**Microwave Engineering 2-5**

# **Chapter 2 Transmission Line**

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# **Learning Objectives**

- Learn circuit parameters measurement with network analyzer
- § Understanding *λ*/4 impedance transformer operation
- § Understanding *λ*/4 impedance transformer with multiple reflections
- What is the condition for impedance matching?

# **Learning contents**

- Circuit Measurements: Vector Network Analyzer (VNA)
- § Quarter-wavelength (*λ*/4) Transformer
- § Multiple Reflection Viewpoint for *λ*/4 Impedance Transformer
- § Impedance Matchings

- § In stead of slotted line, modern network analyzers can measure the related parameters.
- Scalar Network analyzer: only signal magnitude measurable
- Vector Network Analyzer (VNA): Magnitude+ phase
- VNAs are extremely versatile instruments that can characterize source *S*-parameters, match complex impedances, (V)SWR, group delay, insertion phase.
- **Figure**\* shows general block diagram of VNA
- Signal can be sent through the device under test (DUT) from input port to output port.
- Network analyzer's receivers measure the incident, reflected, and transmitted signal to calculate *S*-parameters.



\*https://www.keysight.com/us/en/solutions/measurement-fundamentals/network-analysis.html

power

- Reflection measurement: 1-port device under test (DUT)
- Reflection coefficient (or  $S_{11}$ -parameters):

$$
S_{11} = \Gamma = \frac{V_{reflected}}{V_{incident}} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}
$$

- Return loss (RL):
	-
- (Voltage) Standing wave ratio ((V)SWR):

 $1-\vert\Gamma\vert$  $\text{SWR} = \frac{1}{1 + |S|}$  $1 + |\Gamma|$  $=\frac{1}{4}$   $\frac{|1|}{|1|}$ 

Input impedance:

$$
Z_{\text{in}} = Z_0 \left( \frac{1+\Gamma}{1-\Gamma} \right) = Z_0 \left( \frac{1+S_{11}}{1-S_{11}} \right)
$$





■ Reflection measurement examples: 1-port device under test (DUT)



■ Transmission measurement examples on 2-port devices or circuits





- Transmission measurement: 2-port devices or circuits  $-S$ -parameters:  $S_{11}, S_{21}, S_{12}, S_{22}$ 1  $\boldsymbol{V}$ **rcuit Measurement:** 2-port devices<br>
mission measurement: 2-port devices<br>
ameters:  $S_{11}$ ,  $S_{21}$ ,  $S_{12}$ ,  $S_{22}$ <br>  $S_{11} = \Gamma_{\text{in}} = \frac{V_{reflected}^{Port1}}{V_{reflected}^{Port1}}$   $S_{21} = \frac{V_{transmitted}^{Port2}}{V_{incident}^{Port1}}$ <br>  $S_{12} = \frac{V_{reflected}^{Port1}}{V_{resmitted}^{host1}}$ *Port reflected Port reflected*  $V_{reflected}^{Port1}$   $V_{transmitted}^{Port2}$  $S_{11} = \Gamma_{\text{in}} = \frac{rejected}{\pi r_{\text{Port}}}$   $S_{21} =$  $V_{reflected}^{Port1}$   $V_{incident}^{Port1}$ **Casurements:** Vector Network<br>
ission measurement: 2-port devices or circuits<br>
incident Port 1<br>
incident Port 1<br>  $= \Gamma_{\text{in}} = \frac{V_{reflected}^{Port1}}{V_{reflected}^{Port2}}$ <br>  $= \Gamma_{\text{out}} = \frac{V_{reflected}^{Port2}}{V_{reflected}^{Port2}}$ <br>  $S_{12} = \frac{V_{transmitted}^{Port1}}{V_{incident}^{Port2}}$ <br>  $= \Gamma$ 2  $V^P$ mission measurement: 2-port devices<br>
ameters:  $S_{11}$ ,  $S_{21}$ ,  $S_{12}$ ,  $S_{22}$ <br>  $S_{11} = \Gamma_{in} = \frac{V_{reflected}^{Port1}}{V_{reflected}^{Port1}}$   $S_{21} = \frac{V_{transmitted}^{Port2}}{V_{incident}^{Port1}}$ <br>  $S_{22} = \Gamma_{out} = \frac{V_{reflected}^{Port2}}{V_{reflected}^{Port2}}$   $S_{12} = \frac{V_{transmitted}^{Port1}}{V_{incident}^{Port2}}$ <br>  $S_{$ *Port reflected*  $\mathbf{C}$   $\mathbf{C}$   $\mathbf{r}$  tras *Port reflected*  $V_{reflected}^{Port1}$   $V_{transmitted}^{Port1}$  $S_{22} = \Gamma_{\text{out}} = \frac{rejected}{\pi r^{\text{Port}}}$   $S_{12}$  $V_{reflected}^{Port2}$  12  $V_{incident}^{Port2}$ **Calcular Measurement:** 2-port devices or circuits<br>
ission measurement: 2-port devices or circuits<br>
incident Port 1<br>  $=\Gamma_{\text{in}} = \frac{V_{reduced}^{Port1}}{V_{reduced}^{Port1}}$   $S_{21} = \frac{V_{rcontrol}^{Port1}}{V_{incident}^{Port1}}$ <br>  $=\Gamma_{\text{out}} = \frac{V_{reduced}^{Port1}}{V_{reduced}^{Port2}}$   $S_{1$ 2  $21 - I$ *Port trasmitted Port incident*  $S_{21} = \frac{r_{transmitted}}{r_{\tau}$  $=\frac{r_{transmitted}}{rr^{Port1}}$  Refleo 1  $12 - T$ *Port trasmitted Port incident*  $S_{12} = \frac{r_{\text{transmitted}}}{r_{\text{F}}\text{Port2}}$  $=\frac{7 \text{ transmitted}}{15 P \cdot 2}$ =  $\Gamma_{\text{in}} = \frac{V_{reflected}^{Port1}}{V_{reflected}^{Port1}}$   $S_{21} = \frac{V_{transmitted}^{Port2}}{V_{incident}^{Port1}}$  Reflected<br>
=  $\Gamma_{\text{out}} = \frac{V_{reflected}^{Port2}}{V_{reflected}^{Port2}}$   $S_{12} = \frac{V_{transmitted}^{Port1}}{V_{incident}^{Port2}}$ <br>
and output return losses (RLs): Fransmite<br>  $\Gamma_{\text{in}}(dB) = -20 \log_{10} (|S_{11}|)$ <br> Fortherm  $V_{reflected}^{Port1}$ <br>  $= \Gamma_{out} = \frac{V_{reflected}^{Port1}}{V_{reflected}^{Port2}}$   $S_{12} = \frac{V_{transmitted}^{Port1}}{V_{incident}^{Port2}}$ <br>
and output return losses (RLs):<br>  $\Gamma_{in} (dB) = -20 \log_{10} (|S_{11}|)$ <br>  $\Gamma_{out} (dB) = -20 \log_{10} (|S_{22}|)$ <br>
and output VSWRs:<br>  $\Gamma_{out} = 1 - \Gamma_{in}$  and  $\$ **Measurement:** 2-port devices or circuits<br>  $V_{S11}$ ,  $S_{21}$ ,  $S_{12}$ ,  $S_{22}$ <br>  $V_{reduced}^{Port1}$ <br>  $V_{reduced}^{Port2}$ The phase  $S_{11} = \Gamma_{in} \frac{V_{reflected}^{Port1}}{V_{reflected}^{Port1}}$ <br>  $S_{21} = \Gamma_{in} \frac{V_{reflected}^{Port1}}{V_{reflected}^{Port2}}$ <br>  $S_{22} = \Gamma_{out} \frac{V_{reflected}^{Port1}}{V_{reflected}^{Port2}}$ <br>  $S_{12} = \frac{V_{rguncted}^{Port1}}{V_{inclient}^{port2}}$ <br>  $S_{12} = \frac{V_{rgpartified}^{Port1}}{V_{inclient}^{port2}}$ <br>  $S_{12} = \frac{V_{rgpartified}^{Port1}}{V_{inclient}^{port$  $V_{\text{reflected}}^{Port1}$ <br>  $V_{\text{reflected}}^{Port2}$ <br>  $V_{\text{reflected}}^{Port1}$ <br>  $V_{\text{reflected}}^{Port2}$ <br>  $V_{\text{reflected}}^{Port2}$ <br>  $V_{\text{reflected}}^{Port2}$ <br>  $V_{\text{reflected}}^{Port2}$ <br>  $V_{\text{reflected}}^{Port1}$ <br>  $S_{12} = \frac{V_{\text{resonified}}^{Port1}}{V_{\text{resonified}}^{Port1}}$ <br>  $S_{13} = \frac{V_{\text{resonified}}^{Port1}}{V_{\text{incident}}^{Port1}}$ <br>  $S$ 
	- Input and output return losses (RLs):
- $RL_{in}$  (dB) = -20  $log_{10}$  (|  $S_{11}$  |)  $RL_{out}$  (dB) = -20  $log_{10}$  (|  $S_{22}$  |) V reflected<br>
d output return losses (RLs):<br>  $(dB) = -20 log_{10} (|S_{11}|)$ <br>  $(dB) = -20 log_{10} (|S_{22}|)$ <br>
d output VSWRs:<br>  $\begin{aligned}\n&= \frac{1-\Gamma_{in}}{1+\Gamma_{in}}, \text{ SWR}_{out} = \frac{1-\Gamma_{out}}{1+\Gamma_{out}} \quad &\text{IL}_{dB} = -20\n\end{aligned}$ <br>
mase:  $IL_{phase} = \angle S_{21}$  - Group delay return losses (RLs):<br>
Transmitted<br>
-20  $log_{10} (|S_{11}|)$ <br>  $-20 log_{10} (|S_{22}|)$ <br>
VSWRs:<br>  $\frac{sin}{\pi}$ , SWR<sub>out</sub>  $= \frac{1-\Gamma_{out}}{1+\Gamma_{out}}$ <br>  $\frac{1-\Gamma_{out}}{1+\Gamma_{out}}$ <br>  $\frac{1}{1+\Gamma_{out}}$ <br>  $\frac{1}{1+\Gamma_{out}}$ <br>  $\frac{1}{1+\Gamma_{out}}$ <br>  $\frac{1}{1+\Gamma_{out}}$ <br>  $\frac{1}{1+\Gamma_{out}}$ <br>
	- Input and output VSWRs:

$$
SWR_{in} = \frac{1 - \Gamma_{in}}{1 + \Gamma_{in}}, \quad SWR_{out} = \frac{1 - \Gamma_{out}}{1 + \Gamma_{out}} \qquad \qquad IL_{dB} = -20 \log_{10} (
$$



- Insertion loss (IL) and Gain:

$$
IL_{dB} = -20 log_{10} (|S_{11}|)
$$
 Gain (dB) = 20 log\_{10} (|S\_{21}|)

- Group delay: 
$$
\tau = -\frac{d \angle S_{21}}{d \omega}
$$

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• Transmission examples: 2-port device under test (DUT)



### **Quarter-wavelength (***λ***/4) Transformer 2**

- Useful and practical circuit in microwave system
- Transmission line terminated with resistance load (*R<sub>L</sub>*)<br>
 quarter-wavelength transmission line of *Z*<sub>1</sub>
	-
	- Feedline characteristic impedance: Z<sub>0</sub>
	- Input impedance:



- In order for  $\Gamma = 0$ , we must have  $Z_{\text{in}} = Z_0$ .

- $1 \sqrt{20} \Omega_L$ 2  $1 - 7$   $\rightarrow$  7. 0  $L_1 - \sqrt{L_0} \mathbf{R}$ *L*  $Z_1^2$   $\rightarrow$   $Z$   $\sqrt{Z}$   $\rightarrow$  $Z_0 \Leftrightarrow Z_1 = \sqrt{Z_0 R_L}$  $R_{L}$  <sup>0</sup> <sup>1</sup> **v** <sup>0</sup> <sup>L</sup>
- $\rightarrow$  Geometric mean of load and source impedances
- $\rightarrow$  No standing waves on feedline (SWR = 1)
- $\rightarrow$  Only when the length of the matching section is  $\lambda/4$  or  $(2n + 1)\lambda/4$
- è *A perfect match may be achieved at one frequency (or center frequency)*
- **E** *Reflection tolerance* permits an operating bandwidth, not only center frequency



# **Quarter-wavelength (***λ***/4) Transformer 2**

■ Example quarter-wavelength impedance transformer<br>- *R<sub>L</sub>* = 100 Ω, *Z*<sub>0</sub> = 50 Ω,<br>- Find characteristic impedance of transmission line of *Z*<sub>1</sub> and normalized frequency characteristics

**Solution)** Characteristic impedance of  $Z_1$ :

$$
Z_1 = \sqrt{Z_0 R_L} = \sqrt{50 \times 100} = 70.71 \, [\Omega]
$$

- Reflection coefficient magnitude:

**Quarter-wavelength** (
$$
\lambda/4
$$
) **Transformer**  
\n
$$
L = 100 \Omega, Z_0 = 50 \Omega,
$$
\nind characteristic impedance of transmission line of  $Z_1$   
\n
$$
Z_1 = \sqrt{Z_0 R_L} = \sqrt{50 \times 100} = 70.71 [\Omega]
$$
\n
$$
Z_1 = \sqrt{Z_0 R_L} = \sqrt{50 \times 100} = 70.71 [\Omega]
$$
\n
$$
= \frac{Z_0 = 50 \Omega}{Z_0 + Z_0} = \frac{Z_1 + jZ_0 \tan \beta l}{Z_0 + jZ_1 \tan \beta l}
$$
\n
$$
= \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{Z_0 \frac{Z_1 + jZ_0 \tan \beta l}{Z_0 + jZ_1 \tan \beta l}}{Z_0 + \frac{Z_1}{Z_0} + \frac{Z_2}{Z_0}} = \frac{Z_1 \frac{Z_1 + jZ_0 \tan \beta l}{Z_0}}{Z_0 + \frac{Z_1}{Z_0} + \frac{Z_2}{Z_0}} = \frac{Z_1 \frac{Z_1 + jZ_0 \tan \beta l}{Z_0}}{Z_0 + \frac{Z_1}{Z_0} + \frac{Z_2}{Z_0}} = \frac{Z_1 \frac{Z_2 + jZ_0 \tan \beta l}{Z_0}}{Z_0 + \frac{Z_1}{Z_0} + \frac{Z_2}{Z_0}} = \frac{Z_1 \frac{Z_2 + jZ_0 \tan \beta l}{Z_0}}{Z_0 + \frac{Z_2}{Z_0} + \frac{Z_2}{Z_0}} = \frac{Z_1 \frac{Z_2 + jZ_0 \tan \beta l}{Z_0}}{Z_0 + \frac{Z_1}{Z_0} + \frac{Z_2}{Z_0}} = \frac{Z_1 \frac{Z_1 + jZ_0 \tan \beta l}{Z_0}}{Z_0 + \frac{Z_1}{Z_0} + \frac{Z_2}{Z_0}} = \frac{Z_1 \frac{Z_1 + jZ_0 \tan \beta l}{Z_0}}{Z_0 + \frac{Z_1}{Z_0} + \frac{Z_2}{Z_0}} = \frac{Z_1 \frac{Z_1 + jZ_0 \tan \beta l}{Z_0}}{Z_0 + \frac{Z_1}{Z_0} + \frac{Z_2}{Z
$$



# **Quarter-wavelength (***λ***/4) Transformer 2**

- Reflection coefficient characteristics according to normal frequency  $(f/f_0)$ 



### **Multiple reflection Viewpoint for** *λ***/4 Impedance Transformer 3**

- Reflection and transmission for  $\lambda/4$  impedance transformer
- **-**  $\Gamma$ : overall reflection coefficient of incident wave
- $\Gamma_1$ : partial reflection coefficient of wave incident on load  $Z_1$ from  $Z_0$  line
- $\Gamma_2$ : partial reflection coefficient of wave incident on load  $Z_0$ from  $Z_1$  line
- G<sup>3</sup> : partial reflection coefficient of wave incident on load *R<sup>L</sup>* from  $Z_1$  line
- $T_1$ : partial transmission coefficient of wave from  $Z_0$  line into  $Z_1$  line
- *-*  $T_2$ : partial transmission coefficient of a wave from  $Z_1$  line into  $Z_0$  line  $\hspace{1cm}$  1
- Individual coefficients

from 
$$
Z_0
$$
 line  
\n $\Gamma_2$ : partial reflection coefficient of wave incident on load  $Z_0$   
\nfrom  $Z_1$  line  
\n $\Gamma_3$ : partial reflection coefficient of wave incident on load  $R_L$   
\nfrom  $Z_1$  line  
\n $T_1$ : partial transmission coefficient of wave from  $Z_0$  line into  $Z_1$  line  
\n $T_2$ : partial transmission coefficient of a wave from  $Z_1$  line into  $Z_0$  line  
\n $\Gamma_2$ : partial transmission coefficient of a wave from  $Z_1$  line into  $Z_0$  line  
\nIndividual coefficients  
\n $\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}, \quad \Gamma_2 = \frac{Z_0 - Z_1}{Z_0 + Z_1} = -\Gamma_1, \quad \Gamma_3 = \frac{R_L - Z_1}{R_L + Z_1},$   
\n $T_1 = 1 + \Gamma_1 = \frac{2Z_1}{Z_1 + Z_0}, \quad T_2 = 1 + \Gamma_2 = \frac{2Z_0}{Z_1 + Z_0}$ 



# **Multiple reflection Viewpoint for** *λ***/4 Impedance Transformer** 0 phase shift **3**

Total reflection coefficient

**Multiple reflection Viewpoint for** 
$$
\lambda/4
$$
 **Impedance Transf**  
\ntotal reflection coefficient  
\n
$$
\Gamma = \Gamma_1 - T_1 T_2 \Gamma_3 + T_1 T_2 \Gamma_1^2 - T_1 T_2 \Gamma_2^2 \Gamma_3^3 + \cdots \leftarrow 180^\circ \text{ phase shift}
$$
\n
$$
= \Gamma_1 - T_1 T_2 \Gamma_3 \sum_{n=0}^\infty (-\Gamma_2 \Gamma_3)^n
$$
\nSince  $|\Gamma_3| < 1$  and  $|\Gamma_2| < 1$ ,  
\n
$$
\Gamma = \Gamma_1 - \frac{T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3} \leftarrow \sum_{n=0}^\infty x^n = \frac{1}{1-x} \text{ for } |x| < 1
$$
\n
$$
= \frac{\Gamma_1 + \Gamma_1 \Gamma_2 \Gamma_3 - T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3} = \frac{\Gamma_1 - \Gamma_1^2 \Gamma_3 - T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3}
$$
\nimplified numerator:  $\Gamma_1 - \Gamma_3 (\Gamma_1^2 + T_1 T_2) = \Gamma_1 - \Gamma_3 \left[ \frac{(Z_1 - Z_0)^2}{(Z_1 + Z_0)^2} + \frac{4Z_1 Z_0}{(Z_1 + Z_0)^2} \right] = \Gamma_1 - \Gamma_3$ \n
$$
= \frac{(Z_1 - Z_0) - (R_L - Z_1)}{(Z_1 + Z_0) - (R_L + Z_1)} = \frac{(Z_1 - Z_0)(R_L + Z_1) - (R_L - Z_1)(Z_1 + Z_0)}{(Z_1 + Z_0)(R_L + Z_1)}
$$

- Since  $|\Gamma_3|$  < 1 and  $|\Gamma_2|$  < 1,

3 **Multiple reflection Viewpoint for** 
$$
\lambda/4
$$
 **Impedance Trans**  
\nTotal reflection coefficient  
\n
$$
\Gamma = \Gamma_1 - T_1 T_2 \Gamma_3 + T_1 T_2 \Gamma_2 \Gamma_3^2 - T_1 T_2 \Gamma_2^2 \Gamma_3^3 + \cdots \quad \leftarrow 180^\circ \text{ phase shift}
$$
\n
$$
= \Gamma_1 - T_1 T_2 \Gamma_3 \sum_{n=0}^\infty (-\Gamma_2 \Gamma_3)^n
$$
\n
$$
\text{Since } |\Gamma_3| < 1 \text{ and } |\Gamma_2| < 1,
$$
\n
$$
\Gamma = \Gamma_1 - \frac{T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3} \quad \leftarrow \sum_{n=0}^\infty x^n = \frac{1}{1-x} \quad \text{for } |x| < 1
$$
\n
$$
= \frac{\Gamma_1 + \Gamma_1 \Gamma_2 \Gamma_3 - T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3} = \frac{\Gamma_1 - \Gamma_1^2 \Gamma_3 - T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3}
$$
\n
$$
\text{Simplified numerator: } \Gamma_1 - \Gamma_3 (\Gamma_1^2 + T_1 T_2) = \Gamma_1 - \Gamma_3 [\frac{(Z_1 - Z_0)^2}{(Z_1 + Z_0)^2} + \frac{4Z_1 Z_0}{(Z_1 + Z_0)^2}] = \Gamma_1 - \Gamma_3 \qquad \frac{1}{T_1 T_2 \Gamma_3}
$$
\n
$$
= \frac{(Z_1 - Z_0) - (R_L - Z_1)}{(Z_1 + Z_0)^2} = \frac{(Z_1 - Z_0)(R_L + Z_1) - (R_L - Z_1)(Z_L + Z_0)}{(Z_1 + Z_0)^2}
$$

0

*n*=0

2  $AZ$ <sup>2</sup> 1 0 1 0 1,<br>
1,<br>  $x'' = \frac{1}{1-x}$  for  $|k| < 1$ <br>  $\frac{1}{x_0}$ <br>  $\frac{1}{1+x_1}$ <br>  $\frac{1}{1+x_2}$ <br>  $1+\Gamma_2\Gamma_3$ <br>  $1-\Gamma_3(\Gamma_1^2+T_1T_2) = \Gamma_1 - \Gamma_3[\frac{(Z_1-Z_0)^2}{(Z_1+Z_0)^2} + \frac{4Z_1Z_0}{(Z_1+Z_0)^2}] = \Gamma_1 - \Gamma_3$ <br>  $\frac{1}{1+\Gamma_1T_2}$ <br>  $\frac{1}{1+\Gamma_1T_2}$ <br>  $\frac{1}{$ 

**Section Viewpoint for** 
$$
\lambda/4
$$
 **Impedance Transformer**  
\n
$$
\sum_{j=1}^{n} -T_{j}T_{2}\Gamma_{2}^{2}\Gamma_{3}^{3} + \cdots \leftarrow 180^{\circ} \text{ phase shift}
$$
\n
$$
\sum_{j=1}^{n} \sum_{j=1}^{n} \text{ for } |x| < 1
$$
\n
$$
\sum_{k=1}^{n} x_{k} = \frac{1}{1-x} \text{ for } |x| < 1
$$
\n
$$
\sum_{k=1}^{n} \sum_{
$$

 $\rightarrow$  If  $Z_1 = \sqrt{Z_0 R_L}$ , then  $\Gamma = 0$  and the transmission line is matched.

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 $Z_0$  *Z*<sub>1</sub>

 $100 - 1$  $\Gamma \leftarrow 000$ 

 $1 \quad T_1$ 

 $T_2$ <sup> $\Gamma_2$ </sup>

 $\Gamma_2$ 

 $-T_1T_2\Gamma_3$ 

 $\overline{\Gamma_1^-}$ 

 $T_2$ 

 $T_1T_2\Gamma_3^2\Gamma_2$ 

 $\overline{\neg \rightarrow \Gamma_2}$ 

 $T_{1}$ 

 $\lambda/4$   $-$ 

 $\Gamma_3$ 

 $\Gamma_3$ 

 $\Gamma_3$ 

- Lossless transmission line circuit with arbitrary generator and load impedances
	- Input impedance looking into terminated transmission line from generator:

**Impedance Matchings**  
\nLossless transmission line circuit with arbitrary generator and load impedances  
\n- Input impedance looking into terminated transmission line from generator:  
\n
$$
Z_{\text{in}} = \frac{V_{\text{in}}}{L_{\text{in}}} = \frac{(V_0^+ e^{-j\beta t} + V_0^- e^{-j\beta t})}{(V_0^+ e^{-j\beta t} + V_0^- e^{-j\beta t})/Z_0} = Z_0 \frac{1 + V_0^- e^{-j2\beta t}}{1 - \Gamma_0 e^{-j2\beta t}}
$$
\n
$$
= Z_0 \frac{Z_1 + jZ_0 \tan \beta l}{Z_0 + jZ_1 \tan \beta l}
$$
\nwhere  $\Gamma_i$ : reflection coefficient at load  
\n*l*: length from load  
\n*l*: length from load  
\n- Voltage wave at arbitrary location of transmission line:  
\n
$$
V(z) = (V_0^+ e^{-j\beta z} + V_0^- e^{-j\beta z}) = V_0^+ (e^{-j\beta z} + \Gamma_1 e^{j\beta z}) \leftarrow \Gamma_i = \frac{Z_i - Z_0}{Z_i + Z_0}
$$
\n- Voltage at generator end ( $\omega$   $z = -i)$   
\n
$$
V(-i) = V_g \frac{Z_{\text{in}}}{Z_g + Z_{\text{in}}} = V_0^+ (e^{j\beta t} + \Gamma_1 e^{-j\beta t})
$$
\n
$$
V_0^+ = V_g \frac{Z_{\text{in}}}{Z_{\text{in}}} + \frac{1}{Z_g} \frac{1}{(e^{j\beta t} + \Gamma_1 e^{-j\beta t})}
$$
\n14

0

*-* Voltage at generator end (@ *z* = -*l*)

voltage wave at arbitrary location of transmission line:  
\n
$$
V(z) = (V_0^+ e^{-j\beta z} + V_0^- e^{-j\beta z}) = V_0^+ (e^{-j\beta z} + \Gamma_l e^{j\beta z}) \leftarrow \Gamma_l =
$$
\n
$$
V(\text{diage at generator end } (\text{Q } z = -l)
$$
\n
$$
V(-l) = V_g \frac{Z_{in}}{Z_g + Z_{in}} = V_0^+ (e^{j\beta l} + \Gamma_l e^{-j\beta l})
$$
\n
$$
V_0^+ = V_g \frac{Z_{in}}{Z_{in} + Z_g} \frac{1}{(e^{j\beta l} + \Gamma_l e^{-j\beta l})}
$$

**Impedance Matchings**  
\n- Voltage at generator end 
$$
(\hat{\omega}) z = -l(V_0^+)
$$
  
\n
$$
V_0^+ = V_s \frac{Z_{\text{in}}}{Z_{\text{in}} + Z_s} \frac{1}{(e^{j\beta t} + \Gamma_i e^{-j\beta t})} \leftarrow Z_{\text{in}} = Z_0 \frac{1 + \Gamma_i e^{-j2\beta t}}{1 - \Gamma_i e^{-j2\beta t}}
$$
\n
$$
= V_s \frac{1 + \Gamma_i e^{-2j\beta t}}{Z_0 \frac{1 + \Gamma_i e^{-2j\beta t}}{1 - \Gamma_i e^{-2j\beta t}} + Z_s \frac{e^{j\beta t} + \Gamma_i e^{-j\beta t}}{1 - \Gamma_i e^{-j\beta t}} = V_s \frac{Z_0}{Z_0 + Z_s \frac{1 - \Gamma_i e^{-2j\beta t}}{1 - \Gamma_i e^{-j\beta t}} + \Gamma_i e^{-j\beta t}} = V_s \frac{Z_0}{Z_0 + Z_s \frac{1 - \Gamma_i e^{-2j\beta t}}{1 - \Gamma_i e^{-2j\beta t}} + \Gamma_i e^{-j\beta t}} = V_s \frac{Z_0}{Z_0 + Z_s \frac{1}{e^{j\beta t} + \Gamma_i e^{-j\beta t}} + \Gamma_i e^{-j\beta t}} = V_s \frac{Z_0}{Z_0 e^{j\beta t} + \Gamma_i e^{-j\beta t}} = V_s \frac{Z_0}{Z_0 e^{j\beta t} + \Gamma_i e^{-j\beta t}} = V_s \frac{Z_0}{Z_0 e^{j\beta t} + \Gamma_i e^{-j\beta t}} = V_s \frac{Z_0}{Z_0 + Z_s} \frac{1}{e^{j\beta t} + \Gamma_i e^{-j\beta t}} = V_s \frac{Z_0}{Z_0 + Z_s} \frac{1}{e^{j\beta t} - \Gamma_i e^{-j\beta t}} = V_s \frac{Z_0}{Z_0 + Z_s} \frac{1}{e^{j\beta t} - \Gamma_i e^{-j\beta t}} = V_s \frac{Z_0}{Z_0 + Z_s} \frac{1}{e^{j\beta t} - \Gamma_i e^{-j\beta t}} = V_s \frac{Z_0}{Z_0 + Z_s} \frac{1}{e^{j\beta t} - \Gamma_i e^{-j\beta t}} = V_s \frac{Z_0}{Z_0 + Z_s} \frac{1}{e^{j\beta t} - \Gamma_i e^{-j\beta t}} = V_s \frac{1}{Z_0 + Z_s} \frac{1}{e^{j\
$$

15

- *-* (Averaging) Power delivered to load: *V<sup>g</sup> z -l Z<sup>g</sup>*  $Z_0, \beta$ *Z*in 0 *+ - + -*  $\Gamma$  $I_{\rm in}$  $\Gamma_l$  $\Gamma_{\mathsf{g}}$  $*$  1 and 1  $2$  p  $\begin{array}{c} 1 \end{array}$ edance Matchings<br>g) Power delivered to load:<br> $\left[\frac{I_{\text{in}}^*}{2} \frac{I_{\text{in}}^*}{\sqrt{2}}\right] = \frac{1}{2} \text{Re}[V_{\text{in}} I_{\text{in}}^*] = \frac{1}{2} |V_{\text{in}}|^2 \text{Re}[\frac{1}{Z_{\text{in}}^*}]$ **Iatchings**<br>
ered to load:<br>  $\begin{aligned}\n\sum_{\text{in}} I_{\text{in}}^* &= \frac{1}{2} |V_{\text{in}}|^2 \operatorname{Re}[\frac{1}{Z_{\text{in}}^*}] \\
\frac{1}{2} \end{aligned}$ in  $V$ 2 a set of  $\sim$  2  $\mathcal{L}_{\text{in}}$   $\mathcal{L}_{\text{in}}$   $\mathbb{R}$   $\mathbb{R}$   $\mathbb{R}$   $\mathbb{R}$   $\mathbb{R}$  $g \mid \mathcal{F} \mid \mathcal{F} \mid$   $\mathcal{F}$   $\mathcal{F}$ Power delivered to load:<br>  $\frac{\sin \frac{\pi}{2}}{1} = \frac{1}{2} \text{Re}[V_{in}I_{in}^*] = \frac{1}{2}|V_{in}|^2 \text{Re}[\frac{1}{Z_{in}^*}]$ <br>  $\frac{Z_{in}}{1} + Z_g \bigg|^{2} \text{Re}[\frac{1}{Z_{in}^*}]$ <br>
eneral source and load conditions:  $Z_{in} = R_{in} +$ **If the Contract Contra npedance Matchings**<br>
aging) Power delivered to load:<br>  $Re[\frac{V_{in}}{\sqrt{2}} \frac{I_{in}^*}{\sqrt{2}}] = \frac{1}{2}Re[V_{in}I_{in}^*] = \frac{1}{2}|V_{in}|^2 Re[\frac{1}{Z_{in}^*}]$ <br>  $V_g \bigotimes \frac{Z_g}{Z_{in}^*} = \frac{1}{2}V_g$ <br>  $\frac{1}{Z_{in} + Z_g}$ <br>  $Re[\frac{1}{Z_{in}^*}]$ <br>  $Re[\frac{1}{Z_{in}^*}]$ <br>  $V$  $g \mid \qquad \qquad$   $\qquad$  *in* **Impedance Matchings**<br>
Averaging) Power delivered to load:<br>  $P = \text{Re}\left[\frac{V_{\text{in}}}{\sqrt{2}} \frac{\vec{I_{\text{in}}}}{\sqrt{2}}\right] = \frac{1}{2} \text{Re}[V_{\text{in}} I_{\text{in}}^*] = \frac{1}{2} |V_{\text{in}}|^2 \text{Re}\left[\frac{1}{Z_{\text{in}}^*}\right]$ <br>  $= \frac{1}{2} |V_g|^2 \left|\frac{Z_{\text{in}}}{Z_{\text{in}} + Z_g}\right|^2$  $Z_{\rm in}^*$   $\overline{Y}$  $=\frac{1}{2}|V_{\rm g}|^2\left|\frac{Z_{\rm in}}{Z_{\rm g}}\right|$  Re $[\frac{1}{Z_{\rm g}}]$ **dance Matchings**<br>
Power delivered to load:<br>  $\frac{I_{\text{in}}^*}{\sqrt{2}} = \frac{1}{2} \text{Re}[V_{\text{in}} I_{\text{in}}^*] = \frac{1}{2} |V_{\text{in}}|^2 \text{Re}[\frac{1}{Z_{\text{in}}^*}]$ <br>  $\frac{Z_{\text{in}}}{Z_{\text{in}} + Z_g}|^2 \text{Re}[\frac{1}{Z_{\text{in}}^*}]$ <br>
general source and load conditions:  $Z$ **Impedance Matchings**<br>
veraging) Power delivered to load:<br>
= Re[ $\frac{V_{\text{in}}}{\sqrt{2}} \frac{I_{\text{in}}}{\sqrt{2}}$ ] =  $\frac{1}{2}$  Re[ $V_{\text{in}} I_{\text{in}}^*$ ] =  $\frac{1}{2} |V_{\text{in}}|^2$  Re[ $\frac{1}{Z_{\text{in}}^*}$ ]<br>
=  $\frac{1}{2} |V_g|^2$   $\left| \frac{Z_{\text{in}}}{Z_{\text{in}} + Z$  $+Z_{\rm e}$   $Z_{\rm in}$ <sup>\*</sup>  $V_{\text{in}} I_{\text{in}}^* = \frac{1}{2} |V_{\text{in}}|^2 \text{ Re}[\frac{1}{Z_{\text{in}}^*}]$ <br>  $V_g \bigotimes \frac{Z_g}{Z_{\text{in}}^*} \xrightarrow{V_{\text{in}}} Z_{\text{in}} \xrightarrow{Z_{\text{in}}} Z_{\text{in}}$ <br>  $\frac{1}{Z_{\text{in}}^*}$ <br>  $\therefore$   $Z_{\text{in}} = R_{\text{in}} + jX_{\text{in}}$  and  $Z_g = R_g + jX_g$ <br>  $\frac{2}{X_{\text{in}} + X_g}$ <br>  $\frac{2}{X_{\text{in$  $\sqrt{\frac{1}{2}} \frac{1}{\sqrt{2}} = \frac{1}{2} \text{Re}[V_{in}I_{in}] = \frac{1}{2}|V_{in}|$   $\text{Re}[\frac{1}{Z_{in}^{*}}]$ <br>  $\begin{aligned}\n &\begin{bmatrix}\nZ_{in} \\
Z_{in} + Z_{g}\n\end{bmatrix}^{2}$   $\text{Re}[\frac{1}{Z_{in}^{*}}]$ <br>  $\text{Re}[\frac{1}{Z_{in}^{*}}]^{2}$ <br>  $\text{Re}[\frac{1}{Z_{in}^{*}}]^{2}$ <br>  $\text{Re}[\frac{Z_{in}|^{2}}{(R_{in} + R_{g})^{2} +$ **In pedance Matchings**<br>  $\begin{array}{l}\n\text{r}_2 \frac{1}{\sqrt{2}} \int_{\sqrt{2}}^{\sqrt{2}} \frac{1}{\sqrt{2}} e^{-\frac{1}{2} \int_{\sqrt{2$ raging) Power delivered to load:<br>  $\text{Re}[\frac{V_m}{\sqrt{2}} \frac{I_m^*}{\sqrt{2}}] = \frac{1}{2} \text{Re}[V_{ia}I_m^*] = \frac{1}{2} |V_{ia}|^2 \text{Re}[\frac{1}{Z_m^*}]$ <br>  $V_{ii} \bigotimes \frac{\sqrt{2}}{Z_m} \bigotimes \frac{I_{\infty}^{-1} - I_{ia}}{Z_m}$ <br>  $V_{ii} \bigotimes \frac{Z_m - I_{ia}}{Z_m - Z_m}$ <br>  $V_{ii} \bigotimes \frac{Z_m - I_{ia}}{Z_m - Z$ Example Provident to boat.<br>
Ret $\left[\frac{V_{\text{in}}}{\sqrt{2}}\frac{I_{\text{in}}}{\sqrt{2}}\right] = \frac{1}{2} \text{Re}[V_{\text{in}}I_{\text{in}}^*]=\frac{1}{2}|V_{\text{in}}|^2 \text{Re}[\frac{1}{Z_{\text{in}}^*}]$ <br>  $V_{\text{in}} \left[\frac{Z_{\text{in}}}{Z_{\text{in}}}\right] = \frac{1}{2}|V_{\text{in}}|^2 \text{Re}[\frac{1}{Z_{\text{in}}^*}]$ <br>
Summe genera **Impedance Matchings**<br>
Averaging) Power delivered to load:<br>  $P = \text{Re}[\frac{V_{\text{in}}}{\sqrt{2}}\frac{I_{\text{in}}}{\sqrt{2}}] = \frac{1}{2}\text{Re}[V_{\text{in}}I_{\text{in}}^*] = \frac{1}{2}|V_{\text{in}}|^2 \text{ Re}[\frac{1}{Z_{\text{in}}^*}]$ <br>  $= \frac{1}{2}|V_{\text{in}}|^2 \left|\frac{Z_{\text{in}}}{Z_{\text{in}} + Z_{\text{in}}}\right|^2 \$ **Example 12 Converted to load:**<br> **Power delivered to load:**<br>  $\frac{r_{\text{in}}}{\sqrt{2}} = \frac{1}{2} \text{Re} [Y_{\text{in}} I_{\text{in}}^*] = \frac{1}{2} |V_{\text{in}}|^2 \text{Re} [\frac{1}{Z_{\text{in}}^*}]$ <br>  $\frac{Z_{\text{in}}}{Z_{\text{in}}^* + Z_{\text{in}}} = \frac{1}{2} |V_{\text{in}}|^2 \text{Re} [\frac{1}{Z_{\text{in}}^*}]$ <br> ging) Power delivered to load:<br>  $\left|Y_x\right|^2 \frac{I_w}{\sqrt{2}} \frac{I_w}{\sqrt{2}} = \frac{1}{2} \text{Re}[V_w I_w^*] = \frac{1}{2} |V_w|^2 \text{ Re}[\frac{1}{Z_m^*}]$ <br>  $\left|V_x\right|^2 \left|\frac{Z_m}{Z_m + Z_g}\right|^2 \text{ Re}[\frac{1}{Z_m^*}]$ <br>  $\left|V_x\right|^2 \left|\frac{Z_m}{Z_m + Z_g}\right|^2 \text{ Re}[\frac{1}{Z_m^*}]$ <br>  $\left|V_x\right|^2 \frac{Z_m -$ *P*  $\frac{V_m^2}{\sqrt{2}} = \frac{1}{2} \text{Re}[V_{\text{in}}I_{\text{in}}^{\dagger}] = \frac{1}{2} |V_{\text{in}}|^2 \text{ Re}[\frac{1}{Z_{\text{in}}^2}]$ <br>  $\frac{Z_{\text{in}}}{\sqrt{2}} = \frac{Z_{\text{in}}}{2} \text{ Re}[\frac{1}{Z_{\text{in}}^2}]$ <br>  $\frac{Z_{\text{in}}}{\sqrt{2}} = \frac{Z_{\text{in}}}{2} \text{ Re}[\frac{1}{Z_{\text{in}}^2}]$ <br>  $\frac{Z_{\text{in}}}{\sqrt{2}} = \frac{Z$ **Impedance Matchings**<br>
veraging) Power delivered to load:<br>  $= \text{Re}\left[\frac{Y_n}{\sqrt{2}}\frac{I_n'}{\sqrt{2}}\right] = \frac{1}{2}\text{Re}\left[Y_m I_m' \right] = \frac{1}{2}|V_m|^{2} \text{Re}\left[\frac{1}{Z_m'}\right]$ <br>  $= \frac{1}{2}|V_n|^2 \left|\frac{Z_n}{Z_n + Z_g}\right|^{2} \text{Re}\left[\frac{1}{Z_n'}\right]$ <br>
assume general source **Impedance Matchings**<br>
veraging) Power delivered to load:<br>  $= \text{Re} \left[ \frac{V_n}{\sqrt{2}} \frac{I_{\infty}^*}{\sqrt{2}} \right] = \frac{1}{2} \text{Re} \left[ V_{\text{in}} I_{\text{in}}^* \right] = \frac{1}{2} |V_{\text{in}}|^2 \text{ Re} \left[ \frac{1}{Z_{\text{in}}^*} \right]$ <br>  $= \frac{1}{2} |V_g|^2 \left| \frac{Z_n}{Z_n + Z_g} \right|^2 \text{ Re} \$ ver definited to foad.<br>  $V_{\text{in}} = \frac{1}{2} \text{Re}[V_{\text{in}}V_{\text{in}}^{\dagger}] = \frac{1}{2} |V_{\text{in}}|^2 \text{ Re}[\frac{1}{Z_{\text{in}}^{\dagger}}]$ <br>  $V_{\text{in}} \left( \frac{Z_{\text{in}}}{Z_{\text{in}}^{\dagger}} \right)$ <br>  $V_{\text{in}} \left( \frac{Z_{\text{in}}}{Z_{\text{in}}^{\dagger}} \right)$ <br>  $V_{\text{in}} \left( \frac{Z_{\text{in}}}{Z_{\text{in}}^{\dagger$
- *-* Let assume general source and load conditions:  $Z_{in} = R_{in} + jX_{in}$  and  $Z_g = R_g + jX_g$

$$
V_{\rm g} \bigotimes Z_{\rm in} V_{\rm g} \bigotimes Z_{\rm in} V_{\rm g} \bigotimes Z_{\rm in} Z_{\
$$

*-* Assume that the generator impedance (*Zg*) is fixed and consider three cases according to load impedance.

■ Case study 1: load matched to transmission line

$$
Z_l = Z_0 \longrightarrow \Gamma_l = 0 \& (V)SWR = 1.
$$
  
-  $Z_{in} = Z_0 \neq Z_g$ 

- 
- *-* Power delivered to load:

**1 Impedance Matchings**  
\nCase study 1: load matched to transmission line  
\n
$$
Z_{1} = Z_{0} \rightarrow \Gamma_{I} = 0 \& (V)SWR = 1.
$$
\n
$$
Z_{in} = Z_{0} \neq Z_{g}
$$
\n
$$
= \frac{1}{2} |V_{g}|^{2} \frac{R_{in}}{(R_{in} + R_{g})^{2} + (X_{in} + X_{g})^{2}} \Big|_{Z_{1} = Z_{0} = R_{0}} = \frac{1}{2} |V_{g}|^{2} \frac{Z_{0}}{(Z_{0} + R_{g})^{2} + X_{g}^{2}}
$$
\n**1**\n**1**\n**2**\n**1**\n**2**\n**2**\n**3**\n**3**\n**4**\n**4**\n**5**\n**5**\n**6**\n**6**\n**6**\n**7**\n**7**\n**8**\n**8**\n**9**\n**10**\n**10**\n**11**\n**12**\n**13**\n**14**\n**15**\n**16**\n**17**\n**18**\n**19**\n**10**\n**10**\n**10**\n**11**\n**12**\n**13**\n**14**\n**15**\n**16**\n**17**\n**18**\n**19**\n**10**\n**10**\n**10**\n**11**\n**12**\n**13**\n**14**\n**15**\n**16**\n**17**\n**18**\n**19**\n**10**\n**10**\n**11**\n**11**\n**12**\n**13**\n**13**\n**14**\n**15**\n**16**\n**17**\n**18**\n**19**\n**19**\n**10**\n**10**\n**11**

- § Case study 3: *conjugate matching*
	- Assumption: fixed generator impedance (*Zg*)
	- Variable input impedance  $(Z_{in})$  due to unknown load impedance  $(Z_l)$  and length of transmission line Conditions for maximum power delivered to load:
	-

**Impedance Matchings**  
\nCase study 3: *conjugate matching*  
\nAssumption: fixed generator impedance 
$$
(Z_{\mu})
$$
  
\nVariable input impedance  $(Z_{\mu})$  due to unknown load impedance  $(Z_{l})$  and length of transmission line  
\nCondition for maximum power delivered to load:  
\nCondition 1:  $\frac{\partial P}{\partial R_{in}} = \frac{\partial}{\partial R_{in}} \left[ \frac{1}{2} |V_{\mu}|^{2} \frac{R_{in}}{(R_{in} + R_{g})^{2} + (X_{in} + X_{g})^{2}} \right] = 0 \leftarrow \frac{d}{dx} \left( \frac{f}{g} \right) = \frac{f'g - fg'}{g^{2}}$   
\n
$$
\rightarrow \frac{1}{(R_{in} + R_{g})^{2} + (X_{in} + X_{g})^{2} + (X_{in} + R_{g})^{2} + (X_{in} + R_{g})^{2}} = 0
$$
\n
$$
(R_{in} + R_{g})^{2} + (X_{in} + X_{g})^{2} - 2R_{in} (R_{in} + R_{g}) - 0
$$
\n
$$
R_{g}^{2} - R_{in}^{2} + (X_{in} + X_{g})^{2} = 0 \qquad (*)
$$
\nCondition 2:  $\frac{\partial P}{\partial X_{in}} = 0 \rightarrow \frac{-2R_{in} (X_{in} + X_{g})}{[(R_{in} + R_{g})^{2} + (X_{in} + X_{g})^{2}]^{2}} = 0$   
\n
$$
R_{in} (X_{in} + X_{g}) = 0 \qquad (*)
$$
\n
$$
R_{in} (X_{in} + X_{g}) = 0 \qquad (*)
$$
\n
$$
R_{in} (X_{in} + X_{g}) = 0 \qquad (*)
$$
\n
$$
R_{in} (X_{in} + X_{g}) = 0 \qquad (*)
$$
\n13

- Solving (\*) and (\*\*) simultaneously for  $R_{\text{in}} \neq 0$ ) and  $X_{\text{in}}$ 

 $(R_{\text{in}} = R_g \text{ and } X_{\text{in}} = -X_g) \leftrightarrow Z_{\text{in}} = Z_g^*$ <br>- Maximum power delivered to load:

**Impedance Matchings**  
\nolving (\*) and (\*\*) simultaneously for 
$$
R_{in} \neq 0
$$
) and  $X_{in}$   
\n $(R_{in} = R_g$  and  $X_{in} = -X_g$ )  $\leftrightarrow Z_{in} = Z_g^*$   
\n  
\n**Maximum power delivered to load:**  
\n
$$
P = \frac{1}{2} |V_g|^2 \frac{R_{in}}{(R_{in} + R_g)^2 + (X_{in} + X_g)^2} \Big|_{R_{in} = R_g \& X_{in} = -X_g}
$$
\n
$$
= \frac{1}{2} |V_g|^2 \frac{R_g}{(R_g + R_g)^2 + (-X_g + X_g)^2} = \frac{1}{2} |V_g|^2 \frac{1}{4R_g}
$$
\n  
\n**Conjugate matching**



# è **Conjugate matching**

è **Maximum power transfer** to load for fixed generator impedance

Solving (\*) and (\*) simultaneously for 
$$
R_{in} \neq 0
$$
 and  $X_{in}$   
\n $(R_{in} = R_g$  and  $X_{in} = -X_g$ )  $\Leftrightarrow Z_{in} = Z_g^*$   
\nMaximum power delivered to load:  
\n
$$
P = \frac{1}{2} |V_g|^2 \frac{R_n}{(R_{in} + R_g)^2 + (X_{in} + X_g)^2} \Big|_{R_{in} = R_g \& x_{in} = X_g}
$$
\n
$$
= \frac{1}{2} |V_g|^2 \frac{R_g}{(R_g + R_g)^2 + (-X_g + X_g)^2} = \frac{1}{2} |V_g|^2 \frac{1}{4R_g}
$$
\n
$$
\Leftrightarrow \text{Conjugate matching}
$$
\n
$$
\Leftrightarrow \text{Maximum power transfer to load for fixed generator impedance}
$$
\n
$$
F = \frac{1}{2} |V_g|^2 \frac{R_n}{(R_{in} + R_g)^2 + (X_{in} + X_g)^2} \Big|_{Z_g = Z_g = R_m} = \frac{1}{2} |V_g|^2 \frac{Z_0}{(Z_0 + R_g)^2 + X_g^2}
$$
 (case study #1)\n
$$
P = \frac{1}{2} |V_g|^2 \frac{R_n}{(R_{in} + R_g)^2 + (X_{in} + X_g)^2} \Big|_{Z_g = Z_n} = \frac{1}{2} |V_g|^2 \frac{R_g}{4(R_g^2 + X_g^2)}
$$
 (case study #2)



- Network analyzer
- General and widely used circuit measurement system 1 ~ *<sup>n</sup>* ports network measurement **Review**<br>
twork analyzer<br>
eneral and widely used circuit measurement system<br>  $\sim n$  ports network measurement<br>
agnitude and phase<br>
parameters, RL, IL, GD, etc<br>
4 Impedance Transformer<br>
pedance matching<br>  $Z_{in} = Z_g^*$  or  $Z_g =$ malyzer<br> *Z* and widely used circuit measurement system<br> *z* and phase<br> *Z* and phase<br> *Z* and phase<br> *Z* and **EVIEW**<br>
ork analyzer<br>
ral and widely used circuit measurement system<br>
ports network measurement<br>
nitude and phase<br>
ameters, RL, IL, GD, etc<br>
npedance Transformer<br>
lance matching<br>  $= Z_g^*$  or  $Z_g = Z_m^*$ <br>  $\Leftrightarrow Z_m = R_m + jX_m = R_g -$ 
	-
	- Magnitude and phase
	- *S*-parameters, RL, IL, GD, etc
- § *λ*/4 Impedance Transformer
- § Impedance matching
	- $Z_{\text{in}} = Z_{\text{g}}^{*}$  or  $Z_{\text{g}} = Z_{\text{in}}^{*}$
- de and phase<br>
tters, RL, IL, GD, etc<br>
dance Transformer<br>
e matching<br>  $\int_{g}^{*}$  or  $Z_{g} = Z_{in}^{*}$ <br>  $Z_{in} = R_{in} + jX_{in} = R_{g} jX_{g}^{*}$ gnitude and phase<br>
arameters, RL, IL, GD, etc<br>
Impedance Transformer<br>
edance matching<br>  $\lim_{\text{in}} = Z_g^*$  or  $Z_g = Z_{\text{in}}^*$ <br>  $\Leftrightarrow Z_{\text{in}} = R_{\text{in}} + jX_{\text{in}} = R_g - jX_g^*$ \* rs, RL, IL, GD, etc<br>
nce Transformer<br>
matching<br>
or  $Z_g = Z_{in}^*$ <br>  $\vdots$  =  $R_{in} + jX_{in} = R_g - jX_g^*$ <br>
erence plane de and phase<br>
eters, RL, IL, GD, etc<br>
dance Transformer<br>  $g_{g}$  or  $Z_{g} = Z_{in}^{*}$ <br>  $Z_{in} = R_{in} + jX_{in} = R_{g} - jX_{g}^{*}$ <br>
afore need plane. etc<br> $g - jX_g^*$ 
	- At any reference plane