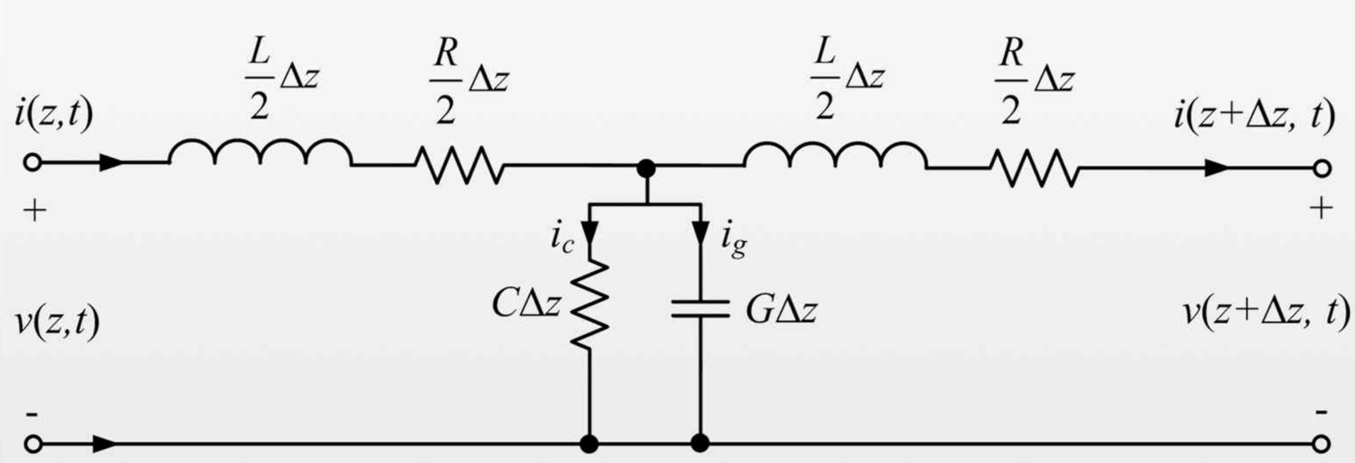
Chapter 2
Transmission Line

Prof. Jeong, Yongchae



Learning Objectives

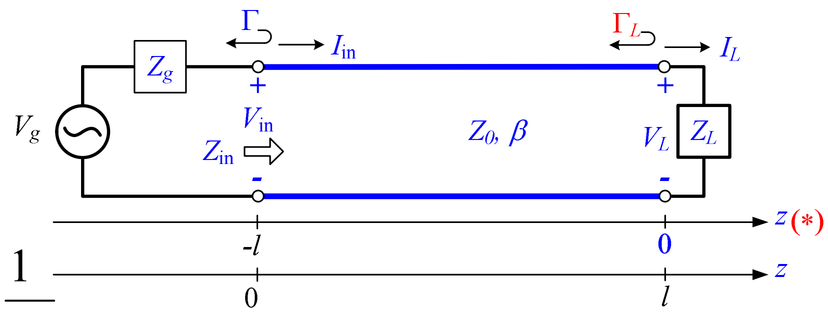
- Understanding combined impedance and admittance Smith chart
- Exercise Smith chart examples
- Learn traveling (or standing) wave on transmission line by using slotted transmission line

Learning contents

- Combined Impedance and Admittance Smith Chart
- Smith Chart Examples
- Slotted Transmission Line

1 Combined Impedance and Admittance Smith Chart

- Input impedance of a load z_L connected to a $\lambda/4$ line:



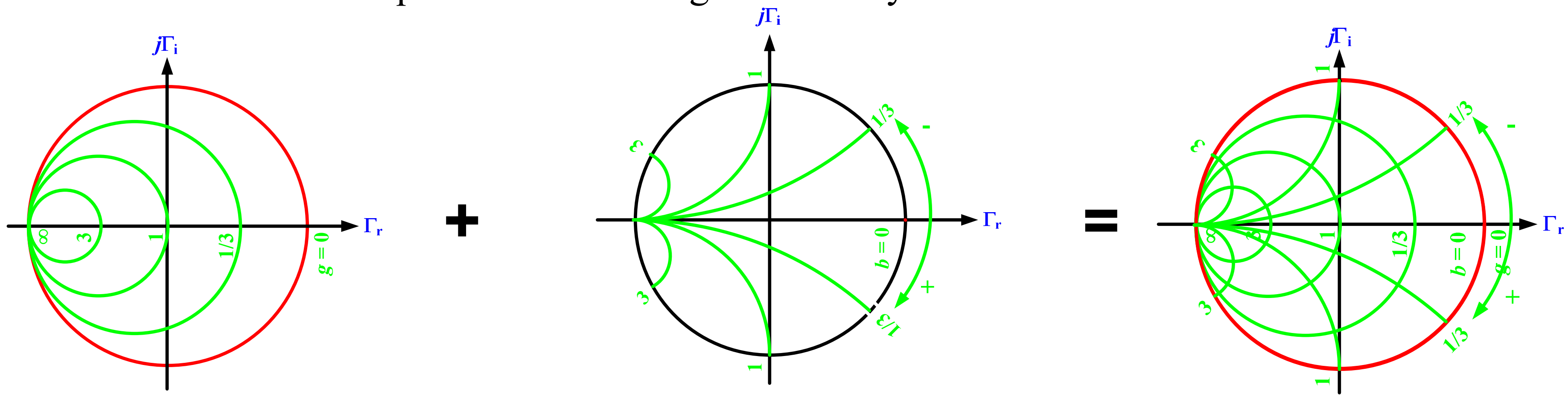
$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta \ell}{Z_0 + jZ_L \tan \beta \ell}$$

$$z_{in}\Big|_{\ell=\lambda/4} = \left.\frac{Z_L + jZ_0 \tan \beta \ell}{Z_0 + jZ_L \tan \beta \ell}\right|_{\ell=\lambda/4} = \frac{Z_L + jZ_0 \tan(2\pi/\lambda \cdot \lambda/4)}{Z_0 + jZ_L \tan(2\pi/\lambda \cdot \lambda/4)} = \frac{Z_0}{Z_L} = \frac{1}{z_L}$$

$$= y_L = g_L + jb_L$$

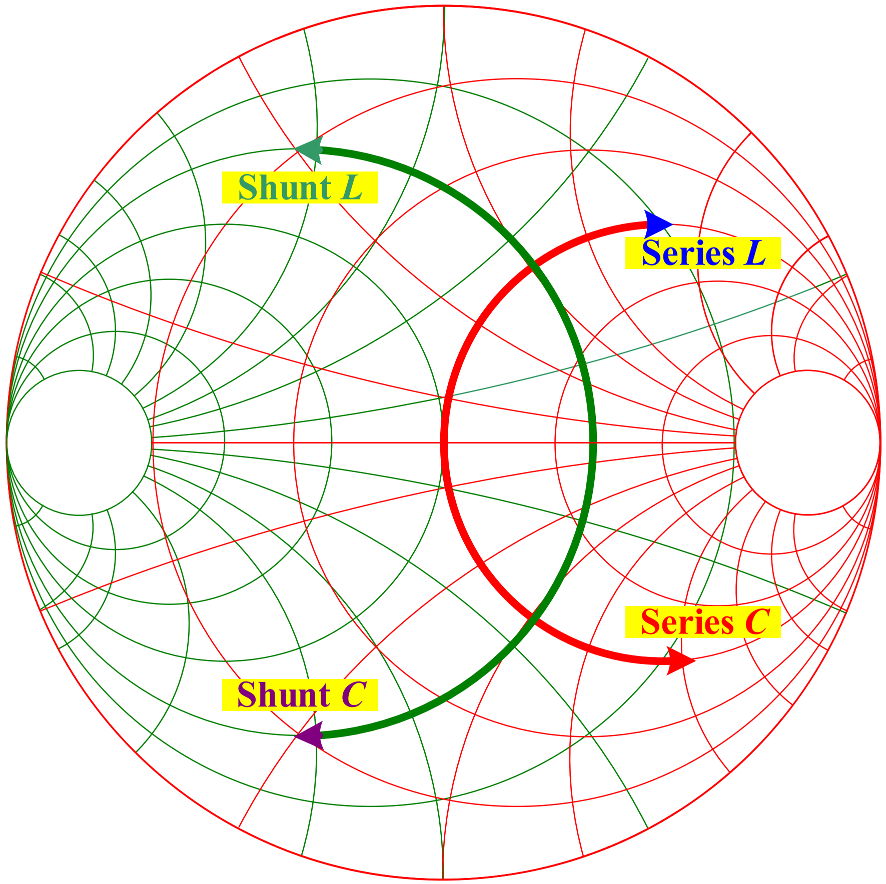
➔ Converting a normalized impedance to a normalized admittance.

➔A $\lambda/4$ transformation is equivalent to rotating the chart by 180º.



1 Combined Impedance and Admittance Smith Chart

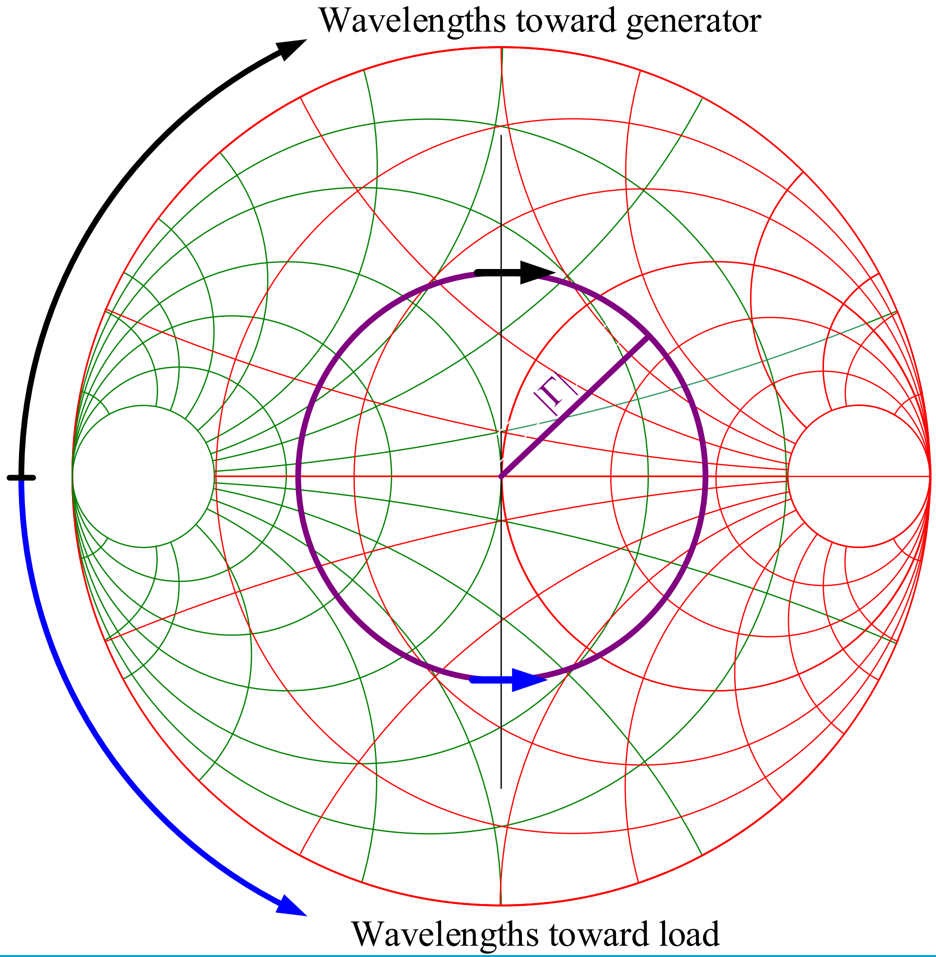
- Impedance trace movement of lumped elements on combined impedance and admittance Smith chart



1 Combined Impedance and Admittance Smith Chart

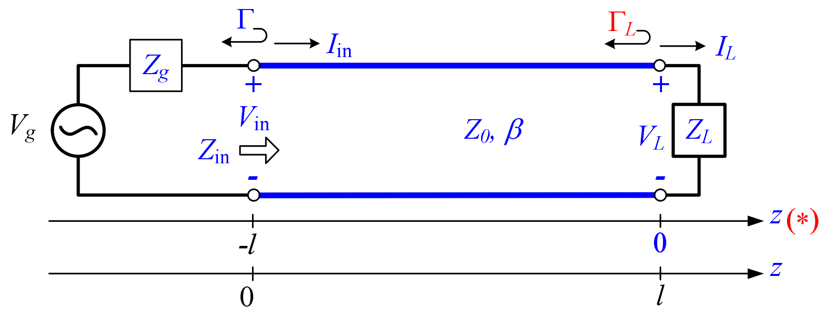
- Impedance trace movement of transmission line on combined impedance and admittance Smith chart

  - Constant reflection coefficient magnitude



2 Smith Chart Example (1)

- Transmission line (Z_0 = 50 [Ω]) circuit terminated with Z_L = 130 + j90[Ω] and line length of $l = 0.3\lambda$
  Find Γ @load, Γ @line input node, Z_{in}, (V)SWR @line input node, and the return loss.



1) Theoretical solutions

- Normalized load impedance: $z_L = \dfrac{Z_L}{Z_0} = 2.60 + j1.8\ [\Omega]$

- Reflection coefficient magnitude at the load:

$$\Gamma_L = \left.\frac{Z_L - Z_0}{Z_L + Z_0}\right|_{z_L} = \left.\frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1}\right|_{z_L} = \left.\frac{z_L - 1}{z_L + 1}\right|_{z_L = 2.6 + j1.8} = \frac{(2.6 + j1.8) - 1}{(2.6 + j1.8) + 1} = \frac{1.6 + j1.8}{3.6 + j1.8} = \frac{(1.6 + j1.8)(3.6 - j1.8)}{(3.6)^2 + (1.8)^2}$$

$$= 0.556 + j0.222 = 0.6\angle 21.8^\circ \qquad \Rightarrow \qquad |\Gamma_L| = 0.6$$

- Γ @ line input node: $\Gamma_{in} = \Gamma_L e^{-j2\beta l}\Big|_{l=0.3\lambda} = \Gamma_L e^{-j2\frac{2\pi}{\lambda}\cdot 0.3\lambda} = \Gamma_L e^{-j1.2\pi} \overset{or}{=} \Gamma_L e^{-j144^\circ} = 0.6\angle -122.2^\circ$

- Input impedance (Z_{in})

$$Z_{in} = Z_0 \left.\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right|_{l=0.3\lambda} = 50\frac{Z_L + jZ_0 \tan\frac{2\pi}{\lambda}\cdot 0.3\lambda}{Z_0 + jZ_L \tan\frac{2\pi}{\lambda}\cdot 0.3\lambda} = 50\frac{(130 + j90) + j50\tan 0.6\pi}{50 + j(130 + j90)\tan 0.6\pi} = 12.75 + j5.83$$

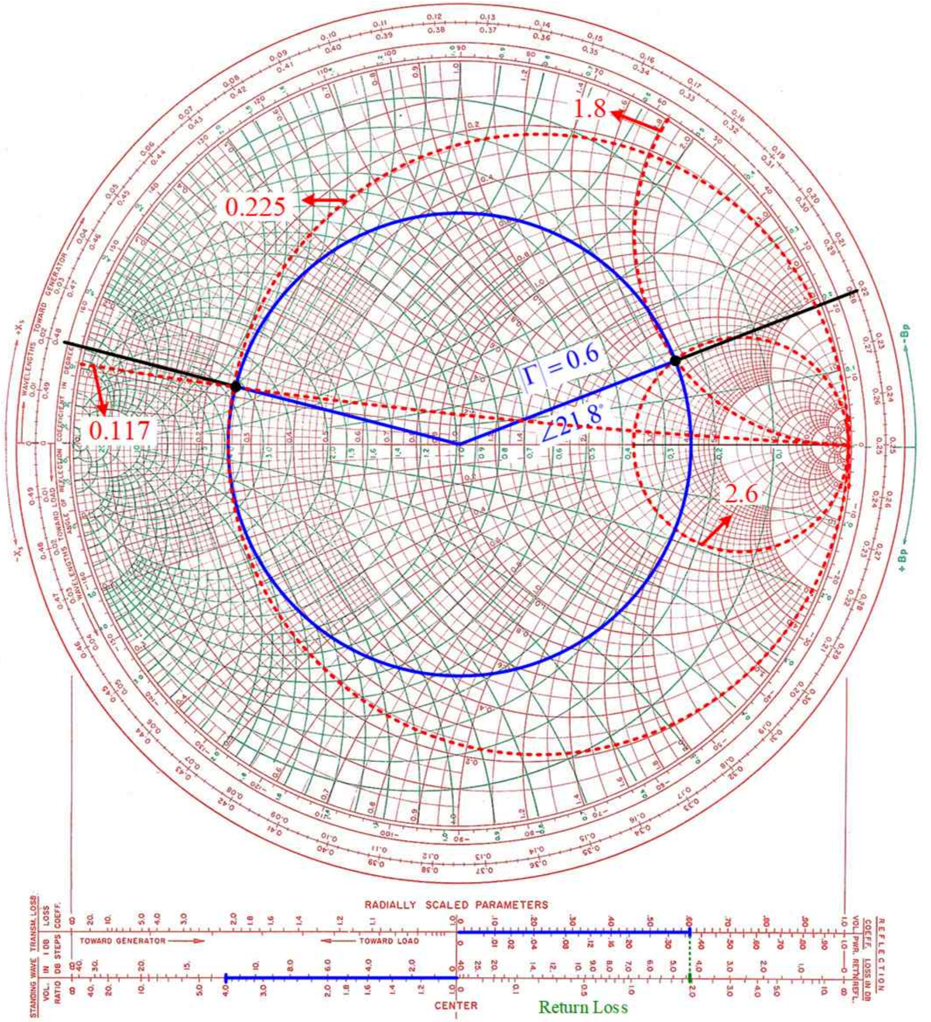
2 Smith Chart Example (1)

- SWR @line input node: $\text{SWR}=\frac{1+|\Gamma|}{1-|\Gamma|}=\frac{1+0.6}{1-0.6}=\frac{1.6}{0.4}=4$

- Return loss: $\text{RL}=-20\log|\Gamma|=-20\log(0.6)=-4.4\text{dB}$

2) Smith chart solutions

- Normalized load impedance: $z_L = 2.60 + j1.8$ [Ω]
- Reflection coefficient magnitude at load: $\Gamma = 0.6\angle 21.8^{o}$
- $|\Gamma| = 0.60$
- SWR = 3.98
- RL = 4.4 dB
- Reading reference position of load: 0.22λ
- Moving down 0.3λ of line according to wavelengths-toward -generator (WTG): 0.22λ+0.3λ=0.52λ=0.02λ
- New normalized input impedance at intersection with SWR circle: $z_{\text{in}} = 0.255 + j0.117$ ➔ $Z_{\text{in}} = Z_0 z_{\text{in}} = 12.7 + j5.8$ [Ω]



2 Smith Chart Example (2)

- Transmission line (Z_0 = 50 [Ω]) circuit terminated with $Z_L = 100 + j50$[Ω] and line length of $l = 0.15\lambda$
  Find load admittance (Y_L) and Z_{in}.

1) Theoretical solutions

- Normalized load impedance: $z_L = 2 + j1$ [Ω]

- Normalized load admittance: $y_L = \dfrac{1}{z_L} = \dfrac{1}{2+j1} = \dfrac{2-j1}{2^2+1^2} = 0.4 - 0.2$

- Load admittance: $Y_L = y_L Y_0 = \dfrac{y_L}{Z_0} = \dfrac{0.4 - j0.2}{50} = 0.008 - j0.004[\text{S}]$

- Input impedance (Z_{in}): $Z_{\text{in}} = Z_0 \dfrac{Z_L + jZ_0 \tan\theta}{Z_0 + jZ_L \tan\theta}\bigg|_{l=0.15\lambda} = 50 \dfrac{(100+j50) + j50\tan\left(\dfrac{2\pi}{\lambda}\times 0.15\lambda\right)}{50 + j(100+j50)\tan\left(\dfrac{2\pi}{\lambda}\times 0.15\lambda\right)}$

$$= 50\frac{100 + j50 + j50\tan(0.3\pi)}{50 - 50\tan(0.3\pi) + j100\tan(0.3\pi)} = 37.50 - j41.45\ [\Omega]$$

❷ Smith Chart Examples (2)

2) Smith chart solutions

- Load admittance:

$$y_L = 0.4 - j0.2$$

$$Y_L = y_L Y_0 = \frac{y_L}{Z_0} = \frac{0.4 - j0.2}{50} = 0.008 - j0.004\,[\mathrm{S}]$$

(electrical length: 0.22λ)

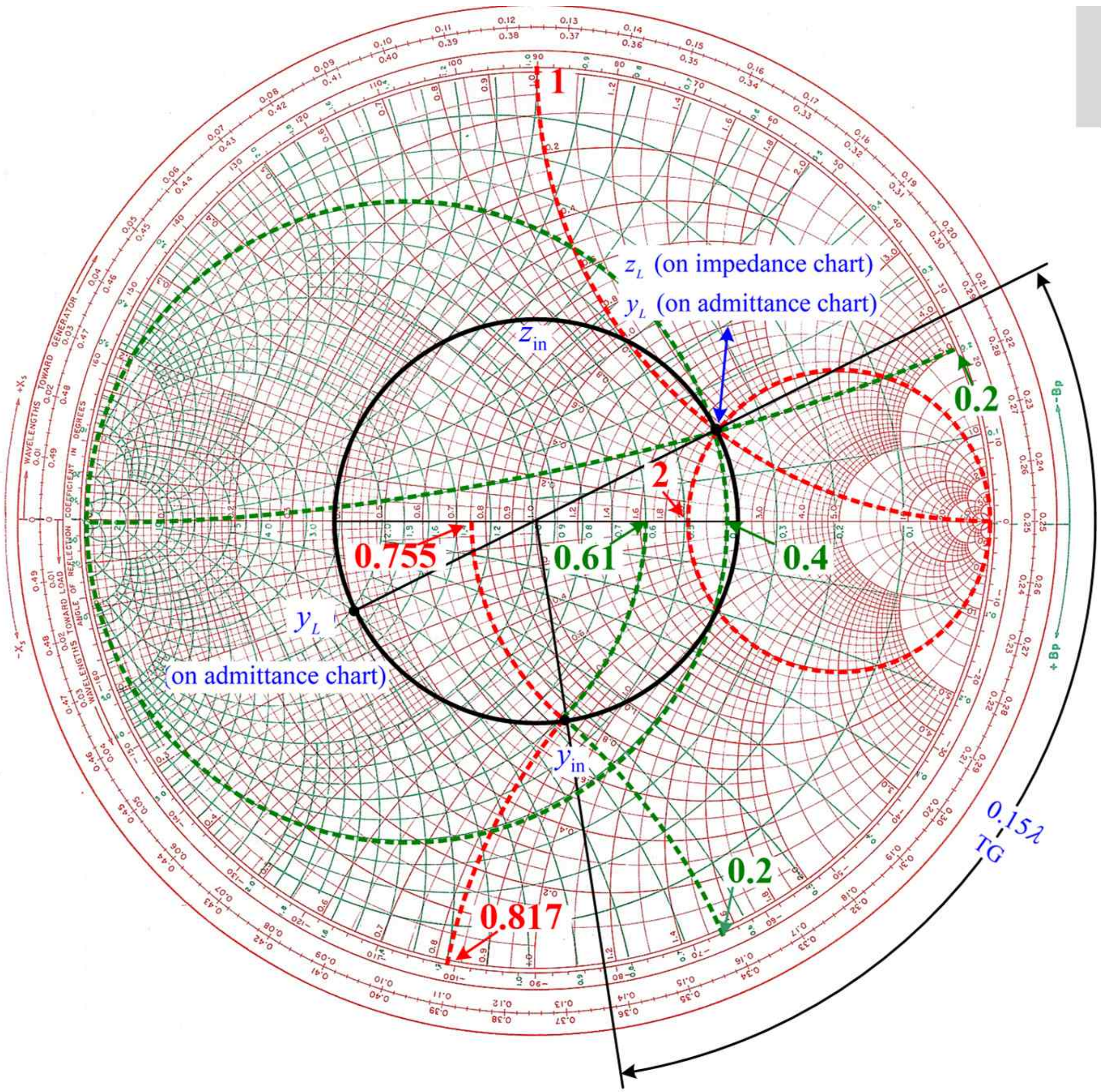
- Moving down 0.15λ of line according to wavelengths-toward-generator (WTG):

$$0.22\lambda + 0.15\lambda = 0.37\lambda$$

$$y_{\text{in}} = 0.61 + j0.66 \text{ or } z_{\text{in}} = \frac{1}{y_{\text{in}}} = 0.755 - j0.817$$

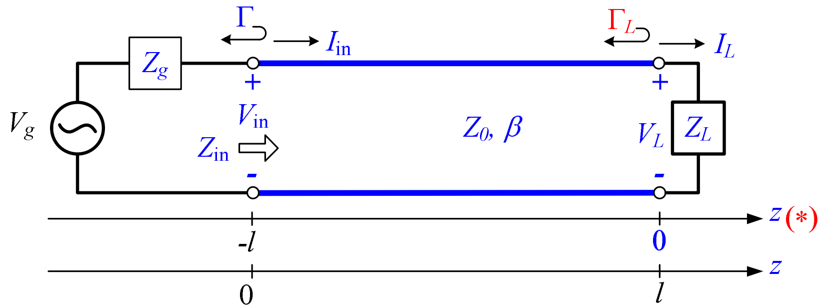
$$\rightarrow Z_{\text{in}} = z_{\text{in}} \times Z_0 = (0.755 - j0.817) \times 50$$

$$\therefore Z_{\text{in}} = 37.75 - j40.85\ [\Omega]$$



2 Smith Chart Examples (3)

- A 10 [mm] long lossless transmission line with Z_0 = 50 [Ω] operating at 3 GHz is terminated with Z_L = 80 + j30 [Ω] If the phase velocity (u) is 0.8c (c: speed of light) on the transmission line, find
  - (a) Reflection coefficient (Γ)
  - (b) (Voltage) Standing wave ratio (s)
  - (c) Input impedance (Z_{in})



1) Theoretical solutions

(a) $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{(80+j30)-50}{(80+j30)+50} = \dfrac{30+j30}{130+j30} = 0.318\angle 32^0$

(b) $s = \dfrac{1+|\Gamma|}{1-|\Gamma|} = \dfrac{1+0.318}{1-0.318} = 1.933$

(c) Since u = 0.8c and l = 0.01 [m], $\beta l = \dfrac{2\pi}{\lambda} l = \dfrac{2\pi f l}{f\lambda} = \dfrac{\omega l}{u} = \dfrac{2\pi(3\times10^9)\times10\times10^{-3}}{0.8(3\times10^8)} = \dfrac{\pi}{4} = 45^0$

$$Z_{\text{in}} = Z_0\left[\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right] = 50\,\frac{(80+j30+j50\tan\beta l)}{[50+j(80+j30)\tan\beta l]} = 50\,\frac{(80+j30+j50\tan 45^0)}{[50+j(80+j30)\tan 45^0]} = 50\,\frac{8+j3+j5}{5+j8-3} = 58.82 - j35.29$$

2 Smith Chart Examples (3)

2) Smith chart solutions

(a) Normalized load impedance: $z_L = \dfrac{Z_L}{Z_0} = \dfrac{80 + j30}{50} = 1.6 + j0.6$

- Locate z_L on Smith chart
- Draw the line from the center of Smith chart to z_L point.
- Get the length of line and then check reflection coefficient magnitude (represented by blue color) in bottom reflection magnitude and angle from the outside of Smith chart.

$$|\Gamma| = 0.318\angle 0.32^0$$

(b) Check line length with of line with bottom standing wave magnitude (represented by green color).

$$s = 1.93$$

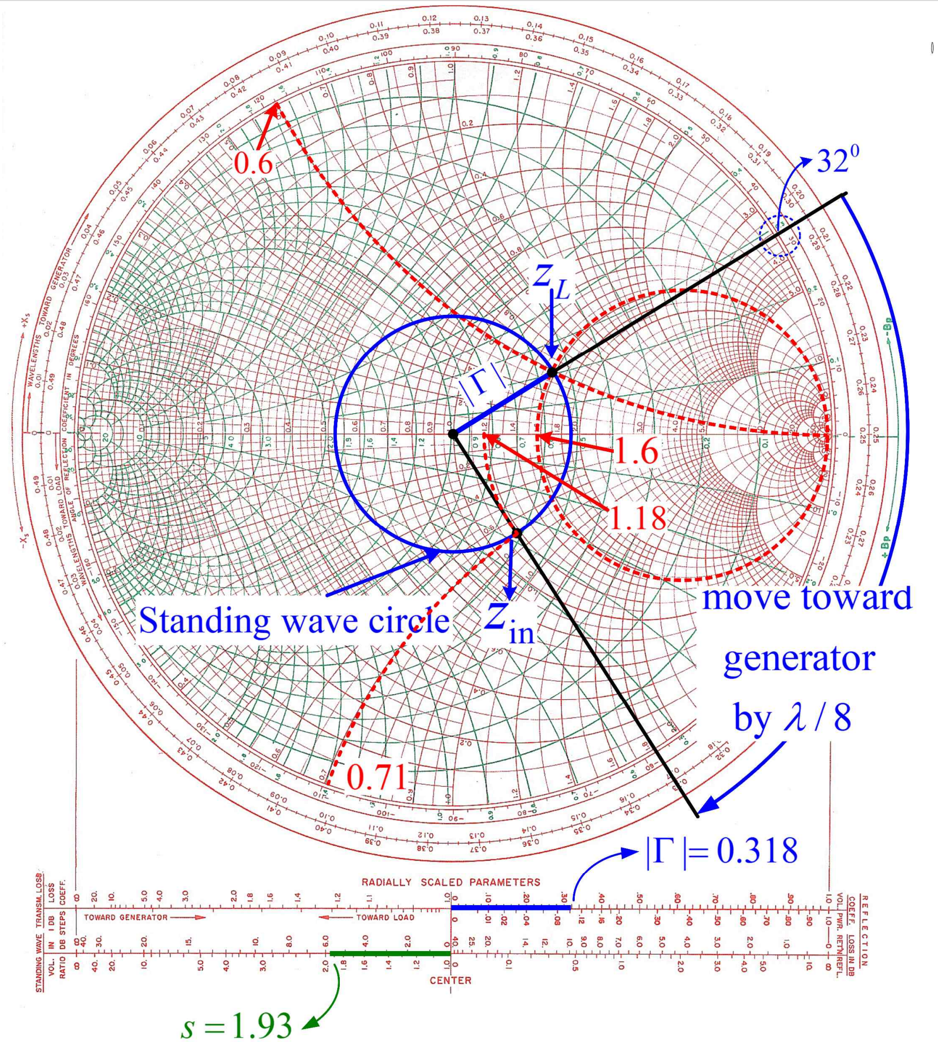
(c) To obtain Z_{in},

- Wavelength and electrical length of l in terms of λ or in degree

$$\lambda = \frac{u}{f} = \frac{0.8(3\times10^8)}{3\times10^9} = 0.08\ [\text{m}] \rightarrow l = 10[\text{mm}] = \frac{10\times10^{-3}}{0.08}\lambda = \frac{\lambda}{8} \leftrightarrow \theta = 45^{\circ}$$

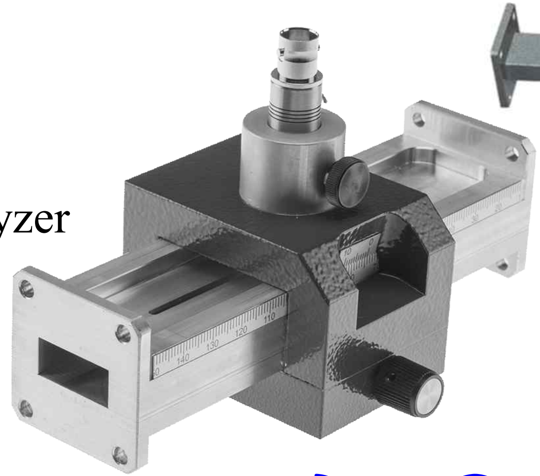
- z_L is moved toward generator about 45º (or $\lambda/8$), then z_{in} = 1.18 - j0.71. $\rightarrow Z_{in} = Z_0 z_{in}$ = 59 - j35.5 [Ω]

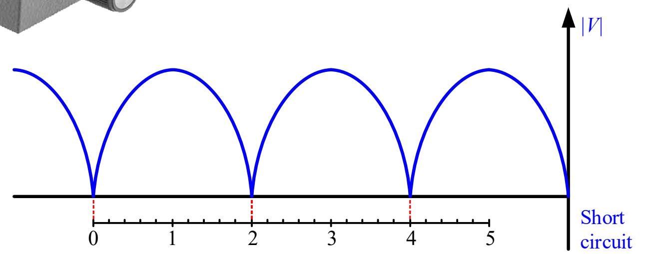
2 Smith Chart Examples (3)

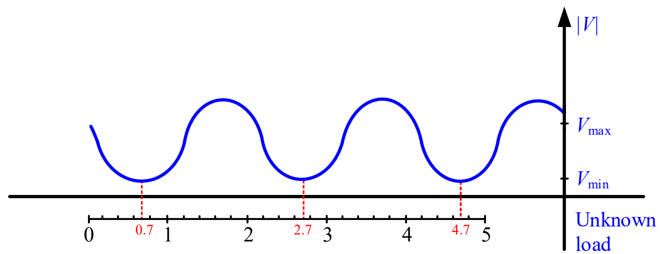


3 Slotted Transmission Line

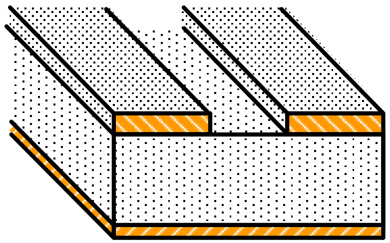
- Sampling of electric (or magnetic) field of standing wave along transmission line
  - Waveguide, coaxial cable
  - Measurement issues: (V)SWR, distance of (first) voltage maximum (or minimum) from load, Z_L, λ
  - Because of the large size, the modern vector network analyzer guaranteeing higher accuracy, versatility, and convenience replaces conventional slotted transmission line.
  - *Educational purposes*

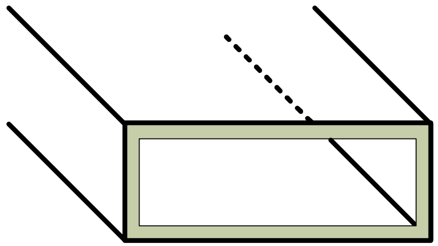






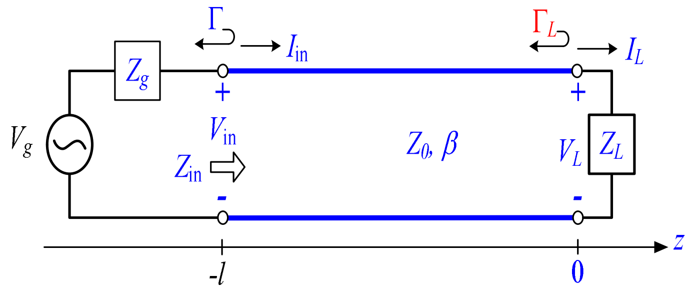
cf.) slot line & rectangular waveguide





3 Slotted Transmission Line

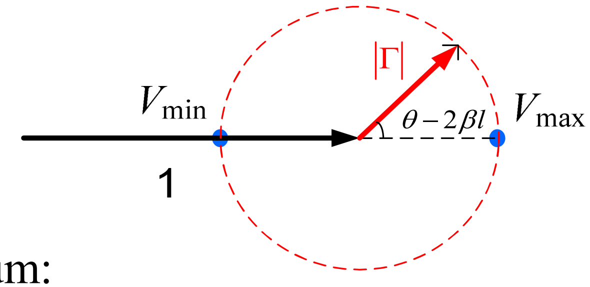
- (V)SWR = $|V_{max}/V_{min}|$
- Magnitude of reflection coefficient along transmission line: $|\Gamma| = \dfrac{SWR-1}{SWR+1}$
- Reflection coefficient: $\Gamma = |\Gamma| e^{j\theta}$



$$|V(-l)| = |V_o^+||e^{j\beta l} + \Gamma e^{-j\beta l}| = |V_o^+||e^{j\beta l}||1+\Gamma e^{-2j\beta l}| = |V_o^+||1+\Gamma e^{-2j\beta l}| \leftarrow \Gamma = |\Gamma| e^{j\theta}$$

$$= |V_o^+||1+|\Gamma| e^{j(\theta-2\beta l)}|$$

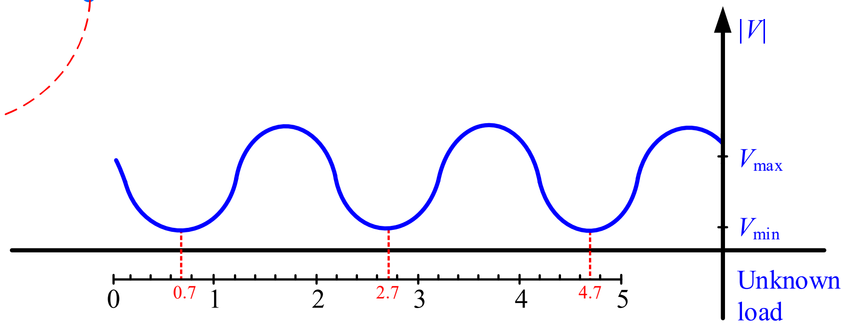
- Voltage minimum: @ $e^{j(\theta-2\beta l)} = -1 = e^{j\pi}$
  Voltage maximum: @ $e^{j(\theta-2\beta l)} = 0 = e^{j0}$



- Distance from load to first voltage minimum:

$$\theta - 2\beta l_{min} = \pi$$

$$\theta = \pi + 2\beta l_{min} \quad \text{or} \quad l_{min} = \frac{1}{2\beta}(\theta - \pi)$$



- **Voltage minimums repeat every $\lambda/2$** where λ is a wavelength on transmission line.

$$2\beta l = 2\frac{2\pi}{\lambda} l = 2\pi \quad \rightarrow l = \frac{\lambda}{2}$$

- Load impedance from Γ with $l = 0$: $Z_L = Z_0 \dfrac{1+\Gamma}{1-\Gamma} \leftarrow \Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

3 Slotted Transmission Line

- Slotted line example for impedance measurement

  - Two-step procedure

    1. A short circuit is placed at load plane

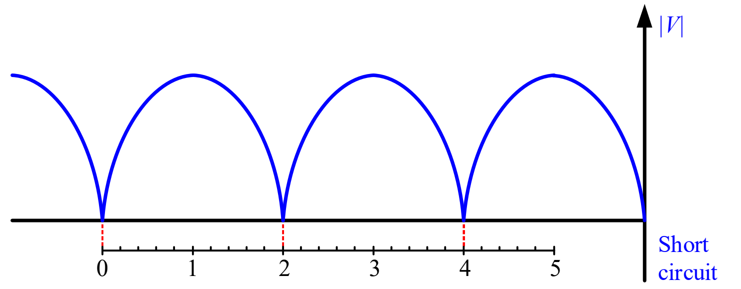
       ➔ Resulting in a standing wave on TL with infinite SWR

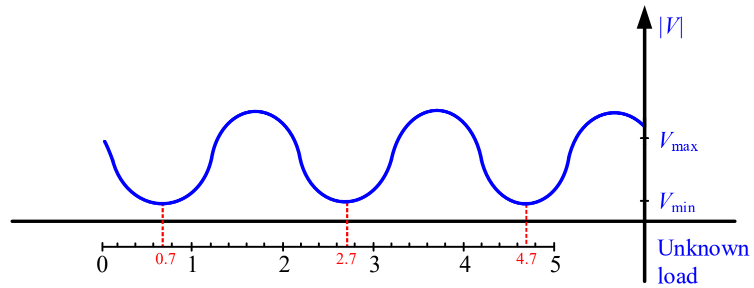
       ➔ Voltage minima at $z = 0$ cm, 2 cm, 4 cm, $\cdots$

    2. After short circuit removal, replace with the unknown load.

       ➔ (V)SWR = 1.5

       ➔ Voltage minima @ $z = 0.7$ cm, 2.7 cm, 4.7 cm, $\cdots$





Solution)

- Wavelength: λ = (distance between consecutive voltage minimum) × 2 = 4 cm
- Since voltage minimum is at 4.7 cm, the next minimum occurs at 2.7 cm.
- Distance from load to first voltage minimum: $l_{\min} = 4 - 2.7 = 1.3 \text{ cm} \rightarrow (1.3/4)\lambda = 0.325\lambda$
- $|\Gamma| = \dfrac{1.5-1}{1.5+1} = 0.2$ $\qquad \theta = \pi + 2\beta l_{\min} = \pi + 2\dfrac{2\pi}{\lambda} l_{\min} = \pi + \dfrac{4\pi}{4}(1.3) = 2.3\pi = 0.3\pi \rightarrow 54^{\circ}$

  $\Gamma = |\Gamma| e^{j\theta} = 0.2 e^{j54^{\circ}} = 0.1176 + j.01618$

3 Slotted Transmission Line

- Slotted line example for impedance measurement (continued)

  - Voltage maximum (or minimum) is occurred at the largest (smallest) resistance.

  - Load impedance:

$$Z_L = 50\frac{1+\Gamma}{1-\Gamma} = 50\frac{1+(0.1176+j0.1618)}{1-(0.1176+j0.1618)}$$

$$= 59.64 + j20.1\ [\Omega]$$

  - From (V)SWR = 1.5 circle on Smith chart, voltage minimum point must be moved 'toward load' (count-clockwise) about 0.325λ.

$$\rightarrow z_L = 1.2 + j0.4$$

$$\rightarrow Z_L = Z_0 \times z_L = 50\times(1.2+j0.4) = 60 + j20\ \ [\Omega]$$

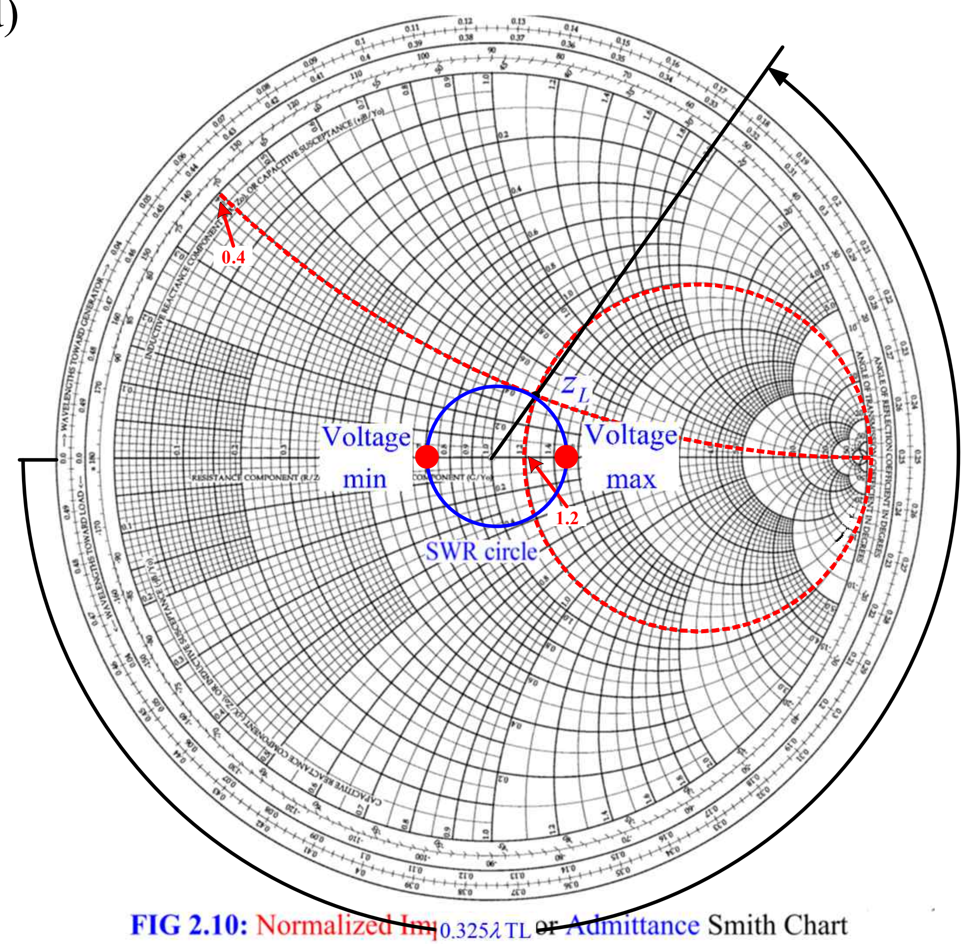


FIG 2.10: Normalized Impedance or Admittance Smith Chart

4 Review

- Smith chart

  - Reflection coefficient plane

  - Impedance and/or admittance chart

  - Transmission calculation tools

- Smith chart examples

  - Reflection coefficient

  - (V)SWR

  - Input impedance, etc.

- Slotted transmission line for sampling of electric (or magnetic) field of standing wave along transmission line

  - (V)SWR, $|\Gamma|$, Γ, Z_L, etc.

  - Educational tools