Chapter 4
Microwave Network Analysis

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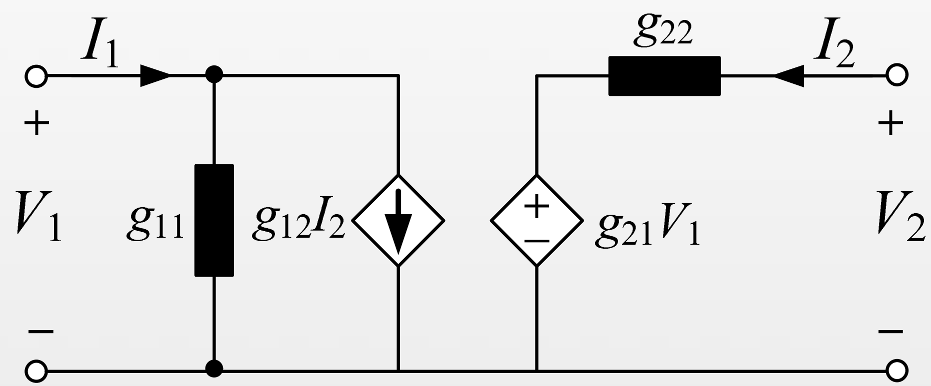


Learning Objectives

- Know about impedance parameters (or Z-parameters)
- Know about admittance parameters (or Y-parameters)
- Know about hybrid and inverse hybrid parameters (or h- and g-parameters)

Learning contents

- Impedance Parameters
- Admittance parameters
- Hybrid and Inverse Hybrid Parameters

1 Impedance Parameters

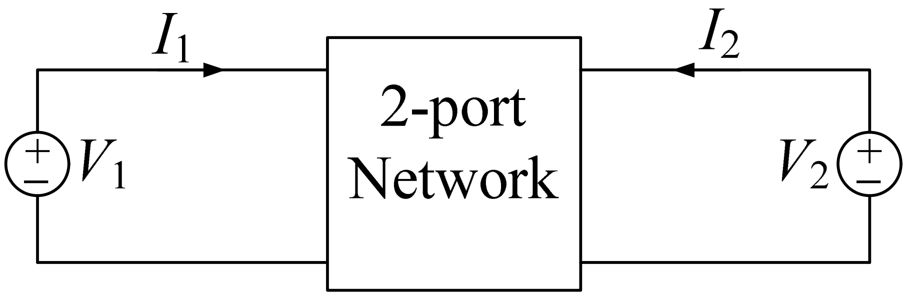
- **Impedance parameter (*Z*-parameters)**

  - *Z*-parameters describe the interactions between voltages and currents in 2-port network under open-circuited condition.

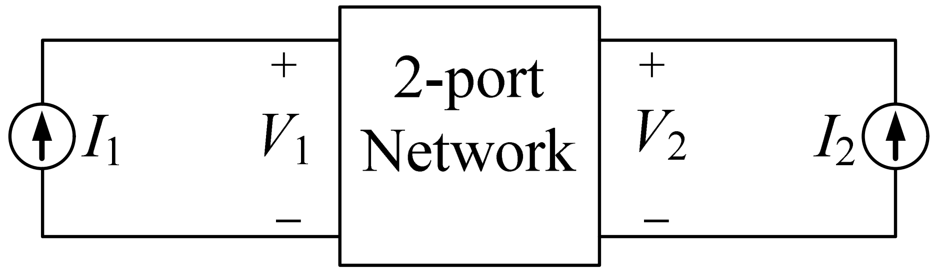
  - By setting input or output ports to be open-circuited, the *Z*-parameter of input and output ports can be calculated.

  - The input and output of a 2-port network can be either voltage or current.

  - Network driven by voltages



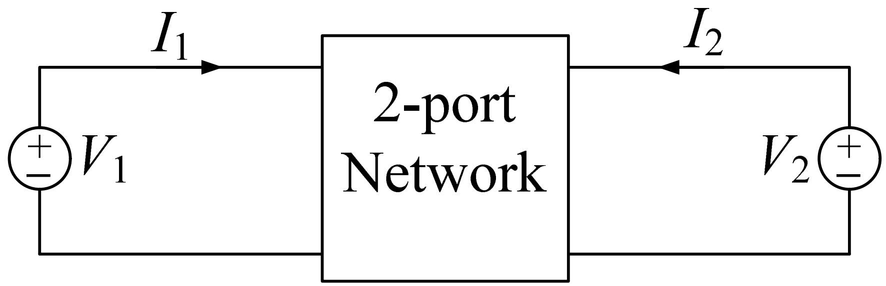
  - Network driven by currents



1 Impedance Parameters

- The ratios of voltage to current can be defined as:

$$\frac{V_1}{I_1},\ \frac{V_1}{I_2},\ \frac{V_2}{I_1},\ \text{and}\ \frac{V_2}{I_2}$$



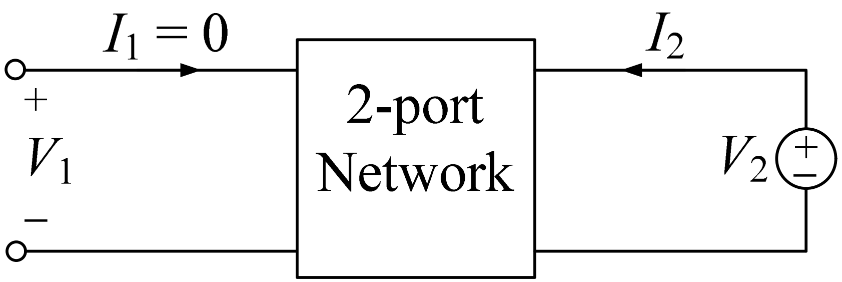
- These four ratios are considered as input, output, trans-impedance of the network:

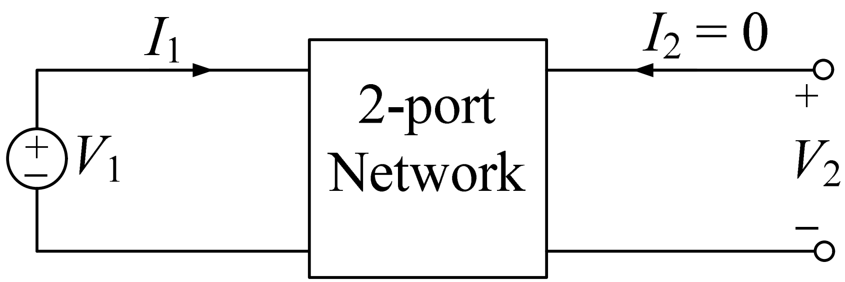
$$Z_{ij} = \frac{V_i}{I_j}$$

@ $i = j$, input and output impedance

@ $i \neq j$, forward or reverse trans-impedance

- The values of Z-parameters in a 2-port network can be determined with the condition of $I_1 = 0$ or $I_2 = 0$.
  ➔ Open conditions



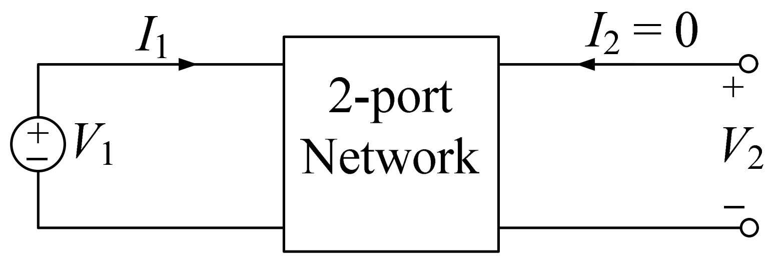


1 Impedance Parameters

- For $I_2 = 0$ or output port is open-circuited:

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$: Open-circuited input impedance

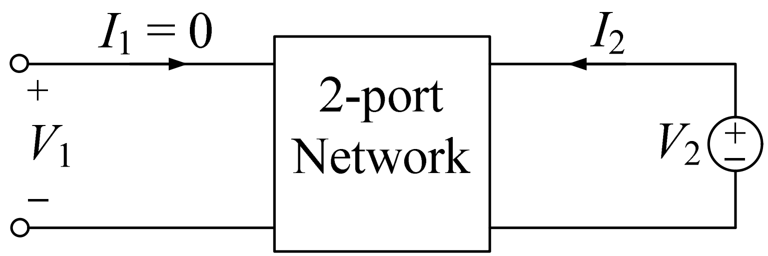
$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$: Open-circuited trans-impedance from input port to output port



- For $I_1 = 0$ or input port is open-circuited:

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$: Open-circuited output impedance

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$: Open-circuited trans-impedance from output port to input port

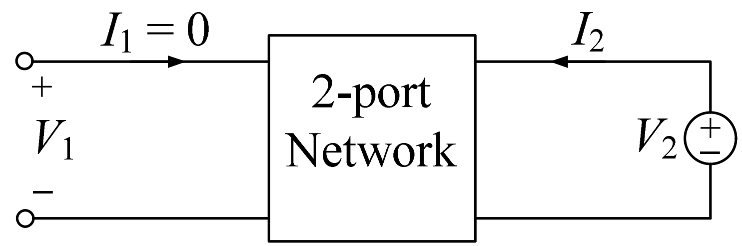


1 Impedance Parameters

- Relation between voltage and current of 2-port network with Z-parameters:

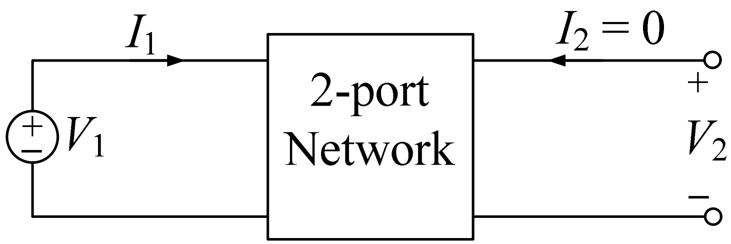
$$V_1 = Z_{11}I_1 + Z_{12}I_2 \qquad (1)$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \qquad (2)$$



- Matrix form of Z-parameters :

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



- For a symmetrical network: $Z_{11} = Z_{22}$

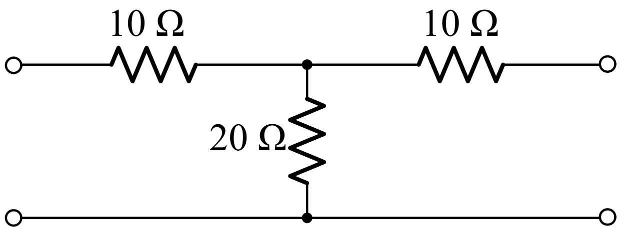
$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

- For reciprocal network: $Z_{21} = Z_{12}$

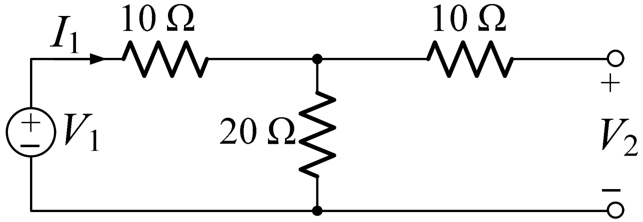
$$\left.\frac{V_2}{I_1}\right|_{I_2=0} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

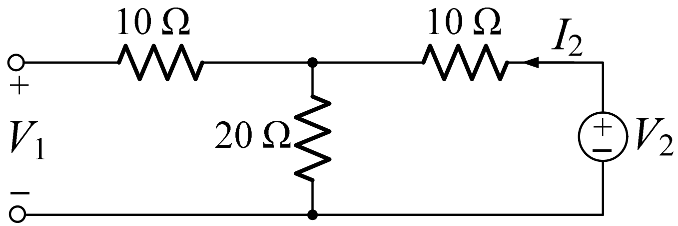
1 Impedance Parameters

- **Example:** Calculate the values of Z-parameter of the following circuit.



- Solution: The values of Z-parameter can be calculated as the following:





$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{(10+20)I_1}{I_1} = 30\ \Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{20I_1}{I_1} = 20\ \Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{(10+20)I_2}{I_2} = 30\ \Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{20I_2}{I_2} = 20\ \Omega$$

2 Admittance Parameters

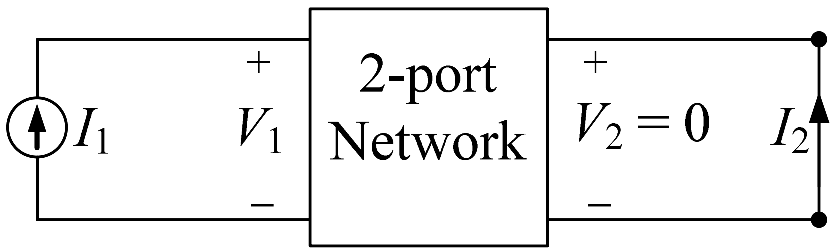
- **Admittance parameters (*Y*-parameters)**

  - *Y*-parameters describe the interactions between voltages and currents in 2-port network under short-circuited condition.

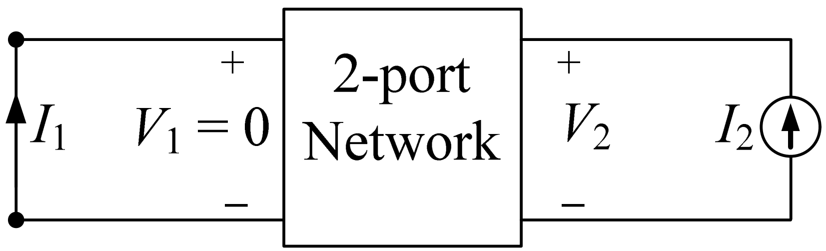
  - By setting input or output port to short-circuited, the *Y*-parameter of input and output ports can be calculated.

  - The input and output of a 2-port network can be either voltage or current.

  - Network driven by current at input port and short-circuited at output port



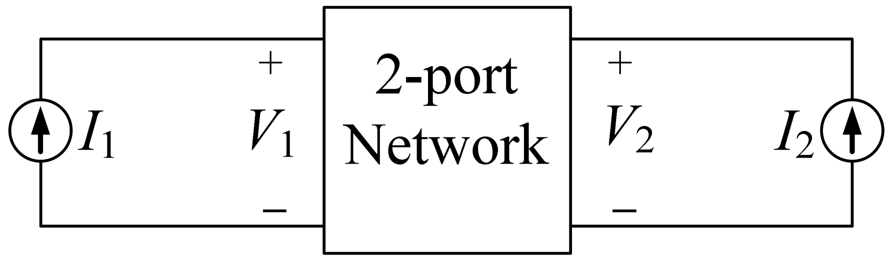
  - Network driven by currents by current at output port and short-circuited at input port



2 Admittance Parameters

- The ratio of current to voltage can be defined as:

$$\frac{I_1}{V_1},\ \frac{I_1}{V_2},\ \frac{I_2}{V_1},\ \text{and}\ \frac{I_2}{V_2}$$

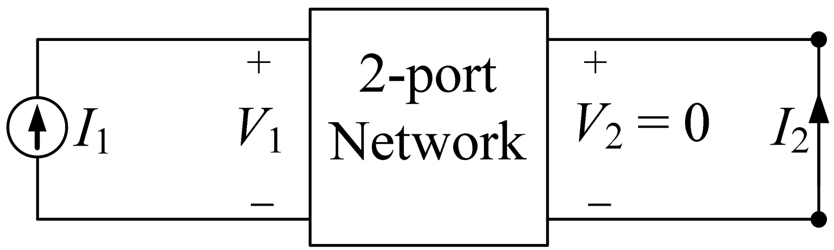


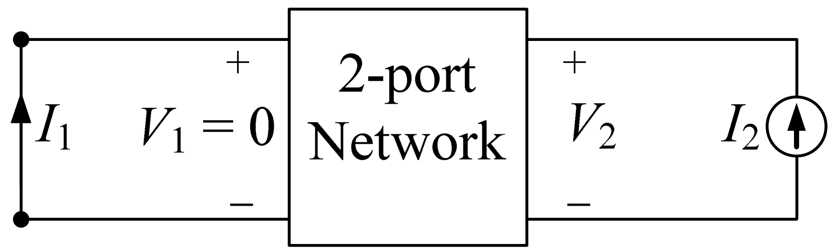
- These four ratios are considered as input, output, trans-admittance of the network:

$$Y_{ij} = \frac{I_i}{V_j}$$

- The values of Y-parameters in a 2-port network can be determined with the condition of $V_1 = 0$ or $V_2 = 0$.

➔ Short conditions



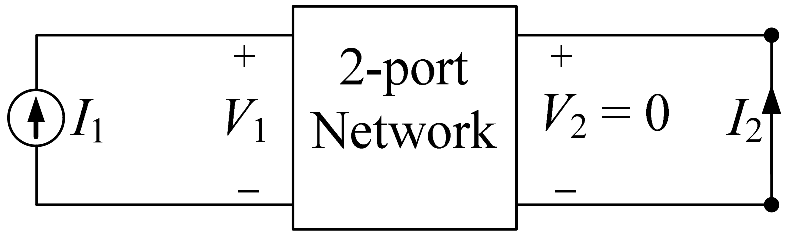


2 Admittance Parameters

- For $V_2 = 0$ or output port is short-circuited:

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$: Short-circuited input admittance

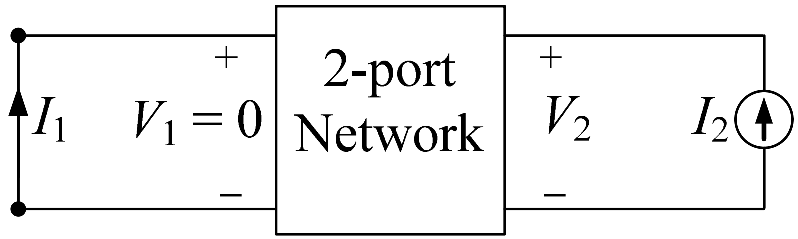
$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$: Short-circuited trans-admittance from input port to output port



- For $V_1 = 0$ or input port is short-circuited:

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$: Short-circuited output admittance

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$: Short-circuited trans-admittance from output port to input port

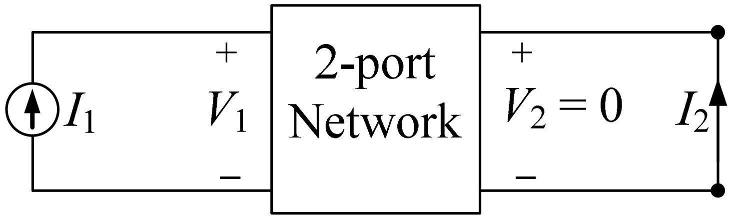


2 Admittance Parameters

- Relation between current and voltage of 2-port network with Y-parameters:

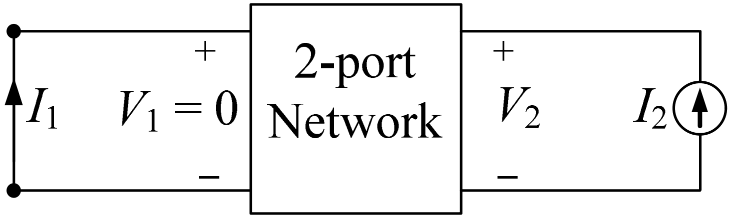
$$I_1 = Y_{11}V_1 + Y_{12}V_2 \qquad (3)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \qquad (4)$$



- Matrix form of Y-parameters:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$



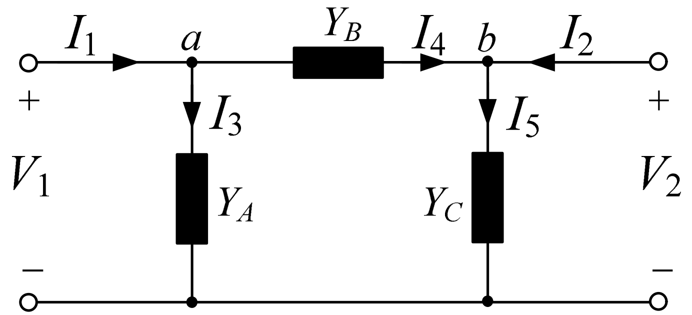
- For a symmetrical network: $Y_{11} = Y_{22}$

$$\left.\frac{I_1}{V_1}\right|_{V_2=0} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

- For reciprocal network: $Y_{21} = Y_{12}$

$$\left.\frac{I_2}{V_1}\right|_{V_2=0} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

- **Example:** Calculate Y-parameter of given network.



- Solution:

Applying Kirchoff's current law at node "*a*":

$$I_1 = I_3 + I_4 = V_1Y_A + (V_1 - V_2)Y_B = \left(Y_A + Y_B\right)V_1 - Y_BV_2 \qquad (5)$$

Apply Kirchoff's current law at node "*b*":

$$I_2 = I_5 - I_4 = V_2Y_C - (V_1 - V_2)Y_B = -Y_BV_1 + \left(Y_B + Y_C\right)V_2 \qquad (6)$$

Comparing equations (3) and (5):

$$Y_{11}V_1 + Y_{12}V_2 = \left(Y_A + Y_B\right)V_1 - Y_BV_2$$

$$\Rightarrow Y_{11} = Y_A + Y_B$$

$$\Rightarrow Y_{12} = -Y_B$$

Comparing equation (4) and (6):

$$Y_{21}V_1 + Y_{22}V_2 = -Y_BV_1 + \left(Y_B + Y_C\right)V_2$$

$$\Rightarrow Y_{21} = -Y_B$$

$$\Rightarrow Y_{22} = Y_B + Y_C$$

3 Hybrid Parameters

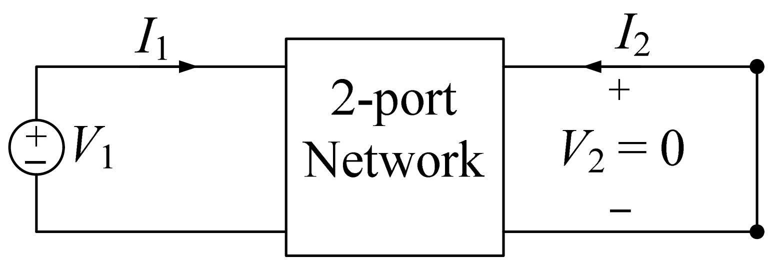
- **Hybrid parameters (*h*-parameters)**

  - *h*-parameters includes short-circuited input impedance and open-circuited output admittance for comprehensive network analysis.

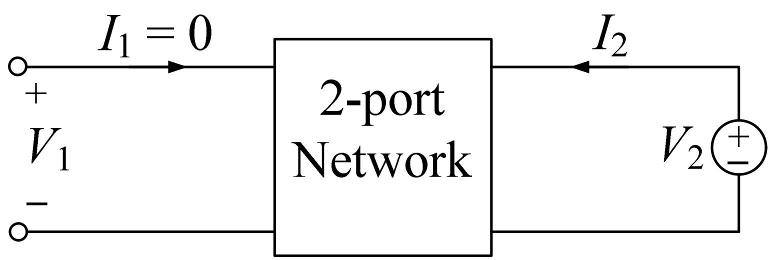
  - *h*-parameters use *Z*-parameter, voltage ratios, *Y*-parameter, and current ratios to shows the relationship between voltage and current in a 2-port network.

  - In cases where *Z*-parameters are impractical such as with the ideal transformer, *h*-parameter provides a viable solution for circuit analysis.

  - Network driven by voltage at input port and short-circuited at output port



  - Network driven by voltage at output port and open-circuited at input port



3 Hybrid Parameters

- Relation between current and voltage of 2-port network with *h*-parameters:

$$V_1 = h_{11} I_1 + h_{12} V_2 \qquad (7)$$

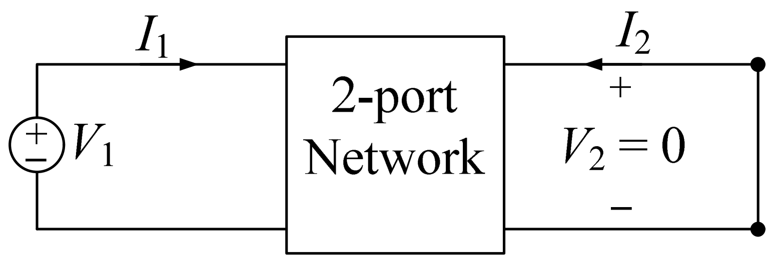
$$I_2 = h_{21} I_1 + h_{22} V_2 \qquad (8)$$

- Matrix form of *h*-parameters:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

- For $V_2 = 0$ or output port is short-circuited:

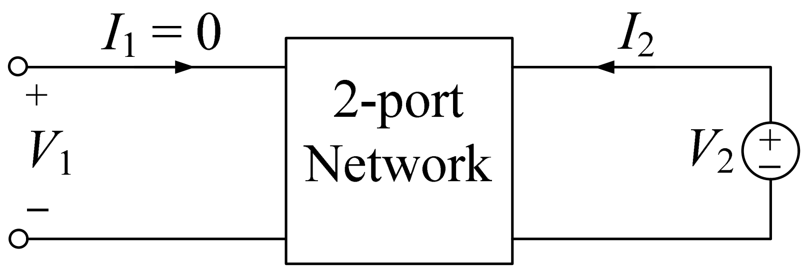
$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$: Short-circuited input impedance



$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$: Short-circuited current gain

3 Hybrid Parameters

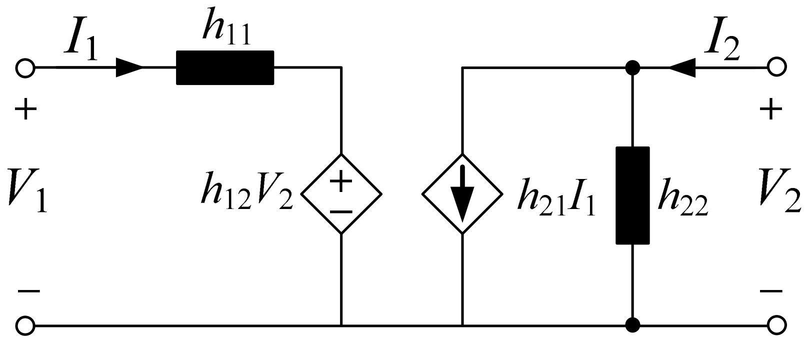
- For $I_1 = 0$ or input port is open-circuited:



$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$: Open-circuited output admittance

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$: Open-circuited reverse voltage gain

- *h*-parameter are crucial for analyzing bipolar junction transistors (BJTs).



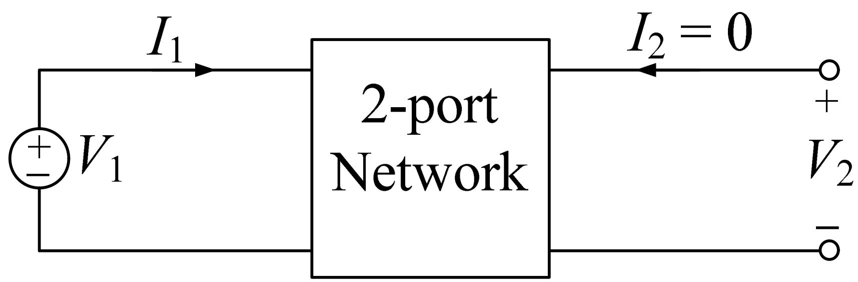
4 Inverse Hybrid Parameters

- **Inverse hybrid parameters (*g*-parameters)**

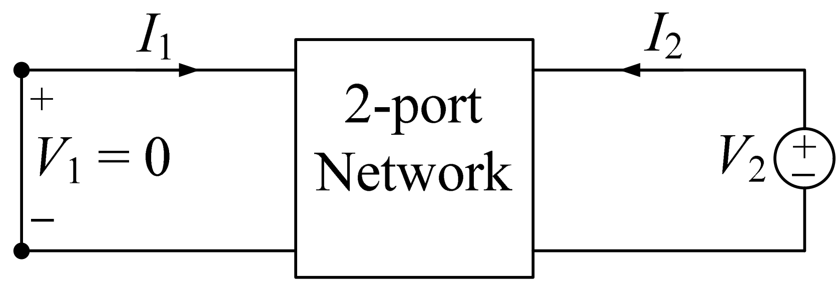
  - *g*-parameters can be obtained in a similar way to *h*-parameters.

  - The port termination conditions of *g*-parameters are inverse to *h*-parameters.

- Network driven by voltage at input port and open-circuited at output port



- Network driven by voltage at output port and short-circuited at input port



4 Inverse Hybrid Parameters

- Relation between current and voltage of 2-port network with g-parameters:

$$I_1 = g_{11}V_1 + g_{12}I_2 \qquad (9)$$

$$V_2 = g_{21}V_1 + g_{22}I_2 \qquad (10)$$

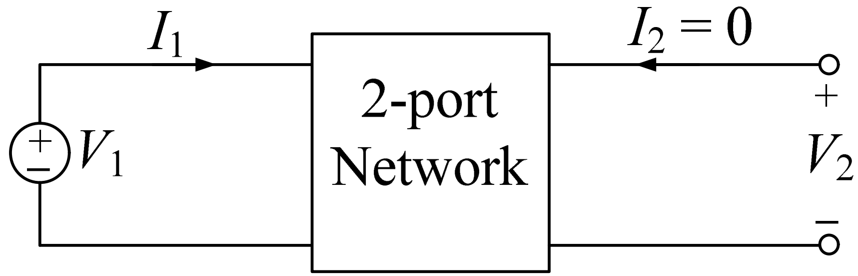
- Matrix form of g-parameters:

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

- For $I_2 = 0$ or output port is open-circuited:

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$$: Open-circuited input admittance

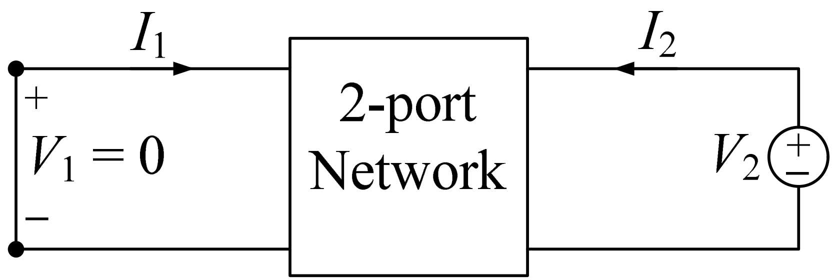
$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$: Open-circuited voltage gain



4 Inverse Hybrid Parameters

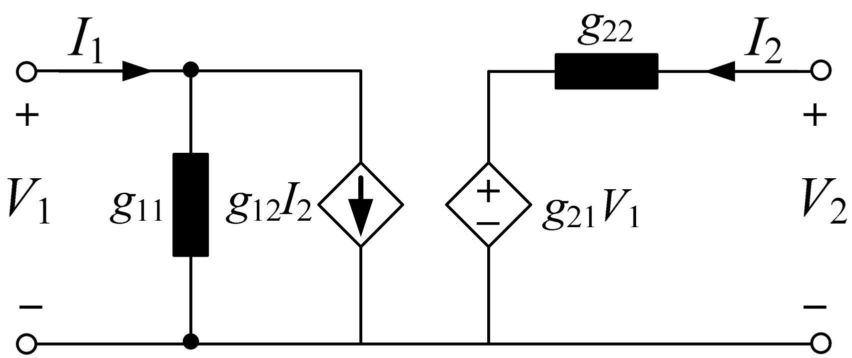
- For $V_1 = 0$ or input port is short-circuited:

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$: Short-circuited output impedance



$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$: Short-circuited reverse current gain

- *g*-parameter are crucial for analyzing junction field effect transistors (JFETs).



5 Review

- Impedance parameters or *Z*-parameters
- Admittance parameters or *Y*-parameters
- Hybrid parameters or *h*-parameters
- Inverse hybrid parameters or *g*-parameters