Transmission line of characteristic impedance Z_0 feeding different characteristic impedance (Z_1) transmission line

- Reflection coefficient

$$\Gamma = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

➔ Some portion of the incident wave is reflected and the remained is transmitted into the second line.



- Voltage waves on each transmission lines

$$V(z) = V_0^+ (e^{-j\beta z} + \Gamma e^{j\beta z}) \qquad @\, z < 0$$

$$V(z) = V_0^+ T e^{-j\beta z} \qquad @\, z > 0$$

- Continuity of transmission coefficient at $z = 0$:

$$T = 1 + \Gamma = 1 + \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{2Z_1}{Z_1 + Z_0}$$

- Insertion loss (IL): $\mathrm{IL} = -20\log|T|\ \mathrm{dB}$

3 Decibel [dB]

- Decibel (dB): **relative unit** of measurement equal to one tenth of a **bel** (**B**)

- Power gain in decibel $= 10\log\frac{P_1}{P_2}$[dB]

Ex.] $P_1/P_2 = 0.5 = 1/2 \rightarrow$ -3 dB, $P_1/P_2 = 10 \rightarrow$ 10 dB

- Voltage gain in decibel: $10\log\frac{P_1}{P_2} = 10\log\frac{V_1^{\,2}/R_1}{V_2^{\,2}/R_2} = 10\log\frac{V_1^{\,2}R_2}{V_2^{\,2}R_1} = 20\log\frac{V_1}{V_2}\sqrt{\frac{R_2}{R_1}}$ [dB]

$= 20\log\frac{V_1}{V_2}$ [dB] $\leftarrow$ If $R_1 = R_2$

- Current gain in decibel: $10\log\frac{P_1}{P_2} = 10\log\frac{I_1^{\,2}R_1}{I_2^{\,2}R_2} = 10\log\frac{I_1^{\,2}}{I_2^{\,2}} = 20\log\frac{I_1}{I_2}$ [dB] $\leftarrow$ If $R_1 = R_2$

where R_1, R_2: load resistances

$V_{1,2}$, $I_{1,2}$: voltages across specific ports and currents passing specific nodes

- Neper [Np]: ratio of voltages across equal load resistances

$$\text{neper} = \ln\frac{V_1}{V_2} = \ln\left[\left(\frac{V_1}{V_2}\right)^2\right]^{1/2} = \ln\left[\left(\frac{V_1^2/R}{V_2^2/R}\right)\right]^{1/2} = \frac{1}{2}\ln\frac{P_1}{P_2} \text{ [Np]}$$

Ex.] $1\text{ Np} = 10\log e^2 = 8.686$ dB

3 Decibel [dB]

- Absolute decibel units: **absolute units** of measurement equal to one tenth to specific value

- If we let $P_2 = 1$ mW, $P_1\,[\text{dBm}] = 10\log\dfrac{P_1\,[\text{mW}]}{1\ \text{mW}}$

  Ex.] $P_1 = 1$ mW → 0 dBm, $P_1 = 1$ W → 30 dBm

- If we let $V_2 = 1$ mV, $V_1 = 20\log\dfrac{V_1}{1\ \text{mV}}$ [dBmV]

  Ex.] $V_1 = 1$ mV → 0 dBmV, $V_1 = 1$ V → 60 dBmV

- If we let $I_2 = 1$ μA, $I_1 = 20\log\dfrac{I_1}{1\ \mu\text{A}}$ [dBμA]

  Ex.] $I_1 = 1$ μA → 0 dBμA, $I_1 = 1$ mA → 60 dBμA

4 Smith Chart

- **Reflection coefficient plane**

  - $\Gamma = |\Gamma| e^{j\theta}$

    where $0 \le |\Gamma| \le 1,\ 0^{\circ} \le \theta \le 360^{\circ}$ (or $0 \le \theta \le 2\pi$)

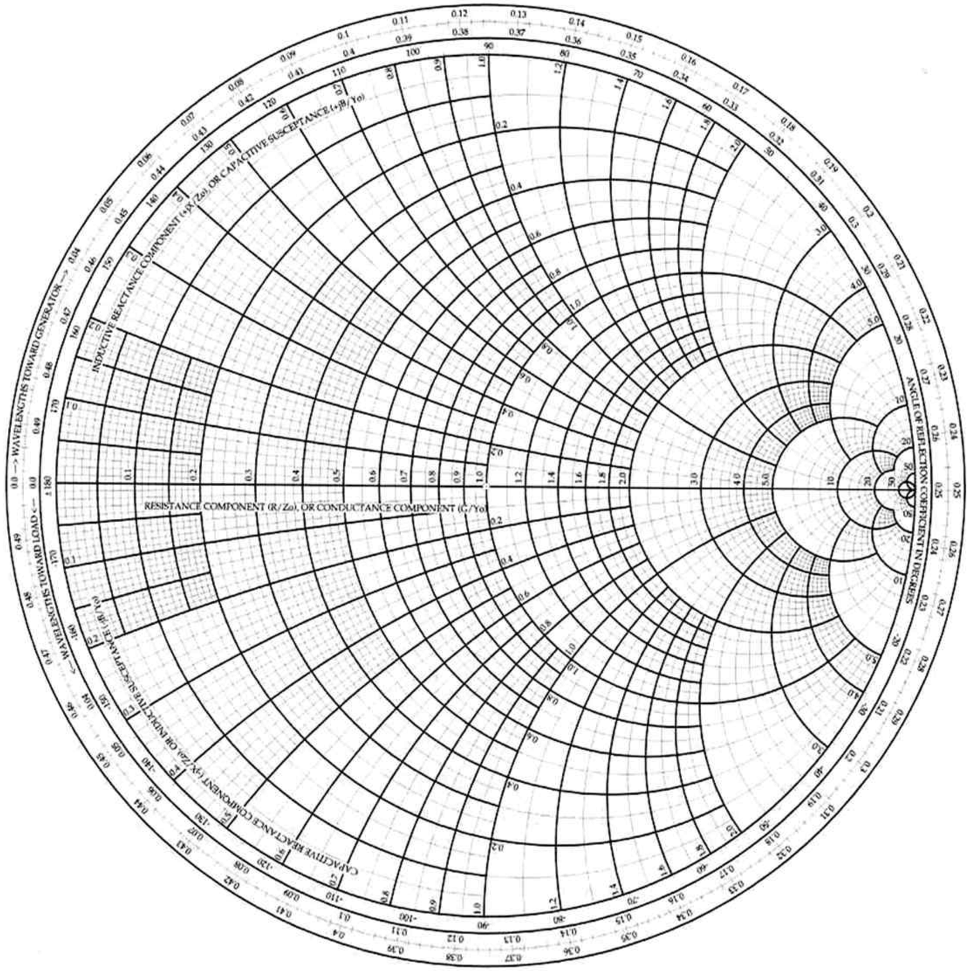
  - Developed by P. Smith
    at Bell Telephone Laboratories in 1939

  - Very useful when solving transmission line problems
    → Visualizing transmission line phenomenon
    → Intuition about transmission line and impedance-matching problems

  - **Normalized** impedance (or admittance):
    $z = Z / Z_0$ (or $y = Y / Y_0$)

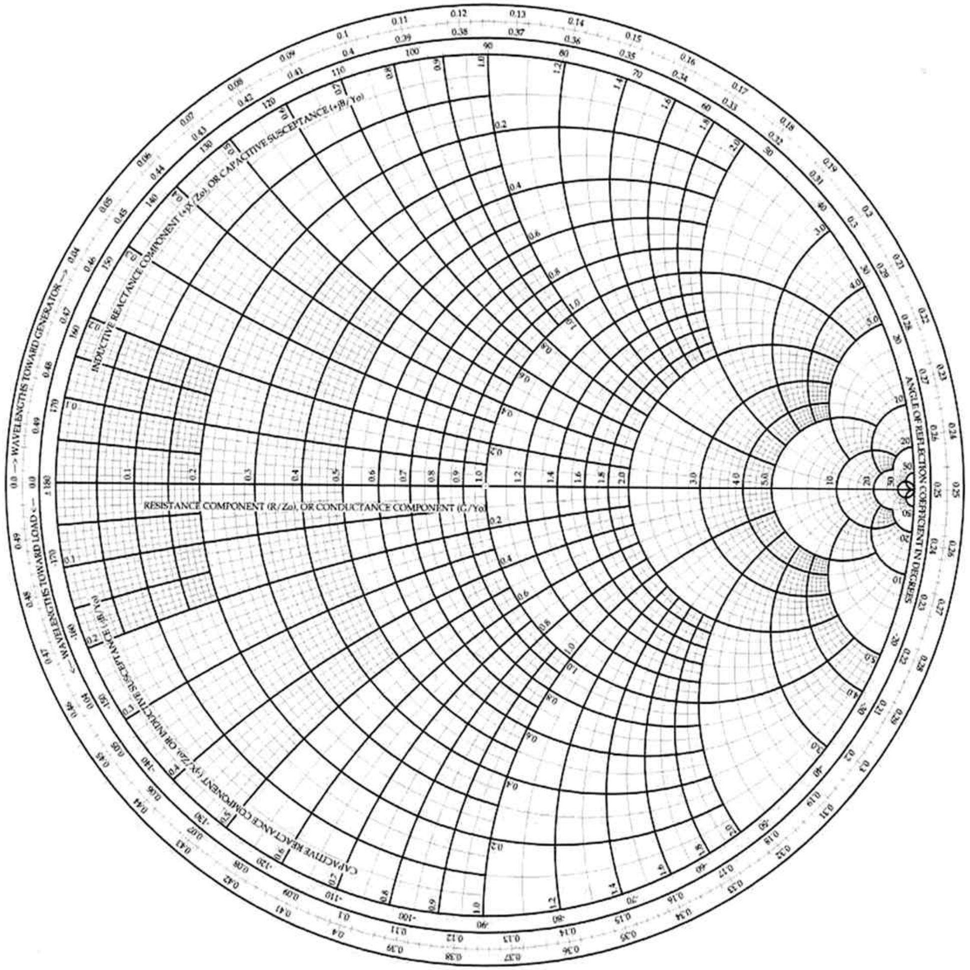
  - Z_0 (or Y_0): arbitrary value

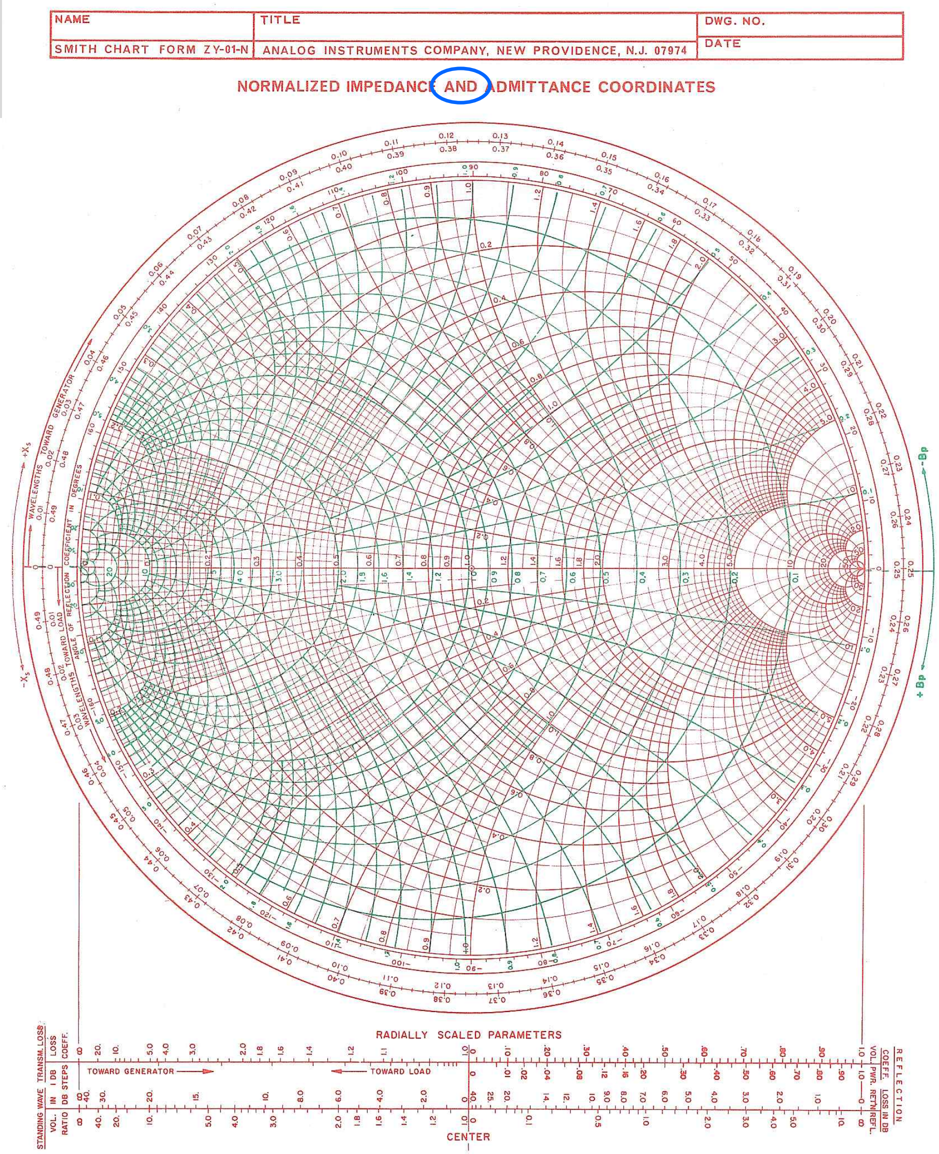
Normalized Impedance **or** Admittance Coordinates



4 Smith Chart

Normalized Impedance **or** Admittance Coordinates





4 Smith Chart

- If a lossless transmission line of characteristic impedance Z_0 is terminated with a load impedance Z_L,

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(Z_L / Z_0) - 1}{(Z_L / Z_0) + 1} = \frac{z_L - 1}{z_L + 1} = |\Gamma| e^{j\theta}, \quad \leftarrow \Gamma = f(Z_L)$$

where $z_L = Z_L/Z_0$: normalized load impedance

- Since $Z_{\text{in}} = \dfrac{V(-l)}{I(-l)} = \dfrac{V_0^+[e^{j\beta l} + \Gamma e^{-j\beta l}]}{V_0^+[e^{j\beta l} - \Gamma e^{-j\beta l}] / Z_0} = \dfrac{e^{j\beta l} + \Gamma e^{-j\beta l}}{e^{j\beta l} - \Gamma e^{-j\beta l}} Z_0 = \dfrac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}} Z_0$,

$$Z_{\text{in}}\big|_{l=0} = Z_L = \left.\frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}} Z_0 \right|_{l=0} = \frac{1+\Gamma}{1-\Gamma} Z_0 \quad \leftarrow \Gamma = |\Gamma| e^{j\theta} = \Gamma_r + j\Gamma_i$$

$$z_L = \frac{Z_L}{Z_0} = \frac{1+\Gamma}{1-\Gamma} = \frac{1 + |\Gamma| e^{j\theta}}{1 - |\Gamma| e^{j\theta}} \quad \leftarrow z_L = g(Z_L)$$

- Let $\Gamma = \Gamma_r + j\Gamma_i$ and $z_L = r_L + jx_L$.

$$z_L = r_L + jx_L = \frac{1+\Gamma}{1-\Gamma} = \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i} = \frac{\{(1+\Gamma_r) + j\Gamma_i\}\{(1-\Gamma_r) + j\Gamma_i\}}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$= \frac{(1-\Gamma_r^2) - \Gamma_i^2 + j\Gamma_i(1+\Gamma_r) + j\Gamma_i(1-\Gamma_r)}{(1-\Gamma_r)^2 + \Gamma_i^2} = \frac{(1-\Gamma_r^2 - \Gamma_i^2) + j2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

4 Smith Chart

- Real and imaginary parts:

$$r_L = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2}, \qquad x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2+\Gamma_i^2}$$

- Rearrangement of real part (or resistance)

$$r_L = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2}$$

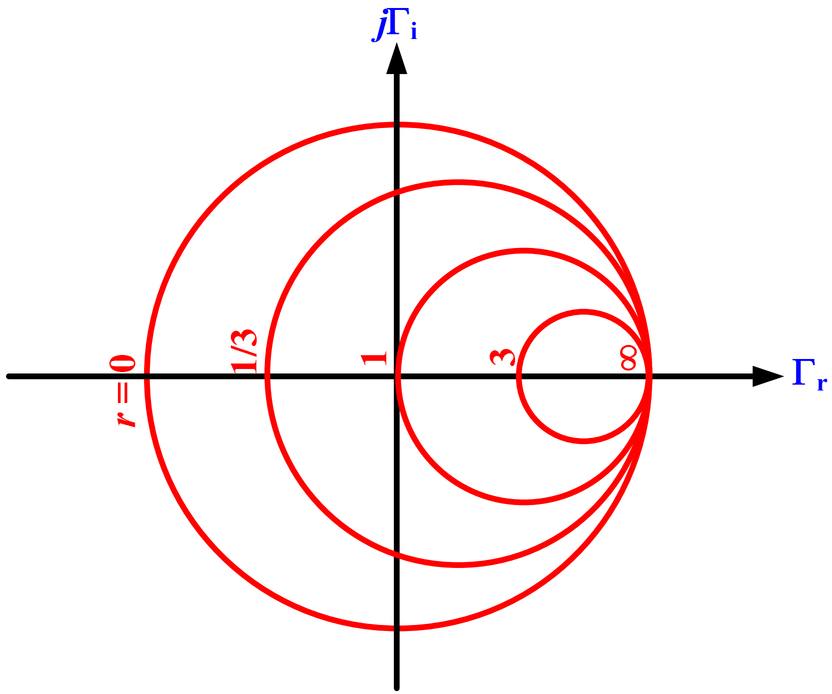
$$r_L(1-\Gamma_r)^2 + r_L\Gamma_i^2 = 1-\Gamma_r^2-\Gamma_i^2$$

$$\Gamma_r^2(r_L+1) - 2r_L\Gamma_r + (r_L+1)\Gamma_i^2 = 1-r_L$$

$$\Gamma_r^2 - \frac{2r_L}{r_L+1}\Gamma_r + \Gamma_i^2 = \frac{1-r_L}{1+r_L} \leftarrow x^2-2ax+y^2=b$$

$$\left(\Gamma_r - \frac{r_L}{r_L+1}\right)^2 + \Gamma_i^2 = \frac{1-r_L}{1+r_L} + \left(\frac{r_L}{r_L+1}\right)^2 = \frac{1}{(1+r_L)^2}$$

- Resistance circles- Center: $\left(\frac{r_L}{r_L+1},\ 0\right)$ Radius: $\frac{1}{1+r_L}$



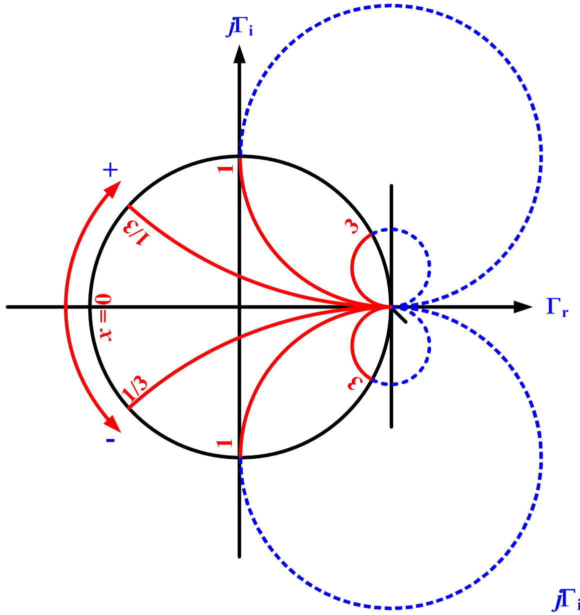
4 Smith Chart

- Rearrangement of imaginary part (or reactance)

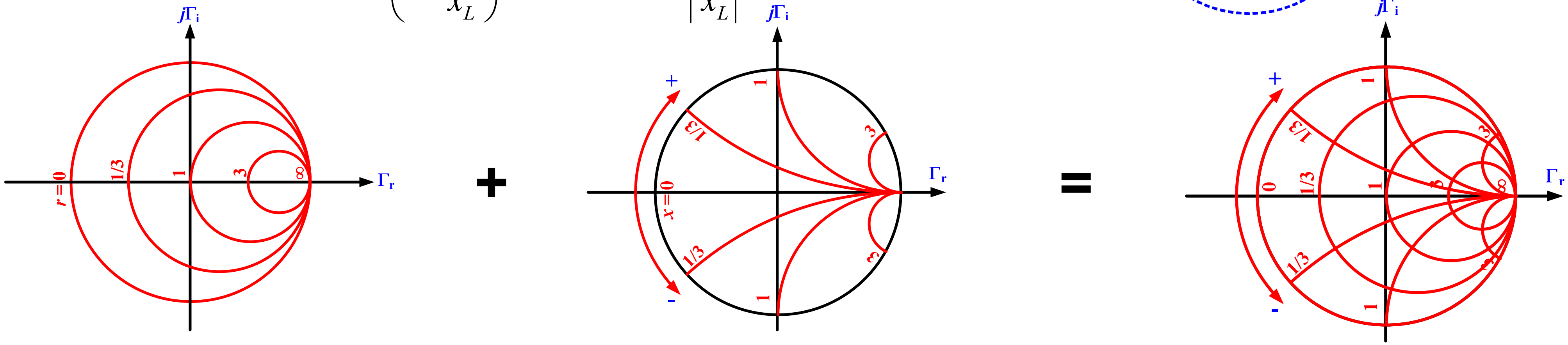
$$x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$(1-\Gamma_r)^2 + \Gamma_i^2 = \frac{2\Gamma_i}{x_L}, \qquad (1-\Gamma_r)^2 + \Gamma_i^2 - \frac{2\Gamma_i}{x_L} = 0$$

$$(\Gamma_r - 1)^2 + \Gamma_i^2 - \frac{2\Gamma_i}{x_L} + \left(\frac{1}{x_L}\right)^2 = (\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$



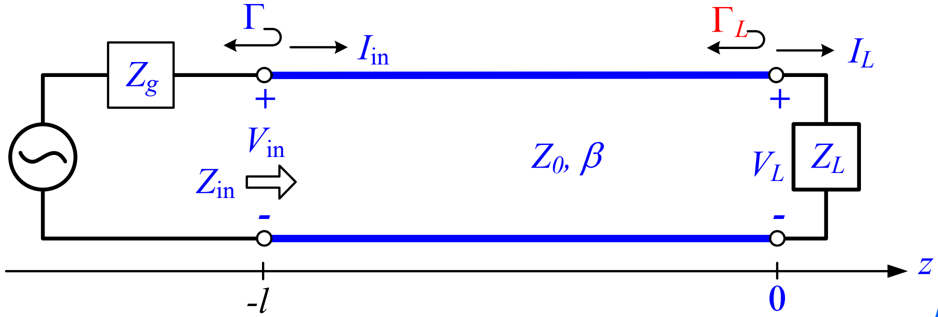
- Reactance circles- Center: $\left(1, \frac{1}{x_L}\right)$ Radius: $\left|\frac{1}{x_L}\right|$



4 Smith Chart

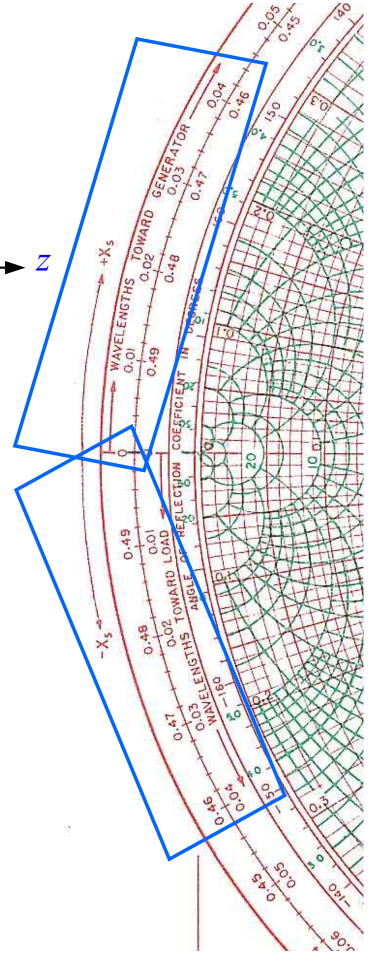
- The Smith chart can also be graphical solution of the transmission line impedance equation in terms of the generalized reflection coefficient as

$$Z_{in} = Z_0 \frac{1+\Gamma e^{-2j\beta l}}{1-\Gamma e^{-2j\beta l}} \qquad \leftarrow \Gamma = |\Gamma| e^{j\theta}$$



where Γ: reflection coefficient at load

l: (positive) length of transmission line from load

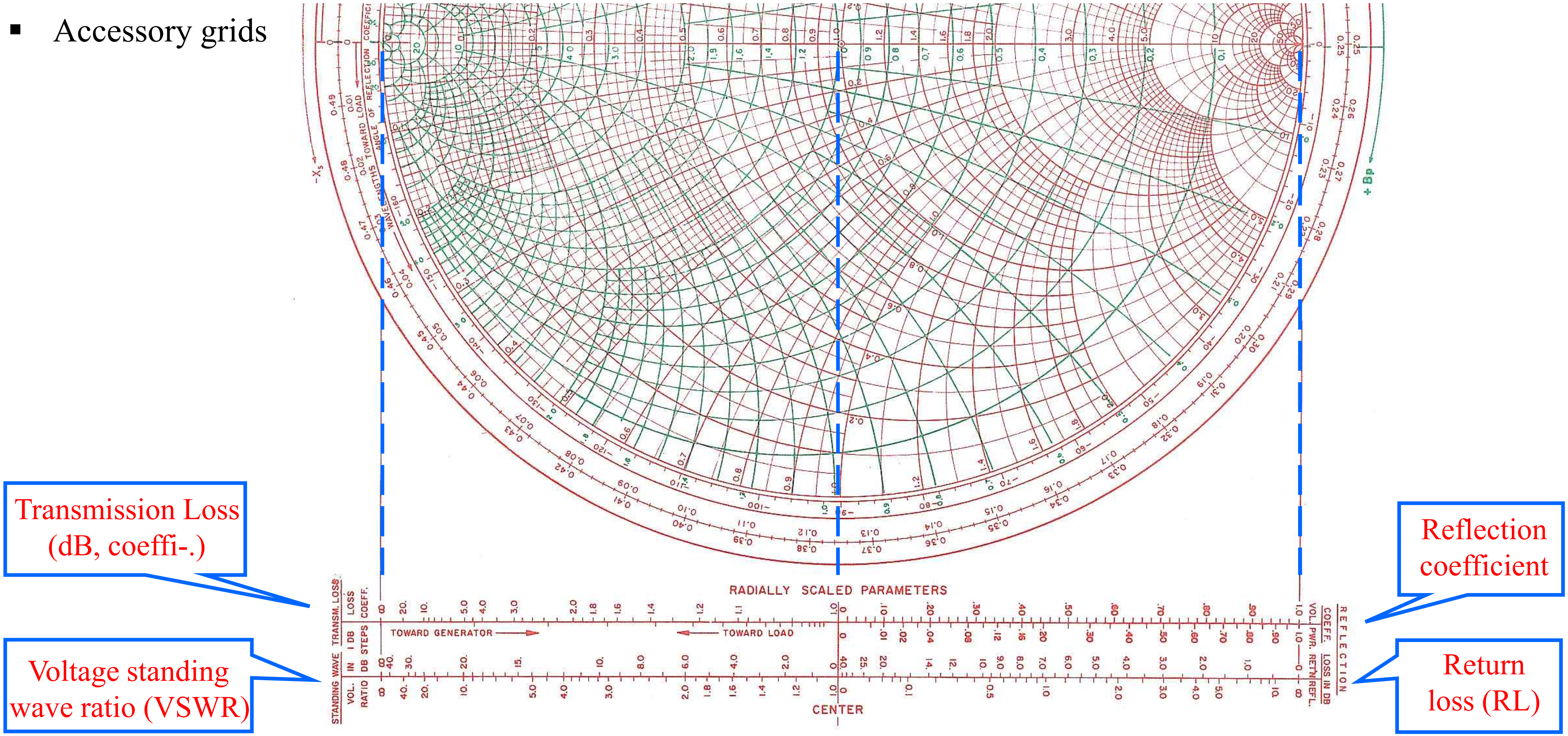


- The normalized input impedance seen looking into a length l of transmission line terminated with z_L can be found by rotating the point **clockwisely** an amount $\mathbf{2\beta l}$ (subtracting $2\beta l$ from θ) around the center of the chart.

➔The same radius is maintained, since the magnitude of Γ does not change with position along the transmission line. (∵ lossless transmission line)

4 Smith Chart

- Accessory grids



5 Review

- Microwave *L* and *C* using Transmission Line
  - Equivalent inductor
  - Equivalent capacitor
- Transmission lines connection
  - Reflection
  - Transmission
- Smith chart
  - Reflection coefficient plane
  - Impedance and/or admittance chart
  - Transmission calculation tools