Chapter 4
Microwave Network Analysis

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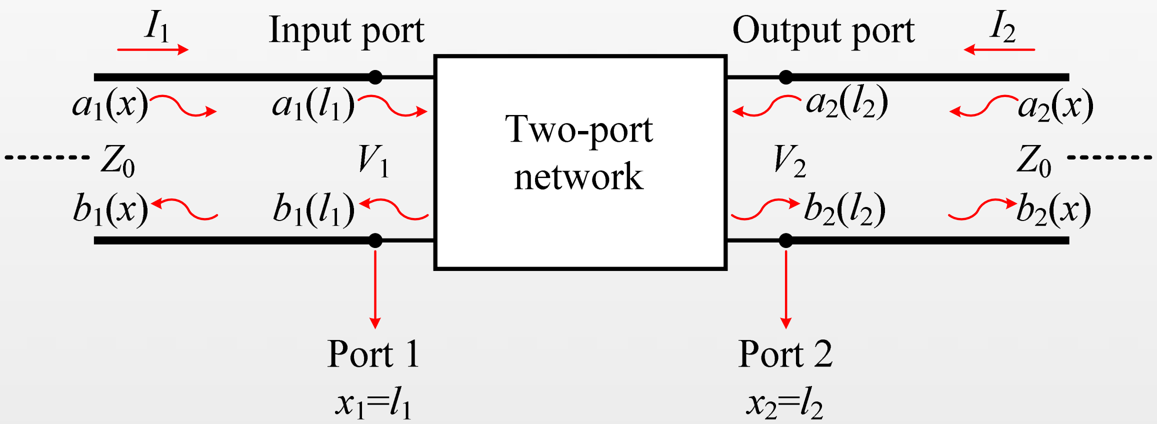


Learning Objectives

- Know about properties of Scattering matrix
- Know about *ABCD*-parameters
- Know about conversion relation between 2-port network parameters

Learning contents

- Properties of Scattering Matrix
- *ABCD*-Parameters
- Conversion between 2-port Network Parameters

1 Properties of Scattering Matrix

- Symmetric matrix: $\boldsymbol{S_{ij} = S_{ji}}$ (reciprocal network)

$$[S] = ([Z]+[U])^{-1}([Z]-[U]) \quad \leftarrow \quad ([A][B])^{-1} = [B]^{-1}[A]^{-1}$$

$$[S]^t = ([Z]-[U])^t \left[([Z]+[U])^{-1}\right]^t = ([Z]-[U])^t \left[([Z]+[U])^t\right]^{-1}$$

$$= ([Z]-[U])([Z]+[U])^{-1} = [S]$$

- **Unitary matrix**: Lossless *n*-port network conservation of energy

Sum of all incident power = Sum of all exiting powers

$$\sum_n |b_i|^2 = [b]^t[b]^* = \sum_n |a_i|^2 = [a]^t[a]^*$$

$$\sum_n |b_i|^2 = [b]^t[b]^* = ([S][a])^t([S][a])^* = [a]^t[S]^t[S]^*[a]^*$$

$$= [a]^t[a]^* = [a]^t[a]^* = \sum_n |a_i|^2$$

$$\therefore [S]^t[S]^* = [U]$$

Ex.] For 2-port network,

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \Rightarrow [b] = [S][a]$$

$$\sum_{i=1}^{2} |b_i|^2 = (b_1 \quad b_2)\begin{pmatrix} b_1^* \\ b_2^* \end{pmatrix} = |b_1|^2 + |b_2|^2$$

1 Properties of Scattering Matrix

- Otherwise

$$\sum_n |b_i|^2 = [b]^{t*}[b] = \sum_n |a_i|^2 = [a]$$

$$|b_i|^2 = [b]^{t*}[b] = ([S][a])^{t*}([S][a]) = \left([a]^t[S]^t\right)^*[S][a] = [a]^{t*}[S]^{t*}[S][a]$$

$$= [a]^{t*}[a] = \sum_n |a_i|^2$$

$$\therefore \quad [S]^{t*}[S] = [U]$$

$$\Leftrightarrow \begin{bmatrix} S_{11}^* & S_{21}^* \\ S_{12}^* & S_{22}^* \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{cases} S_{11}^*S_{11} + S_{21}^*S_{21} = |S_{11}|^2 + |S_{21}|^2 = 1 & \text{-------- (1)} \\ S_{12}^*S_{12} + S_{22}^*S_{22} = |S_{12}|^2 + |S_{22}|^2 = 1 & \text{-------- (2)} \\ S_{11}^*S_{12} + S_{21}^*S_{22} = 0 & \text{-------- (3)} \\ S_{12}^*S_{11} + S_{22}^*S_{21} = 0 & \text{-------- (4)} \end{cases}$$

1 Properties of Scattering Matrix

$$|b_1|^2 = b_1^* b_1 = \left(S_{11}^* a_1^* + S_{12}^* a_2^*\right)\left(S_{11} a_1 + S_{12} a_2\right)$$

$$|b_2|^2 = b_2^* b_2 = \left(S_{21}^* a_1^* + S_{22}^* a_2^*\right)\left(S_{21} a_1 + S_{22} a_2\right)$$

$$\Rightarrow |b_1|^2 + |b_2|^2 = \left(|S_{11}|^2 |a_1|^2 + |S_{12}|^2 |a_2|^2 + S_{11}^* S_{12} a_1^* a_2 + S_{11} S_{12}^* a_1 a_2^*\right)$$

$$+\left(|S_{21}|^2 |a_1|^2 + |S_{22}|^2 |a_2|^2 + S_{21}^* S_{22} a_1^* a_2 + S_{21} S_{22}^* a_1 a_2^*\right)$$

$$= \left(|S_{11}|^2 + |S_{21}|^2\right)|a_1|^2 + \left(|S_{12}|^2 + |S_{22}|^2\right)|a_2|^2 \quad \leftarrow (1)\ \&\ (2)$$

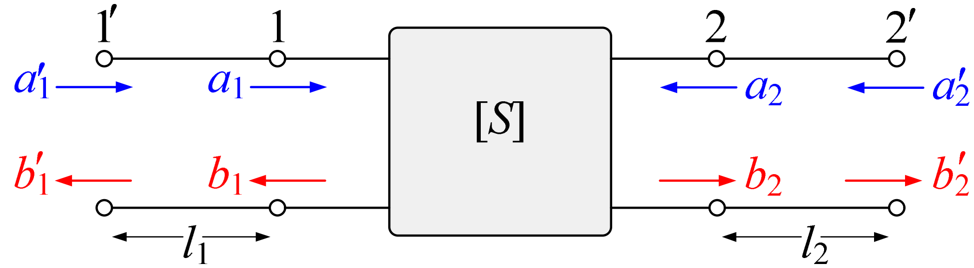
$$+\left(S_{11}^* S_{12} + S_{21}^* S_{22}\right)a_1^* a_2 + \left(S_{11} S_{12}^* + S_{21} S_{22}^*\right)a_1 a_2^* \quad \leftarrow (3)\ \&\ (4)$$

$$= |a_1|^2 + |a_2|^2$$

: Conservation of energy

1 Properties of Scattering Matrix

- S-parameter variation due to change of reference plane



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} a_1' e^{-j\beta l_1} \\ a_2' e^{-j\beta l_2} \end{bmatrix} = \begin{bmatrix} e^{-j\beta l_1} & 0 \\ 0 & e^{-j\beta l_2} \end{bmatrix} \begin{bmatrix} a_1' \\ a_2' \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} b_1' e^{j\beta l_1} \\ b_2' e^{j\beta l_2} \end{bmatrix} = \begin{bmatrix} e^{j\beta l_1} & 0 \\ 0 & e^{j\beta l_2} \end{bmatrix} \begin{bmatrix} b_1' \\ b_2' \end{bmatrix}$$

$$\begin{bmatrix} e^{j\beta l_1} & 0 \\ 0 & e^{j\beta l_2} \end{bmatrix} \begin{bmatrix} b_1' \\ b_2' \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} e^{-j\beta l_1} & 0 \\ 0 & e^{-j\beta l_2} \end{bmatrix} \begin{bmatrix} a_1' \\ a_2' \end{bmatrix}$$

$$\begin{bmatrix} b_1' \\ b_2' \end{bmatrix} = \begin{bmatrix} e^{j\beta l_1} & 0 \\ 0 & e^{j\beta l_2} \end{bmatrix}^{-1} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} e^{-j\beta l_1} & 0 \\ 0 & e^{-j\beta l_2} \end{bmatrix} \begin{bmatrix} a_1' \\ a_2' \end{bmatrix}$$

$$= \begin{bmatrix} e^{-j\beta l_1} & 0 \\ 0 & e^{-j\beta l_2} \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} e^{-j\beta l_1} & 0 \\ 0 & e^{-j\beta l_2} \end{bmatrix} \begin{bmatrix} a_1' \\ a_2' \end{bmatrix}$$

$$\begin{bmatrix} b_1' \\ b_2' \end{bmatrix} = \begin{bmatrix} S_{11} e^{-j2\beta l_1} & S_{12} e^{-j\beta(l_1+l_2)} \\ S_{21} e^{-j\beta(l_1+l_2)} & S_{22} e^{-j2\beta l_2} \end{bmatrix} \begin{bmatrix} a_1' \\ a_2' \end{bmatrix}$$

$$= \begin{bmatrix} S' \end{bmatrix} \begin{bmatrix} a_1' \\ a_2' \end{bmatrix}$$

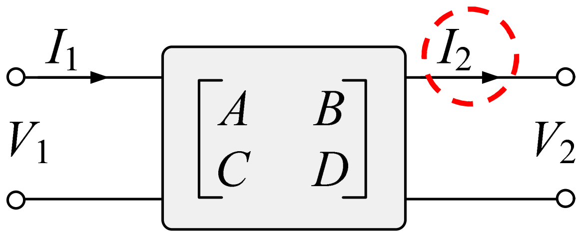
$$\begin{bmatrix} S' \end{bmatrix} = \begin{bmatrix} S_{11} e^{-j2\beta l_1} & S_{12} e^{-j\beta(l_1+l_2)} \\ S_{21} e^{-j\beta(l_1+l_2)} & S_{22} e^{-j2\beta l_2} \end{bmatrix}$$

New S-parameters: ***only phase shift***

2 *ABCD*-Parameters

- *ABCD*-parameters are used to model two-port network, linking input and output voltages and currents.
  - Relation between voltages and currents as the function of *ABCD*-parameters:

$$\begin{cases} V_1 = AV_2 + BI_2 \\ V_2 = CV_2 + DI_2 \end{cases}$$



  - Matrix form:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

  - Define for *ABCD*-parameters:

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$: Open circuit *reverse* voltage gain

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$$: Short-circuited reverse trans-impedance

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$: Open-circuited reverse trans-admittance

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$: Short-circuited reverse current gain

2 *ABCD*-Parameters

- Relation to impedance matrix [Z]:

$$\begin{cases} V_1 = Z_{11} I_1 - Z_{12} I_2 \\ V_2 = Z_{21} I_1 - Z_{22} I_2 \end{cases} \Rightarrow \begin{cases} I_1 = \dfrac{1}{Z_{21}} V_2 + \dfrac{Z_{22}}{Z_{21}} I_2 \\ V_1 = Z_{11}\left(\dfrac{1}{Z_{21}} V_2 + \dfrac{Z_{22}}{Z_{21}} I_2\right) - Z_{12} I_2 = \dfrac{Z_{11}}{Z_{21}} V_2 + \left(\dfrac{Z_{11} Z_{22}}{Z_{21}} - Z_{12}\right) I_2 \end{cases}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{Z_{11} I_1}{Z_{21} I_1} = \frac{Z_{11}}{Z_{21}} \qquad B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \left.\frac{Z_{11} I_1 - Z_{12} I_2}{I_2}\right|_{V_2=0} = Z_{11} \left.\frac{I_1}{I_2}\right|_{V_2=0} - Z_{12} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{I_1}{Z_{21} I_1} = \frac{1}{Z_{21}} \qquad D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{Z_{22}}{Z_{21}}$$

– $Z_{12} = Z_{21}$ for reciprocal network:

$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} Z_{11}/Z_{12} & \left(Z_{11} Z_{22} - Z_{12}^2\right)/Z_{12} \\ 1/Z_{12} & Z_{22}/Z_{12} \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

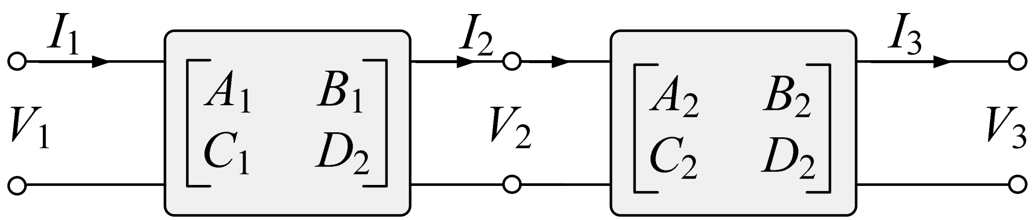
→ Reciprocal circuit: AD – BC = 1

2 *ABCD*-Parameters

- In practice, many microwave networks consist of a cascade connection of two or more 2-port networks.
  - Convenient to analyze cascade connection of two or more 2-port networks by using *ABCD*-parameters

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

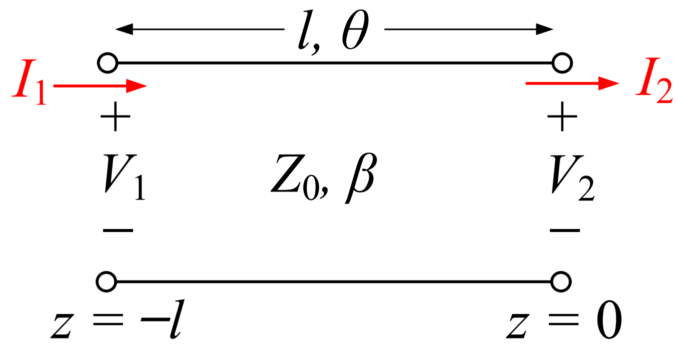
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} = \begin{bmatrix} A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2 \\ C_1A_2 + D_1C_2 & C_1B_2 + D_1D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_1 \end{bmatrix}$$



2 *ABCD*-Parameters

- *ABCD*-parameters of transmission line:

$$\begin{cases} V_1 = AV_2 + BI_2 \\ I_1 = CV_2 + DI_2 \end{cases}$$



$$\begin{cases} V(z) = V^+ e^{-\gamma z} + V^- e^{\gamma z}, & V(0) = V_2, \quad V(-l) = V_1 \\ I(z) = \dfrac{1}{Z_0}\left(V^+ e^{-\gamma z} - V^- e^{\gamma z}\right), & I(0) = I_2, \quad I(-l) = I_1 \end{cases}$$

- $\gamma = \alpha + j\beta = j\beta$:for lossless condition

$$\begin{cases} V(0) = V_2 = V^+ + V^- \\ Z_0 I(0) = Z_0 I_2 = V^+ - V^- \end{cases} \Rightarrow \begin{cases} V^+ = \dfrac{1}{2}\left(V_2 + Z_0 I_2\right) \\ V^- = \dfrac{1}{2}\left(V_2 - Z_0 I_2\right) \end{cases}$$

$$\begin{aligned} \Rightarrow V(z) &= V^+ e^{-\gamma z} + V^- e^{\gamma z} \\ &= \frac{1}{2}\left(V_2 + Z_0 I_2\right) e^{-\gamma z} + \frac{1}{2}\left(V_2 - Z_0 I_2\right) e^{\gamma z} \\ &= V_2 \left(\frac{e^{\gamma z} + e^{-\gamma z}}{2}\right) + I_2 Z_0 \left(\frac{e^{-\gamma z} - e^{\gamma z}}{2}\right) \\ &= V_2 \cosh \gamma z - I_2 Z_0 \sinh \gamma z \\ &= V_2 \cos \beta z - j I_2 Z_0 \sin \beta z \end{aligned}$$

cf.) $$\begin{cases} \cosh \gamma z = \dfrac{e^{j\beta z} + e^{-j\beta z}}{2} = \cos \beta z \\ \sinh \gamma z = \dfrac{e^{j\beta z} - e^{-j\beta z}}{2} = j \sin \beta z \end{cases}$$

2 *ABCD*-Parameters

$$\Rightarrow \quad I(z) = \frac{V^+ e^{-\gamma z} - V^- e^{\gamma z}}{Z_0}$$

$$= \frac{1}{Z_0}\left(\frac{V_2 + Z_0 I_2}{2}\right) e^{-\gamma z} - \frac{1}{Z_0}\left(\frac{V_2 - Z_0 I_2}{2}\right) e^{\gamma z}$$

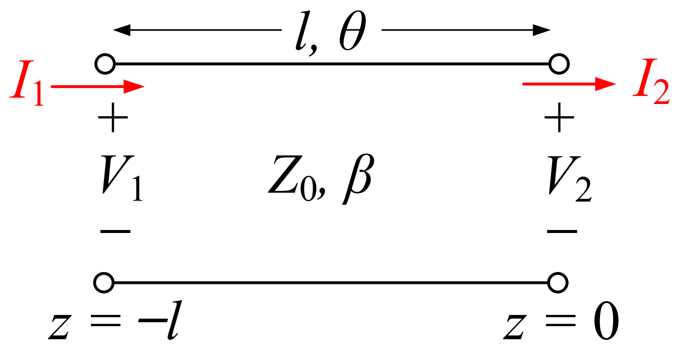
$$= \frac{V_2}{Z_0}\left(\frac{e^{-\gamma z} - e^{\gamma z}}{2}\right) + I_2\left(\frac{e^{\gamma z} + e^{-\gamma z}}{2}\right)$$

$$= -\frac{V_2}{Z_0}\sinh \gamma z + I_2 \cosh \gamma z$$

$$= -j\frac{V_2}{Z_0}\sin \beta z + I_2 \cos \beta z$$

$$\begin{cases} V(-l) = V_1 = V_2 \cos \beta l + j I_2 Z_0 \sin \beta l \\ I(-l) = I_1 = j\dfrac{V_2}{Z_0}\sin \beta l + I_2 \cos \beta l \end{cases}$$

$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_0 \sin\theta \\ jY_0 \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad \leftarrow \theta = \beta l$$

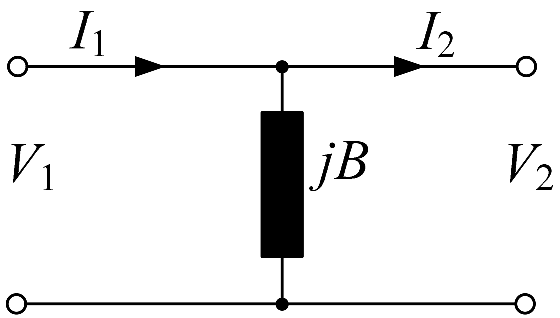


$$\therefore \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_0 \sin\theta \\ jY_0 \sin\theta & \cos\theta \end{bmatrix}$$

- For lossless case: AD – BC = 1

2 *ABCD*-Parameters

- *ABCD*-parameters of 2-port network consisting of a shunt susceptance between ports 1 and 2:



$$\begin{cases} V_1 = AV_2 + BI_2 \\ I_1 = CV_2 + DI_2 \end{cases}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1 \quad \text{(Voltage ratio)}$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = 0 \quad \text{(Resistive)}$$

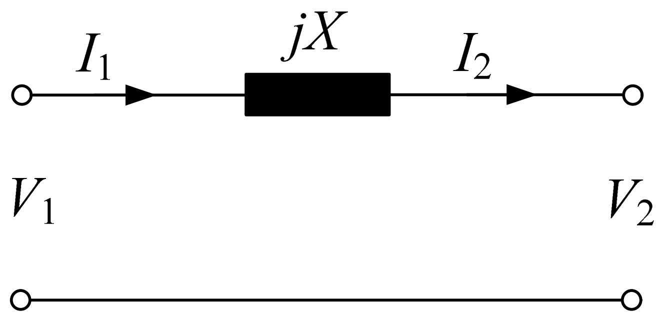
$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \left.\frac{I_1}{V_1}\right|_{I_2=0} = jB \quad \text{(Conductive)}$$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = 1 \quad \text{(Current ratio)}$$

$$\Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ jB & 1 \end{bmatrix}$$

2 *ABCD*-Parameters

- *ABCD*-parameters of 2-port network consisting of a series reactance between ports 1 and 2:



$$\begin{cases} V_1 = AV_2 + BI_2 \\ I_1 = CV_2 + DI_2 \end{cases}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = jX$$

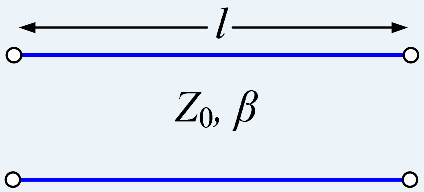
$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \left.\frac{I_1}{V_1}\right|_{I_2=0} = 0$$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = 1$$

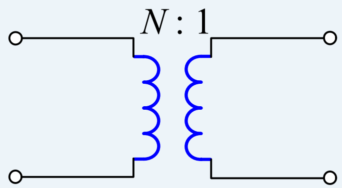
$$\Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & jX \\ 0 & 1 \end{bmatrix}$$

2 *ABCD*-Parameters

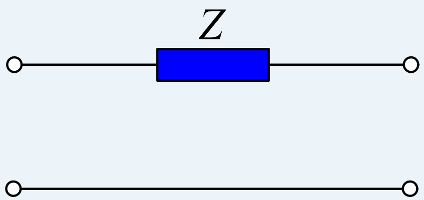
- *ABCD*-parameters of some useful 2-port networks



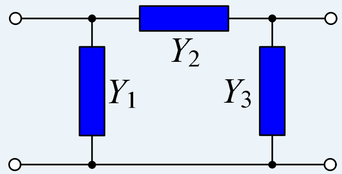
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_0 \sin\theta \\ jY_0 \sin\theta & \cos\theta \end{bmatrix}$$



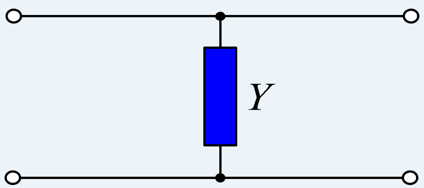
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} N & 0 \\ 0 & \frac{1}{N} \end{bmatrix}$$



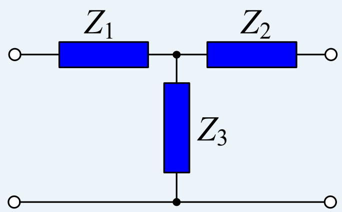
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & jX \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+\frac{Y_2}{Y_3} & \frac{1}{Y_3} \\ Y_1+Y_2+\frac{Y_1 Y_2}{Y_3} & 1+\frac{Y_1}{Y_3} \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ jB & 1 \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+\frac{Z_1}{Z_3} & Z_1+Z_2+\frac{Z_1 Z_2}{Z_3} \\ \frac{1}{Z_3} & 1+\frac{Z_2}{Z_3} \end{bmatrix}$$

3 Conversion between 2-port Network Parameters

	S	***Z***	***Y***	***ABCD***
S_{11}	S_{11}	$\frac{(Z_{11}-Z_0)(Z_{22}+Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\frac{(Y_0-Y_{11})(Y_0+Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\frac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D}$
S_{12}	S_{12}	$\frac{2Z_{12}Z_0}{\Delta Z}$	$\frac{-2Y_{12}Y_0}{\Delta Y}$	$\frac{2(AD-BC)}{A+B/Z_0+CZ_0+D}$
S_{21}	S_{21}	$\frac{2Z_{21}Z_0}{\Delta Z}$	$\frac{-2Y_{21}Y_0}{\Delta Z}$	$\frac{2}{A+B/Z_0+CZ_0+D}$
S_{22}	S_{22}	$\frac{(Z_{11}+Z_0)(Z_{22}-Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\frac{(Y_0+Y_{11})(Y_0-Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\frac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D}$
Z_{11}	$Z_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{11}	$\frac{Y_{22}}{\lvert Y\rvert}$	$\frac{A}{C}$
Z_{12}	$Z_0\frac{2S_{12}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{12}	$\frac{-Y_{12}}{\lvert Y\rvert}$	$\frac{AD-BC}{C}$
Z_{21}	$Z_0\frac{2S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{21}	$\frac{-Y_{21}}{\lvert Y\rvert}$	$\frac{1}{C}$
Z_{22}	$Z_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{22}	$\frac{Y_{11}}{\lvert Y\rvert}$	$\frac{D}{C}$

$\lvert Z\rvert = Z_{11}Z_{22}-Z_{12}Z_{21}$, $\lvert Y\rvert = Y_{11}Y_{22}-Y_{12}Y_{21}$, $\Delta Z=(Z_{11}+Z_0)(Z_{22}+Z_0)-Z_{12}Z_{21}$, $\Delta Y=(Y_{11}+Z_0)(Y_{22}+Z_0)-Y_{12}Y_{21}$, $Y_0=1/Z_0$

3 Conversion between 2-port Network Parameters

	S	Z	Y	ABCD
Y_{11}	$Y_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{Z_{22}}{\|Z\|}$	Y_{11}	$\frac{D}{B}$
Y_{12}	$Y_0\frac{-2S_{12}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{-Z_{12}}{\|Z\|}$	Y_{12}	$\frac{BC-AD}{B}$
Y_{21}	$Y_0\frac{-2S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{-Z_{21}}{\|Z\|}$	Y_{21}	$\frac{-1}{B}$
Y_{22}	$Y_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{Z_{11}}{\|Z\|}$	Y_{22}	$\frac{A}{B}$
A	$\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{11}}{Z_{21}}$	$\frac{-Y_{22}}{Y_{21}}$	A
B	$Z_0\frac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$	$\frac{\|Z\|}{Z_{21}}$	$\frac{-1}{Y_{21}}$	B
C	$\frac{1}{Z_0}\frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}}$	$\frac{1}{Z_{21}}$	$\frac{-\|Y\|}{Y_{21}}$	C
D	$\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{22}}{Z_{21}}$	$\frac{-Y_{11}}{Y_{21}}$	D

$|Z| = Z_{11}Z_{22} - Z_{12}Z_{21}$, $|Y| = Y_{11}Y_{22} - Y_{12}Y_{21}$, $\Delta Z = (Z_{11}+Z_0)(Z_{22}+Z_0) - Z_{12}Z_{21}$, $\Delta Y = (Y_{11}+Z_0)(Y_{22}+Z_0) - Y_{12}Y_{21}$, $Y_0 = 1/Z_0$

4 Review

- Properties of Scattering matrix
- *ABCD*-parameters
- Conversion between 2-port network different parameters