Chapter 2
Transmission Line

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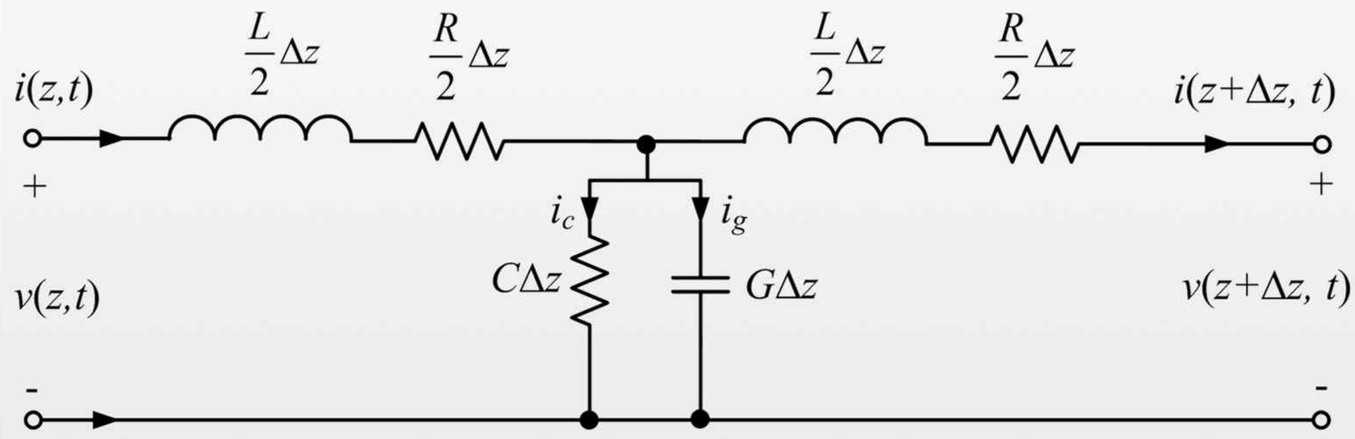


Learning Objectives

- Learn what the maximum power transmission condition is.
- Understanding real low loss transmission line
- Learn what is different for using lossy transmission instead of lossless transmission line

Learning contents

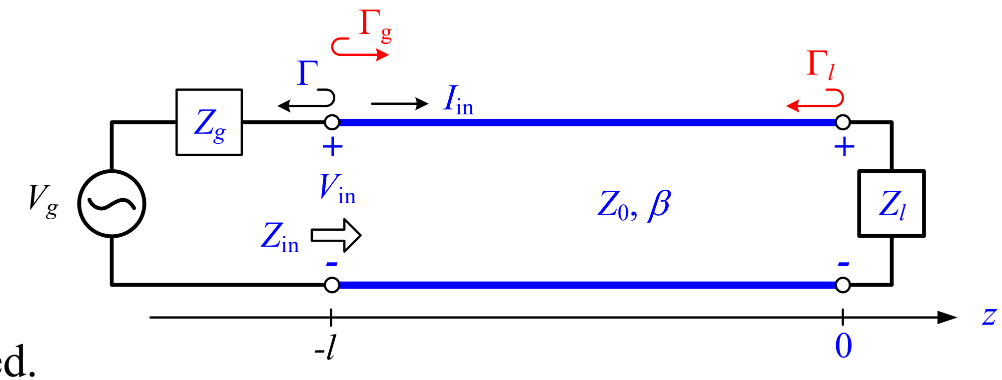
- Maximum Power Transmission Condition
- Lossy Transmission Lines
- Terminated Lossy Transmission Line

1 Maximum Power Transmission Condition

- Lossless transmission line circuit with arbitrary generator and load impedances

  - Input impedance looking into terminated transmission line from generator:

$$Z_{in} = \frac{V_{in}}{I_{in}} = Z_0 \frac{Z_l + jZ_0 \tan \beta l}{Z_0 + jZ_l \tan \beta l}$$



  - Let assume general source and load conditions:

$Z_{in} = R_{in} + jX_{in}$ and $Z_g = R_g + jX_g$

$$P = \frac{1}{2}\left|V_g\right|^2 \frac{R_{in}}{(R_{in}+R_g)^2 + (X_{in}+X_g)^2}$$

  - Let's assume that the generator impedance (Z_g) is only fixed.
  - Power transmission for load matched to transmission line (case study 1)

$$P = \frac{1}{2}\left|V_g\right|^2 \frac{R_{in}}{(R_{in}+R_g)^2 + (X_{in}+X_g)^2}\Bigg|_{Z_l=Z_0=R_{in}} = \frac{1}{2}\left|V_g\right|^2 \frac{Z_0}{(Z_0+R_g)^2 + X_g^2}$$

  - Power transmission for generator matched to transmission line (case study 2)

$$P = \frac{1}{2}\left|V_g\right|^2 \frac{R_{in}}{(R_{in}+R_g)^2 + (X_{in}+X_g)^2}\Bigg|_{Z_g=Z_{in}} = \frac{1}{8}\left|V_g\right|^2 \frac{R_g}{R_g^2 + X_g^2}$$

1 Maximum Power Transmission Condition

- Power transmission for generator matched to transmission line (case study 3)

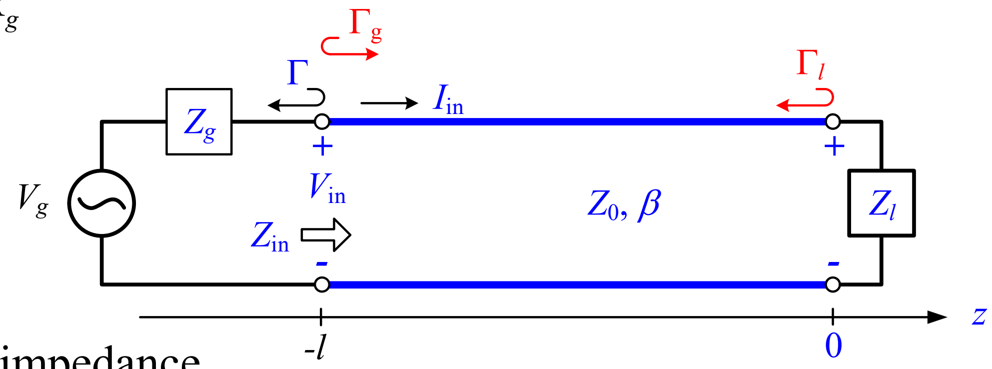
$$P = \frac{1}{2}|V_g|^2 \frac{R_{\text{in}}}{(R_{\text{in}}+R_g)^2+(X_{\text{in}}+X_g)^2}\Bigg|_{R_{\text{in}}=R_g \;\&\; X_{\text{in}}=-X_g} = \frac{1}{2}|V_g|^2 \frac{1}{4R_g}$$

- Maximum power condition: case study 3

$$Z_{\text{in}} = Z_g^*$$

➔ **Conjugate matching**

➔ **Maximum power transfer** to load for fixed generator impedance



cf.) $$P = \frac{1}{2}|V_g|^2 \frac{R_{\text{in}}}{(R_{\text{in}}+R_g)^2+(X_{\text{in}}+X_g)^2}\Bigg|_{Z_l=Z_0=R_{\text{in}}} = \frac{1}{2}|V_g|^2 \frac{Z_0}{(Z_0+R_g)^2+X_g^2}$$ (case study 1)

$$P = \frac{1}{2}|V_g|^2 \frac{R_{\text{in}}}{(R_{\text{in}}+R_g)^2+(X_{\text{in}}+X_g)^2}\Bigg|_{Z_g=Z_{\text{in}}} = \frac{1}{2}|V_g|^2 \frac{R_g}{4(R_g^2+X_g^2)}$$ (case study 2)

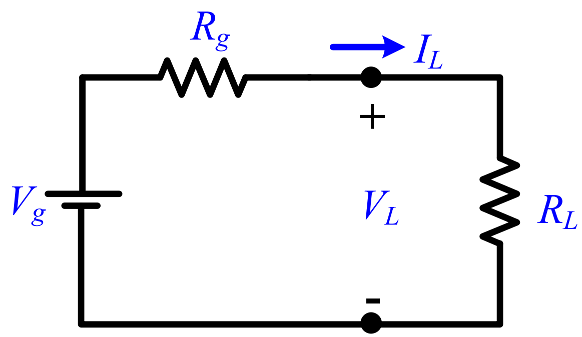
1 Maximum Power Transmission Condition

- **Maximum power transfer condition for DC related circuit** in condition of fixed source resistance

$$V_L = V_g \frac{R_L}{R_g + R_L}, \qquad I_L = \frac{V_g}{R_g + R_L}$$

$$P_L = I_L V_L = V_g^2 \frac{R_L}{(R_g + R_L)^2}$$

$$P_{L,\max} = \frac{dP_L}{dR_L} = V_g^2 \frac{(R_g + R_L)^2 - 2R_L(R_g + R_L)}{(R_g + R_L)^4} = V_g^2 \frac{R_g^2 - R_L^2}{(R_g + R_L)^4} = 0 \Leftrightarrow R_g = R_L$$



➔ The condition for **the maximum DC power transferring** to load is for ***same source and load resistances***.

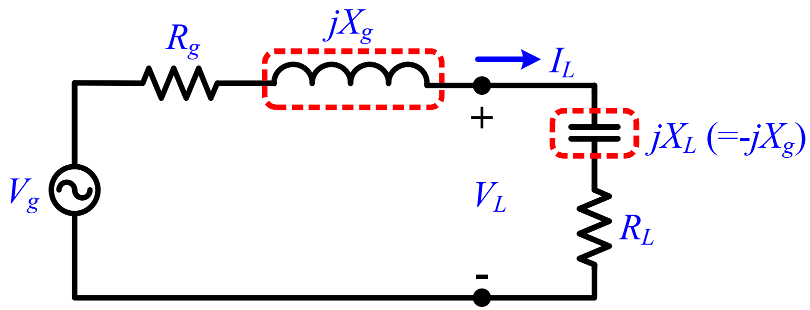
- **Maximum power transfer condition for AC or microwave circuit**

$$Z_L = Z_g^* \quad \text{or} \quad Z_g = Z_L^*$$

$$\Leftrightarrow \; Z_L = R_L + jX_L = R_g - jX_g^*$$

➔ The **maximum microwave power transfer** condition is same with the maximum DC power transfer condition **except resonance condition**.

➔ ***Frequency selective maximum power delivery condition***



2 Lossy Transmission Lines

- Effects of loss on transmission line behavior
- How many the attenuation constant can be calculated?

- General complex propagation constant:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{(j\omega L)(j\omega C)(1 + \frac{R}{j\omega L})(1 + \frac{G}{j\omega C})} = j\omega\sqrt{LC}\sqrt{1 - j(\frac{R}{\omega L} + \frac{G}{\omega C}) - \frac{RG}{\omega^2 LC}}$$

$$\beta = j\omega\sqrt{LC}\sqrt{1 - \frac{RG}{\omega^2 LC}}$$

- β is not a linear function of frequency (e.g., $\beta \neq a\omega$).

➔ Phase velocity ($v_p = \omega / \beta$) will be different for different frequencies ω.

➔ The various frequency components of a wideband signal will travel with different phase velocities.

➔ The arrival at the receiver end of the transmission line is at slightly different times. ➔ **<u>Dispersion</u>** (: a distortion of the signal)

➔ Undesirable effect

2 Lossy Transmission Lines

- For low-loss transmission line ($R << \omega L$, $G << \omega C$) (↔ low conductor loss and low dielectric loss),

$$\gamma \cong j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)} \quad \leftarrow RG << \omega^2 LC$$

By using $\sqrt{1+x} \approx 1 + x/2 + \cdots$,

$$\gamma = \alpha + j\beta \approx j\omega\sqrt{LC}\left[1 - \frac{j}{2}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)\right]$$

$$\alpha \approx \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) = \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right), \qquad \beta = \omega\sqrt{LC}$$

➔ β of low loss transmission line is almost similar to lossless transmission line.

where $Z_0 = (L/C)^{0.5}$: characteristic impedance of lossless transmission line

α: attenuation constant

β: phase constant

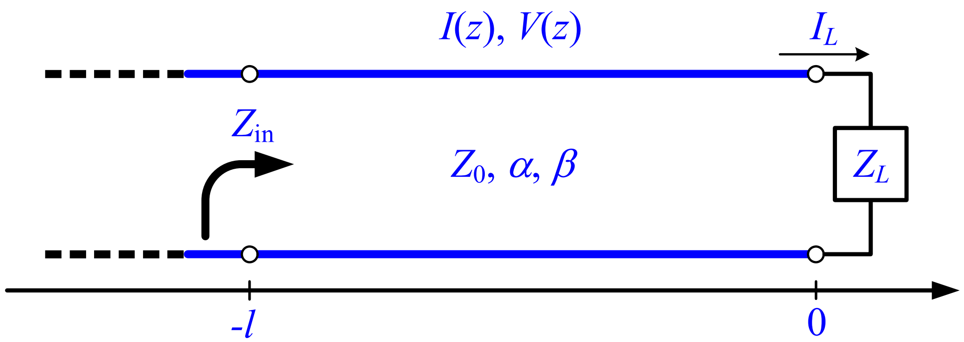
3 Terminated Lossy Transmission Line

- Lossy transmission line terminated in a load impedance Z_L

  - Complex propagation constant : $\gamma = \alpha + j\beta$

  - Assumption: low loss transmission line ($Z_0 \approx \sqrt{\frac{L}{C}}$)

  - Voltage and current waves on low loss transmission line



$$V(z) = V_0^+[e^{-\gamma z} + \Gamma e^{\gamma z}]$$

$$I(z) = \frac{V_0^+}{Z_0}[e^{-\gamma z} - \Gamma e^{\gamma z}]$$

where Γ: reflection coefficient at load (= (Z_L - Z_0) / (Z_L + Z_0)),

V_0^+: incident voltage amplitude referenced at z = -l

  - Reflection coefficient at a distance l from load:

$$\Gamma(-l) = \Gamma e^{-2j\beta l} e^{-2\alpha l} = \Gamma e^{-2\gamma l}$$

  - Input impedance Z_{in} at a distance l from load:

$$Z_{in} = \frac{V(-l)}{I(-l)} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$

3 Terminated Lossy Transmission Line

- Power delivered to input of terminated line at $z = -l$:

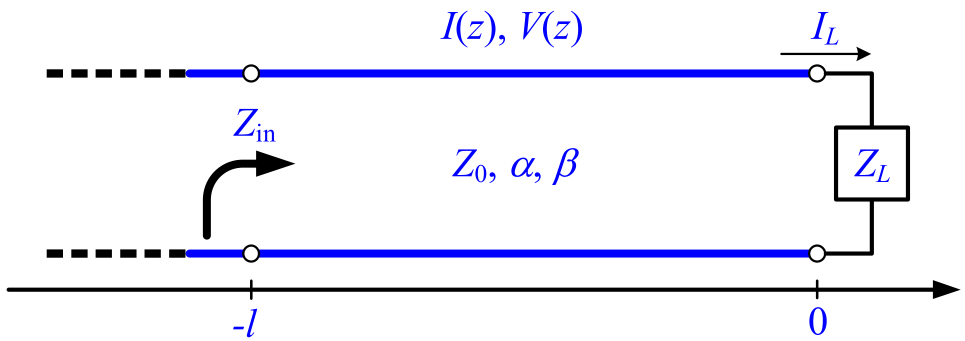
$$P_{\text{in}} = \frac{1}{2}\text{Re}\left[V(-l)I^*(-l)\right] = \frac{|V_0^+|^2}{2Z_0}\text{Re}\left[(e^{\gamma l} + \Gamma e^{-\gamma l})(e^{\gamma^* l} - \Gamma^* e^{-\gamma^* l})\right] \leftarrow \gamma = \alpha + j\beta$$

$$= \frac{|V_0^+|^2}{2Z_0}\text{Re}\left[(e^{\alpha l}e^{j\beta l} + \Gamma e^{-\alpha l}e^{-j\beta l})(e^{\alpha l}e^{-j\beta l} - \Gamma^* e^{-\alpha l}e^{j\beta l})\right]$$

$$= \frac{|V_0^+|^2}{2Z_0}\text{Re}\left\{(e^{2\alpha l} - |\Gamma|^2 e^{-2\alpha l} + \Gamma e^{-2j\beta l} - \Gamma^* e^{j2\beta l})\right\}$$

$$= \frac{|V_0^+|^2}{2Z_0}[e^{2\alpha l} - |\Gamma|^2 e^{-2\alpha l}] \leftarrow |\Gamma|^2 = \Gamma(0)\cdot\Gamma^*(0) = |\Gamma(-l)|^2 e^{4\alpha l},\ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{|V_0^+|^2}{2Z_0}\left[1 - |\Gamma(-l)|^2\right]e^{2\alpha l}$$



- Power delivered to load: $P_L = \frac{1}{2}\text{Re}\left[V(0)I^*(0)\right] = \frac{|V_0^+|^2}{2Z_0}(1 - |\Gamma|^2)$

Incident wave power loss

Reflected wave power loss

- Power loss through transmission line: $P_{\text{loss}} = P_{\text{in}} - P_L = \frac{|V_0^+|^2}{2Z_0}\left[(e^{2\alpha l} - 1) + |\Gamma|^2 (1 - e^{-2\alpha l})\right]$

- If $\alpha \uparrow$, also $P_{\text{loss}} \uparrow$

4 Review

- Impedance matching

  - $Z_{\text{in}} = Z_g^*$ or $Z_g = Z_{\text{in}}^*$

    $\Leftrightarrow \; Z_{\text{in}} = R_{\text{in}} + jX_{\text{in}} = R_g - jX_g^*$

  - At any reference plane

- Low loss transmission line

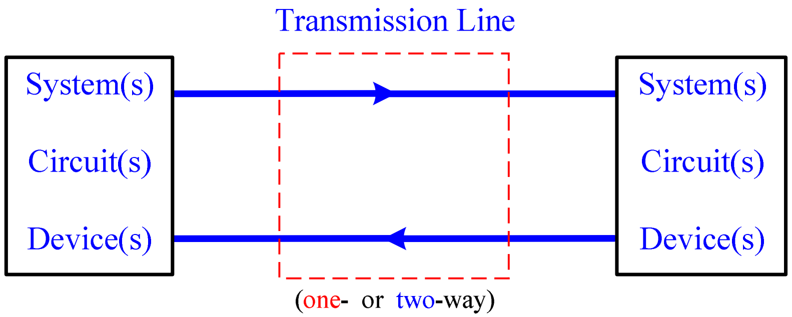
  - It is important to understand why smallest transmission line is used.

  - Signal loss and distortion

  - Incident and reflected wave power loss

5 Chapter Review

- **Transmission line (TRL)**: carrier to transmit electric energy, signals, data, and information from one point to another point with small insertion loss



- Analyzing approaches

  1) Maxwell's equations: electromagnetic field method (complete & accurate)
  2) Circuit equations: equivalent voltage and current analysis method (intuitive & inaccurate)

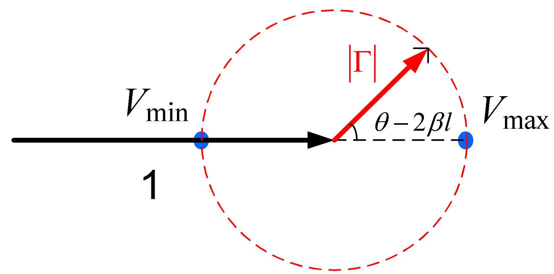
- General lumped elements (*R*, *L*, *C*, etc.) frequency dependent characteristics ➔ SRF

- Characteristic impedance: a ratio of voltage wave to current wave $Z_0 = \frac{V_o^+}{I_o^+} = -\frac{V_o^-}{I_o^-} = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \neq R_0$

- Complex propagation constant: $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} = f(\omega)$

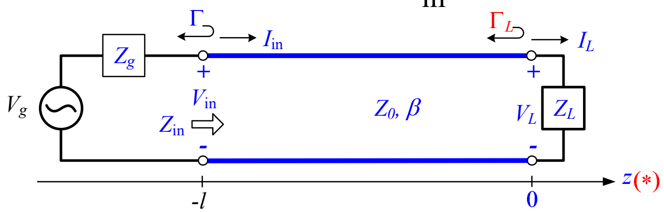
- Voltage reflection coefficient (Γ) = Reflected voltage wave amplitude / Incident voltage wave amplitude

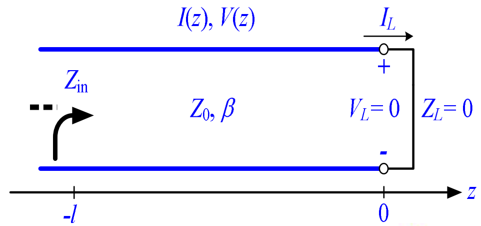
$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{|V_0^-|e^{j\phi^-}}{|V_0^+|e^{j\phi^+}} = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma|e^{j\theta}$$



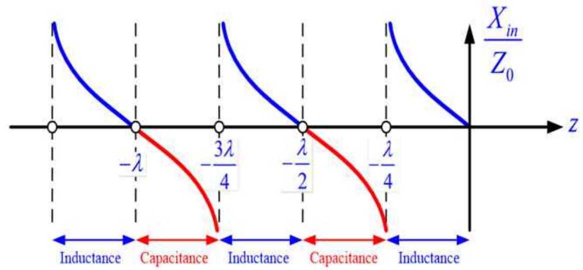
- **If $\Gamma = 0$, no reflected wave.** ➔ $Z_L = Z_0$ (**(Impedance) Matching!!**)
- (Voltage) Standing wave ratio (V**SWR**): $\text{SWR} = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$
- Two successive voltage maxima (or minima) on TRL: $l = \lambda / 2$
- Input impedance seen looking toward the load at $z = -l$: Z_{in}

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$





- TRL terminated with short or open circuit can be used inductor or capacitor in microwave frequency range.
  - Frequency and transmission line length dependent
  - Periodic characteristics

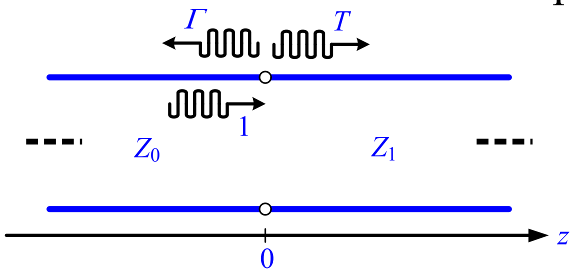


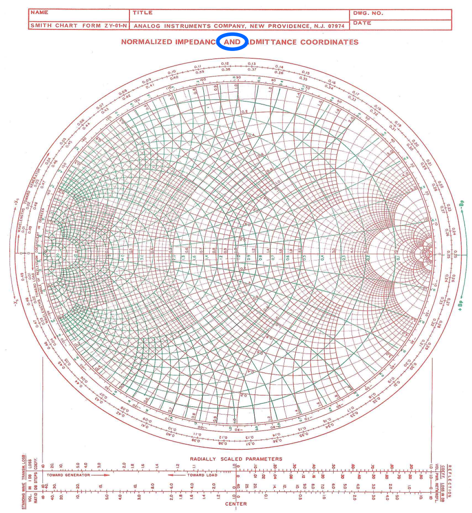
5 Chapter Review

- TRL of characteristic impedance Z_0 connected different characteristic impedance (Z_1) transmission line

  - Reflection coefficient: $\Gamma = \dfrac{Z_1 - Z_0}{Z_1 + Z_0}$

  - Reflection and transmission





- Smith chart
  - Reflection coefficient plane
  - Analyzing approaches for visualizing transmission line phenomenon
  - Intuition about transmission line and impedance-matching problems

- Slotted TRL
  - Waveguide, coaxial cable
  - Measurement issues: (V)SWR, distance of (first) voltage maximum (or minimum) from load, Z_L, λ, etc.
  - In real condition, network analyzer is used generally.

- Maximum power transmission condition: impedance conjugate matching

  $Z_{\text{in}} = Z_g^*$

- Lossy TRL: $P_{\text{loss}} = P_{\text{in}} - P_L = \dfrac{|V_0^+|^2}{2Z_0}\left[(e^{2\alpha l} - 1) + |\Gamma|^2 (1 - e^{-2\alpha l})\right]$