Chapter 5
Impedance Matching

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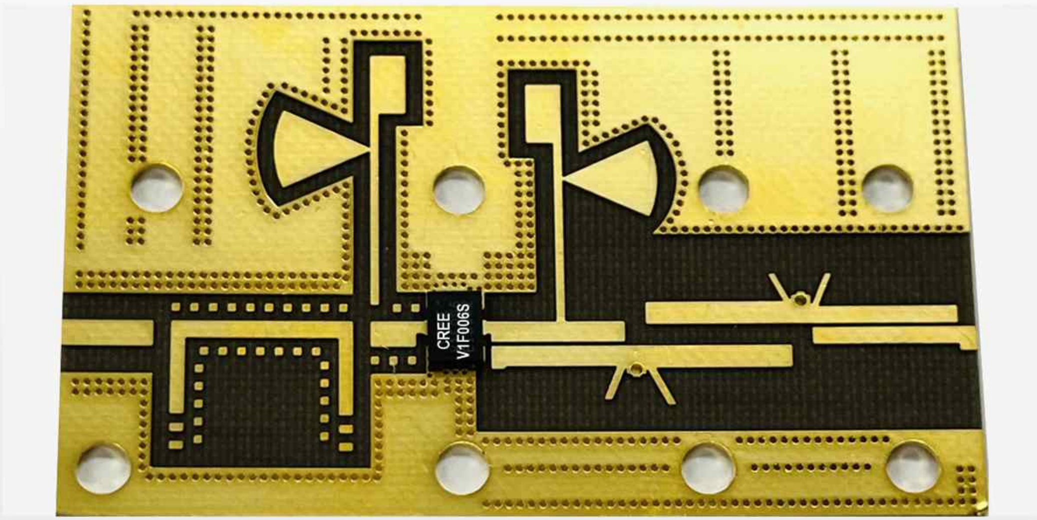


Learning Objectives

- Learn why impedance matching is important in system viewpoints.
- Learn how impedance matching can be done.
- Understanding real microwave lumped elements

Learning contents

- Matching Network with Lumped Elements (L-section)
- Impedance Matching Example using Calculation and Smith Chart
- Microwave Lumped Elements

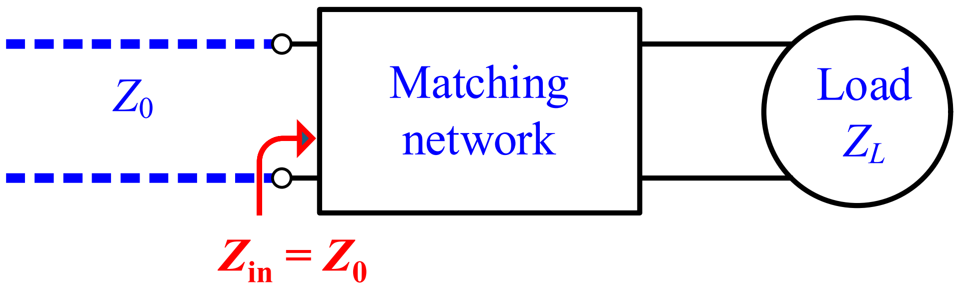
1 Usefulness of Impedance Matchings

- Matching network usefulness

  - The ***maximum power*** is delivered when the load is matched to the transmission line (assuming the generator is matched), and power loss in the feed line is minimized.
  - Impedance matching sensitive receiver components (antenna, low-noise amplifier, etc.) ***improves the signal-to-noise ratio(SNR)*** of the receiving system.
  - Impedance matching in a power distribution network (such as an antenna array feed network) will ***reduce amplitude and phase errors spatially***.

- Important factors in selection of particular matching network

  - Complexity: simple
  - Bandwidth: restricted/broad/multi frequency band
  - Implementation: easy
  - Adjustability: tunable



2 Matching Network with Lumped Elements (*L*-section)

- L-section matching network
  - Simplest matching network including two reactive elements
  - Normalized load impedance: $z_L = Z_L / Z_0$ and $Z_L = R_L + jX_L$
  - X and B: lumped or distributed (reactive and susceptive) element depending on operating frequency
  - Matching network for (a) z_L <u>inside of 1 + *jx* circle</u> and (b) z_L <u>outside of 1 + *jx* circle</u> on Smith chart

- For (a),

$$Z_{in}' = Z_L // \frac{1}{jB} = \frac{Z_L / jB}{Z_L + \frac{1}{jB}} = \frac{Z_L}{1 + jBZ_L}$$

$$\mathrm{Re}[Z_{in}'] < R_L$$

For (b),

$$Z_{in}' = (Z_L + jX) // \frac{1}{jB} = \frac{(Z_L + jX) / jB}{(Z_L + jX) + \frac{1}{jB}} = \frac{Z_L + jX}{1 + jB(Z_L + jX)}$$

$$\mathrm{Re}[Z_{in}'] > R_L$$

2 Matching Network with Lumped Elements (*L*-section)

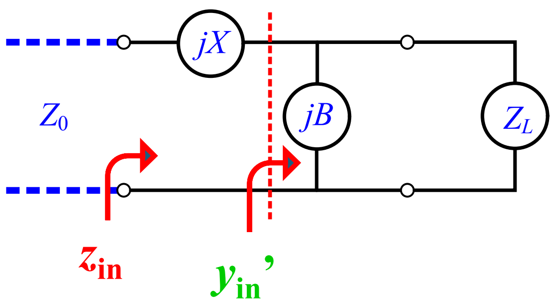
- Smith chart: reflection coefficient plane

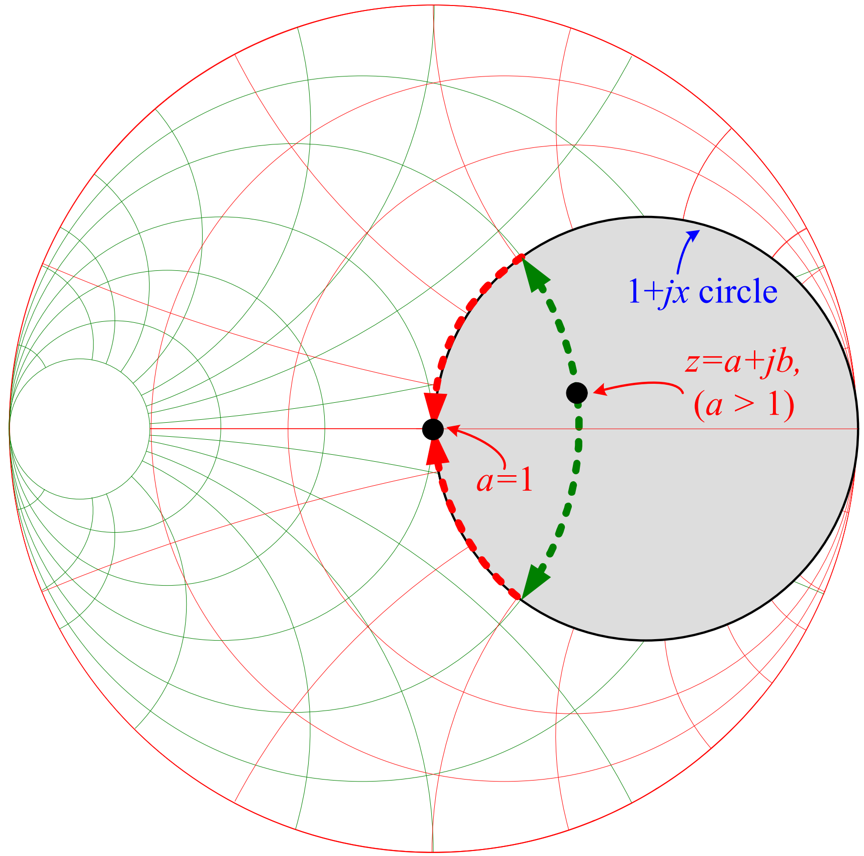
$$\Gamma = \Gamma_r + j\Gamma_i Z_L \overset{\text{or}}{=} |\Gamma| e^{j\phi}$$

$$z = r + jx$$

- Matching condition: $|\Gamma| = 0$ or $z = 1 + j0$ ($\leftrightarrow y = 1 + j0$)
- For z_L inside of $1 + jx$ circle ($r_L > 1$),

  $\boldsymbol{z_L \rightarrow y_{in}' = 1 + jx}$ **circle** $\boldsymbol{\rightarrow z_{in} = 1 + j0}$





2 Matching Network with Lumped Elements (*L*-section)

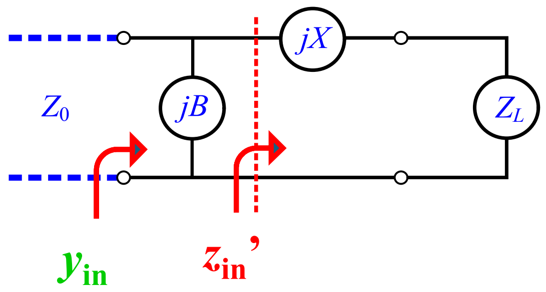
- Smith chart: reflection coefficient plane

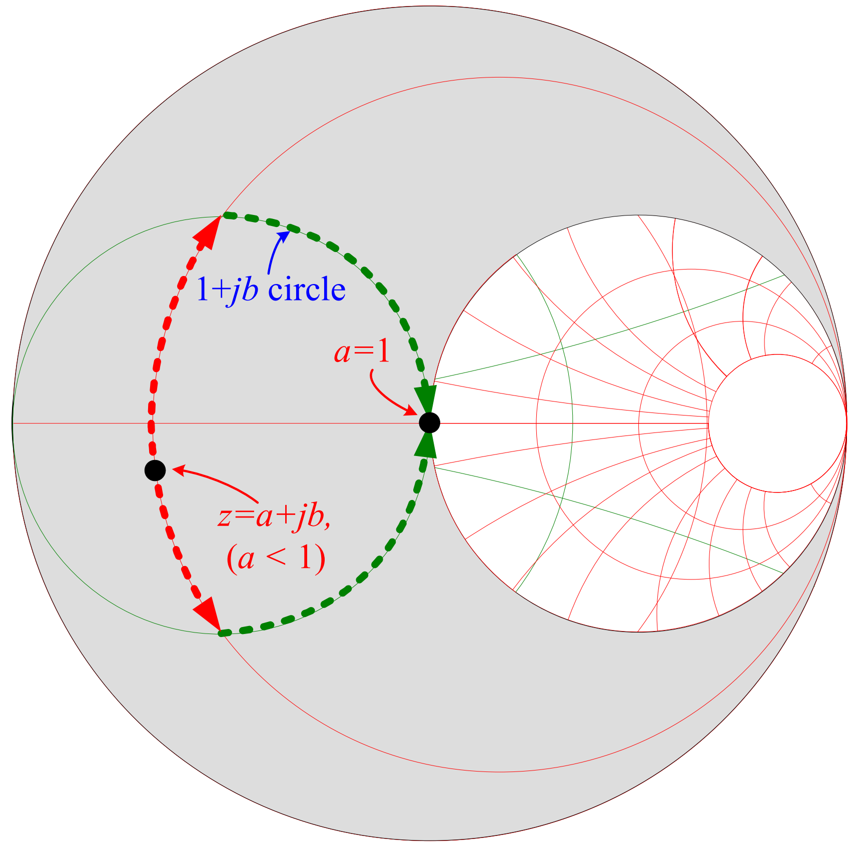
$$\Gamma = \Gamma_r + j\Gamma_i Z_L \overset{\text{or}}{=} |\Gamma| e^{j\phi}$$

$$z = r + jx$$

- Matching condition: $|\Gamma| = 0$ or $z = 1 + j0$ ($\leftrightarrow y = 1 + j0$)
- For z_L outside of $1 + jx$ circle ($r_L < 1$),

$\boldsymbol{z_L \rightarrow z_{in}' = 1 + jb}$ **circle** $\boldsymbol{\rightarrow y_{in} = 1 + j0}$

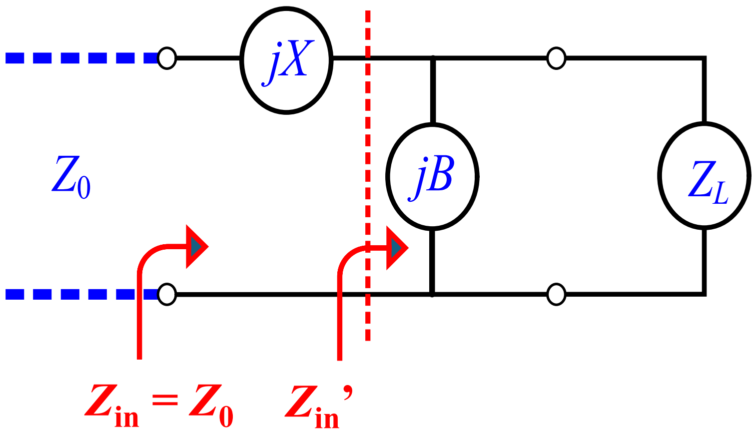




2 Matching Network with Lumped Elements (L-section)

- Analytic solution **for z_L inside $1 + jx$ circle ($r_L > 1$)**

  - $Z_L = R_L + jX_L$ (assumption: $Z_0 < R_L$)
  - Input impedance seen looking into the matching network:



$$Z_{in} = Z_0$$
$$= jX + \frac{1}{jB + 1/(R_L + jX_L)}$$
$$Z_0 - jX = \frac{R_L + jX_L}{(1 - BX_L) + jBR_L}$$
$$(Z_0 - jX)\{(1 - BX_L) + jBR_L\} = R_L + jX_L$$
$$\{Z_0 + B(R_L X - X_L Z_0)\} + j\{BR_L Z_0 - X(1 - BX_L)\} = R_L + jX_L$$

  - Rearranging and separating into real and imaginary parts:

real part: $B(R_L X - X_L Z_0) = R_L - Z_0$

imaginary part: $X(1 - BX_L) = BR_L Z_0 - X_L$

2 Matching Network with Lumped Elements (L-section)

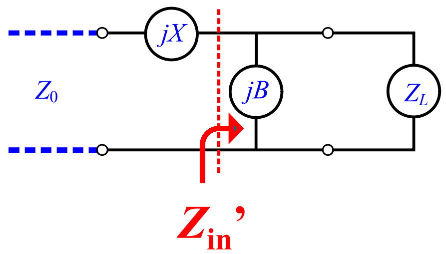
- Solving and substituting X for B

$$B = \frac{X_L \pm \sqrt{R_L / Z_0}\sqrt{R_L^2 + X_L^2 - Z_0 R_L}}{R_L^2 + X_L^2}$$

where $R_L^2 + X_L^2 - Z_0 R_L > 0 \ (\because R_L > Z_0)$

$$X = \frac{B X_L Z_0 + R_L - Z_0}{B R_L} = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{B R_L}$$

➔ Two solutions: dual valued components (B, X)

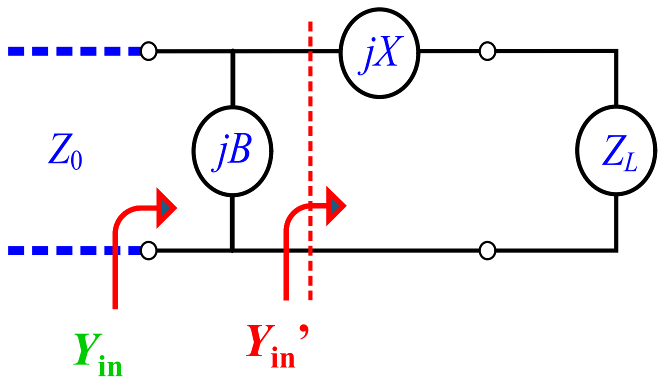


- Analytic solution **for z_L outside $1 + jx$ circle ($r_L < 1$)**

- Input admittance seen looking into the matching network:

$$Y_{in} = jB + \frac{1}{R_L + j(X + X_L)} \quad \leftarrow Z_L = R_L + jX_L$$

$$= \frac{1}{Z_0}$$



2 Matching Network with Lumped Elements (*L*-section)

- Rearranging and separating into real and imaginary parts:

$$BZ_0(X+X_L)=Z_0-R_L$$

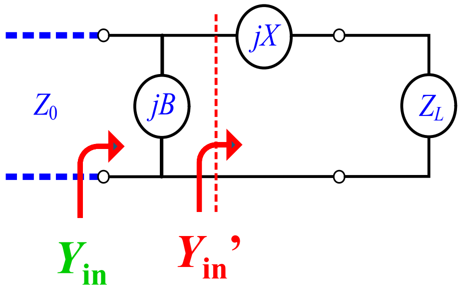
$$(X+X_L)=BZ_0R_L$$

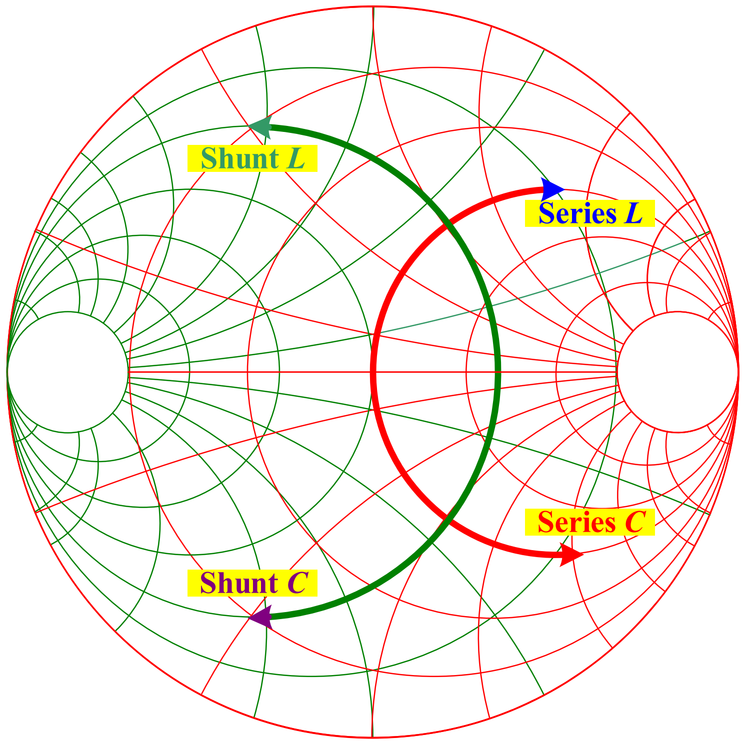
- Solving for *X* and *B*:

$$X=\pm\sqrt{R_L(Z_0-R_L)}-X_L$$

$$B=\pm\frac{\sqrt{(Z_0-R_L)/R_L}}{Z_0}$$

➔ Two solutions: dual valued components (*B*, *X*)





- Addition series L/C on fixed impedance and shunt L/C on fixed admittance

$$Z+j\omega L\ (\text{or } -j\frac{1}{\omega C})=R+j(X+\omega L)(\text{or } R+j(X-\frac{1}{\omega C}))$$

$$Y+j\omega C\ (\text{or } -j\frac{1}{\omega L})=G+j(B+\omega C)(\text{or } G+j(B-\frac{1}{\omega L}))$$

3 Impedance Matching Example using Calculation and Smith Chart

- Design an L-section matching network to match a series RC load with an impedance $Z_L = 100 + j200$ [Ω] to 50 [Ω] transmission line at a frequency of 1 GHz.

1) Theoretical solutions

- Characteristic admittance: $Y_0 = \frac{1}{Z_0} = \frac{1}{50} = 0.02$

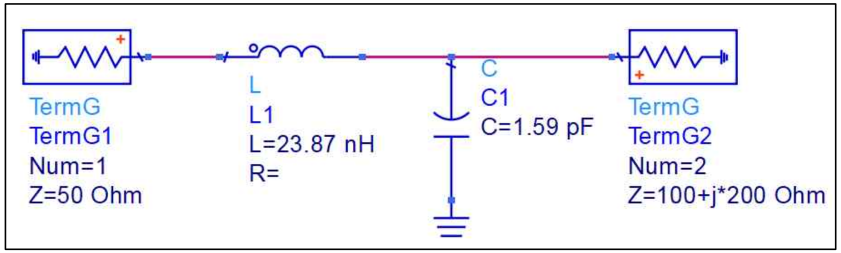
⇒ **Solution #01**: (series L & shunt C) can be found

$$- B = \frac{X_L + \sqrt{R_L / Z_0}\sqrt{R_L^2 + X_L^2 - Z_0 R_L}}{R_L^2 + X_L^2} = \frac{200 + \sqrt{100/50} \times \sqrt{100^2 + 200^2 - 50 \times 100}}{100^2 + 200^2} = 0.01 \rightarrow b = \frac{B}{Y_0} = \frac{0.01}{0.02} = 0.5$$

$$- X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{B R_L} = \frac{1}{0.01} + \frac{200 \times 50}{100} - \frac{50}{0.01 \times 100} = 150 \rightarrow x = \frac{X}{Z_0} = \frac{150}{50} = 3$$

$$\rightarrow C = \frac{b}{2\pi f Z_0} = \frac{0.5}{2\pi \times 10^9 \times 50} = 1.59\,\text{pF} \leftarrow B = bY_0 = \frac{b}{Z_0} = \omega C,\ C = \frac{b}{\omega Z_0}$$

$$\rightarrow L = \frac{x Z_0}{2\pi f} = \frac{3 \times 50}{2\pi \times 10^9} = 23.87\ \text{nH} \quad \leftarrow X = x Z_0 = \omega L,\ L = \frac{x Z_0}{\omega}$$



3 Impedance Matching Example using Calculation and Smith Chart

1) Theoretical solutions

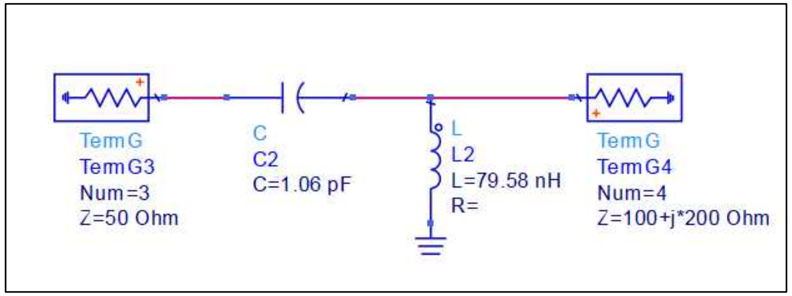
⇒ **Solution #02**: (series C & shunt L) can be found

- $B = \dfrac{X_L - \sqrt{R_L / Z_0}\sqrt{R_L^2 + X_L^2 - Z_0 R_L}}{R_L^2 + X_L^2} = \dfrac{200 + \sqrt{100/50} \times \sqrt{100^2 + 200^2 - 50 \times 100}}{100^2 + 200^2} = -0.002 \rightarrow b = \dfrac{B}{Y_0} = \dfrac{-0.002}{0.02} = -0.1$

- $X = \dfrac{1}{B} + \dfrac{X_L Z_0}{R_L} - \dfrac{Z_0}{BR_L} = \dfrac{1}{-0.002} + \dfrac{200 \times 50}{100} - \dfrac{50}{-0.002 \times 100} = -150 \rightarrow x = \dfrac{X}{Z_0} = \dfrac{150}{50} = -3$

$\rightarrow C = -\dfrac{1}{2\pi f x Z_0} = -\dfrac{1}{2\pi \times 10^9 \times (-3) \times 50} = 1.06 \text{ pF} \quad \leftarrow jX = jxZ_0 = -j\dfrac{1}{\omega C}, \quad C = -\dfrac{1}{\omega x Z_0}$

$\rightarrow L = -\dfrac{Z_0}{2\pi f b} = -\dfrac{50}{2\pi \times 10^9 \times (-0.1)} = 79.58 \text{ nH} \quad \leftarrow jB = jbY_0 = \dfrac{jb}{Z_0} = -j\dfrac{1}{\omega L}, \quad L = -\dfrac{Z_0}{\omega b}$



3 Impedance Matching Example using Calculation and Smith Chart

2) Smith chart solutions

- $Z_L = 100 + j200 \rightarrow z_L = \frac{Z_L}{Z_0} = 2 + j4 \rightarrow y_L = \frac{1}{z_L} = 0.1 - j0.2$

⇒ **Solution #01:** (Denoted by blue color on Smith chart)

- $y = y_L + jb = \frac{1}{z} = (0.1 - j0.2) + j0.5 = 0.1 + j0.3 \quad \leftarrow b = 0.5$
- Interconnection with impedance circle of $1 - jx$

$$z_{\text{in}} = z + jx = (1 - j3) + j3 = 1 - j0 \quad \rightarrow \quad x = 3$$

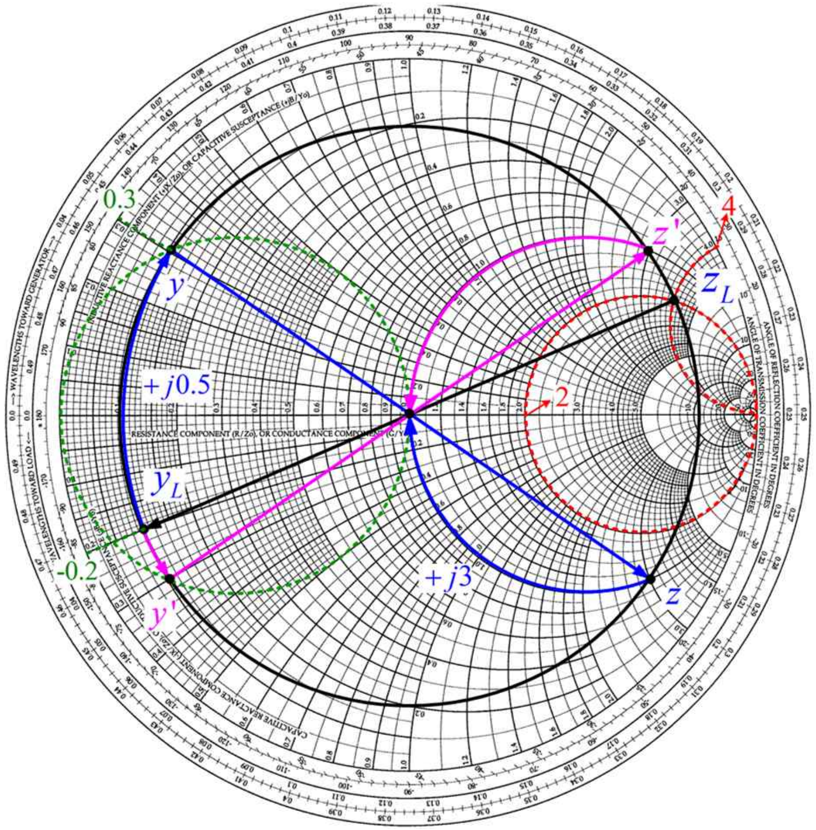
⇒ **Solution #02:** (Denoted by pink color on Smith chart)

- $y' = y_L + jb' = \frac{1}{z'} = (0.1 - j0.2) - j0.1 = 0.1 - j0.3 \quad \leftarrow jb = -j0.1$
- Interconnection with impedance circle of $1 + jx$

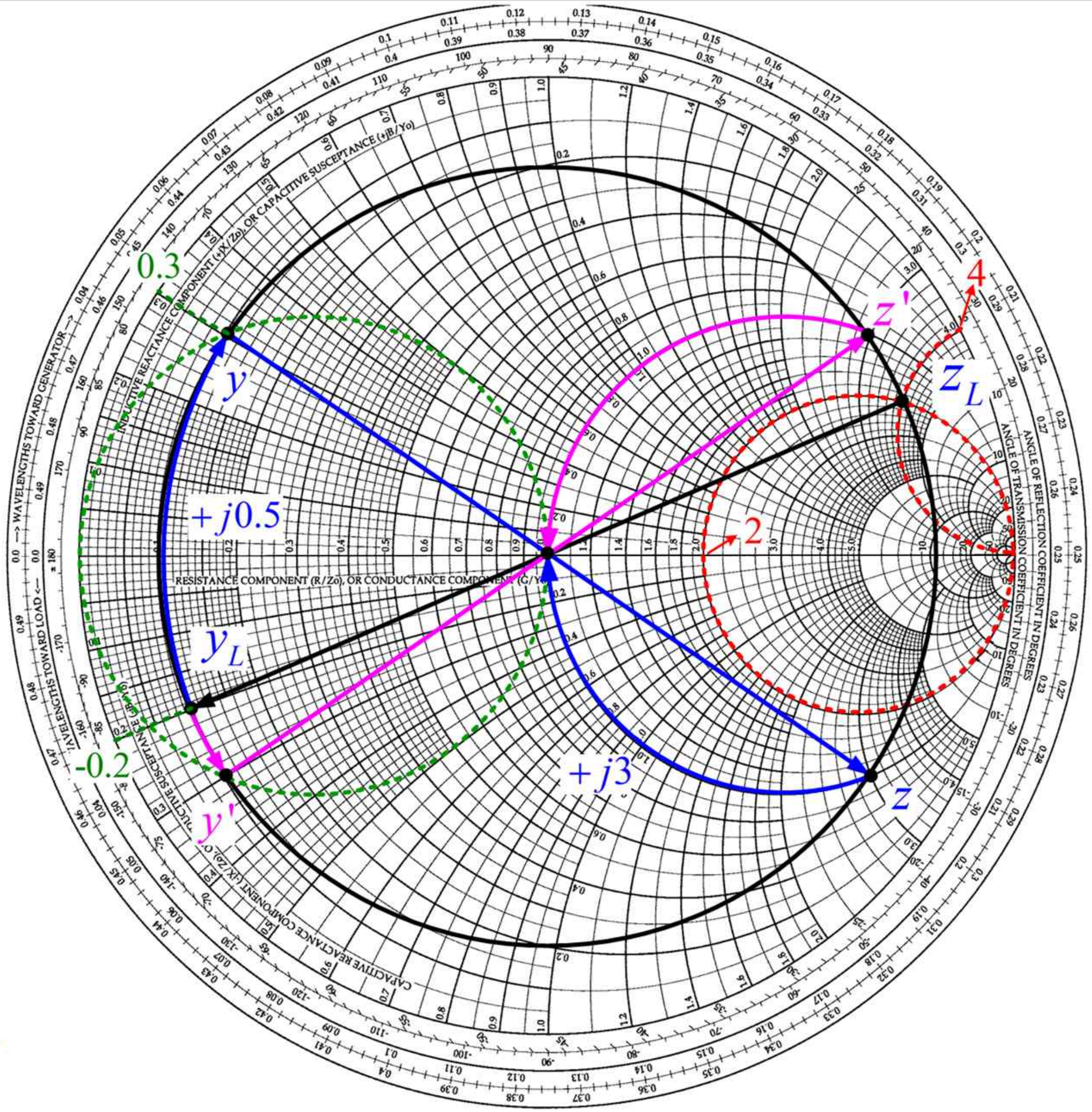
$$z_{\text{in}} = z' + jx = (1 + j3) - j3 = 1 + j0 \quad \rightarrow \quad x = -3$$

⇒ Finally, L & C can be found by using these formula:

$$C = \frac{b}{2\pi f Z_0}, \quad L = \frac{x Z_0}{2\pi f}$$: approximately same with theorical solution

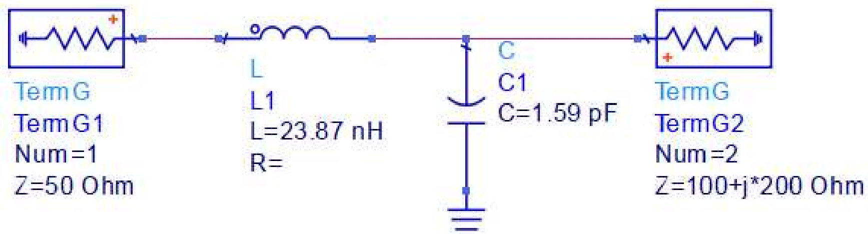


3 Impedance Matching Example using Calculation and Smith Chart

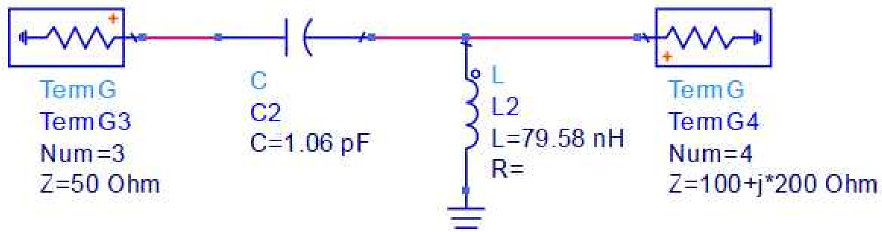


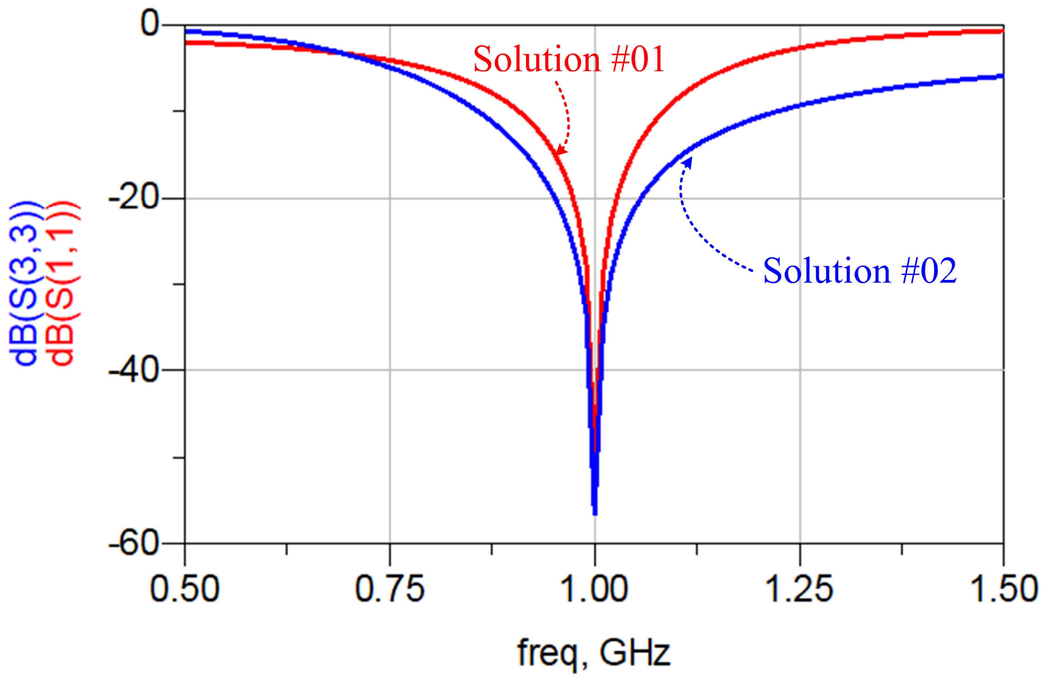
3 Impedance Matching Example using Calculation and Smith Chart

- Solution #01

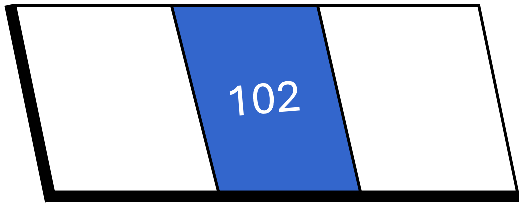


- Solution #02

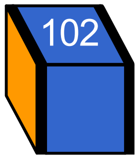




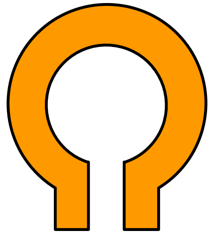
4 Microwave Lumped Elements



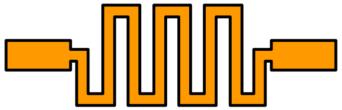
planar resistor



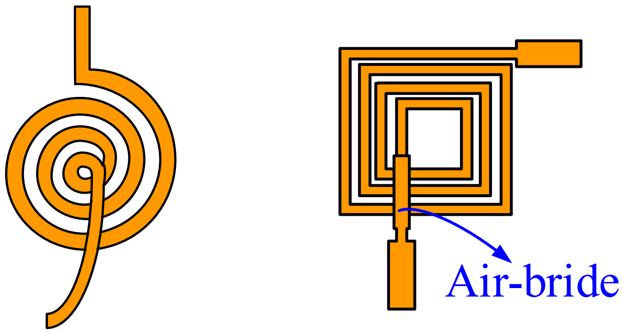
chip resistor



loop inductor

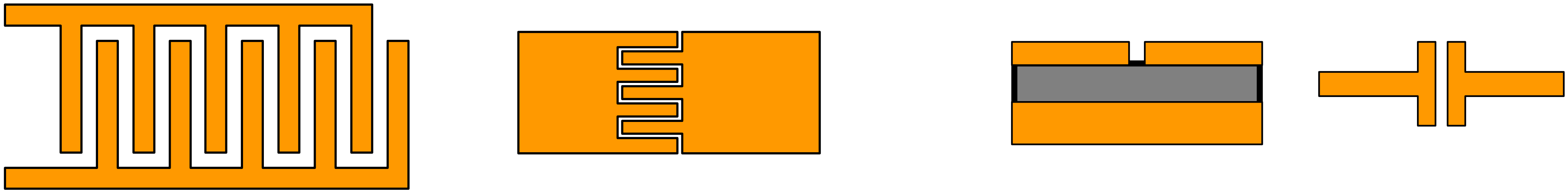


(high impedance)
meander line inductor

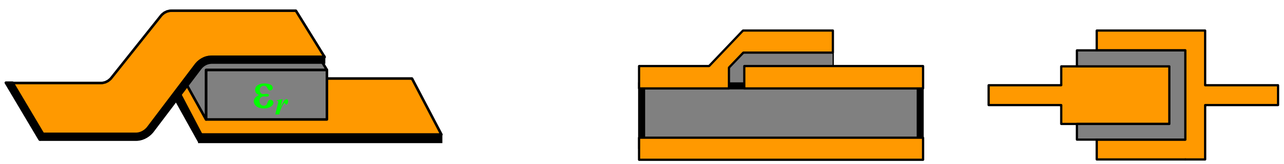


spiral inductors

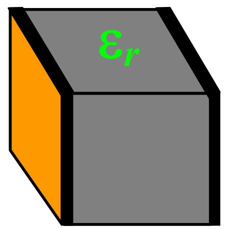
4 Microwave Lumped Elements



interdigital gap capacitors



Metal-**i**nsulator-**m**etal capacitors

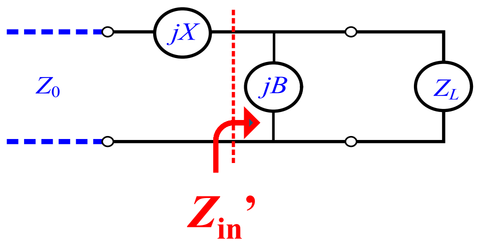


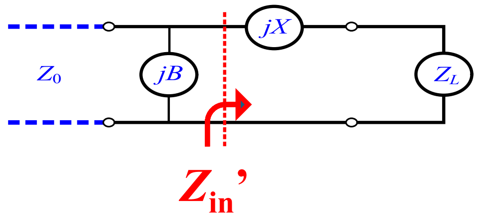
chip capacitor

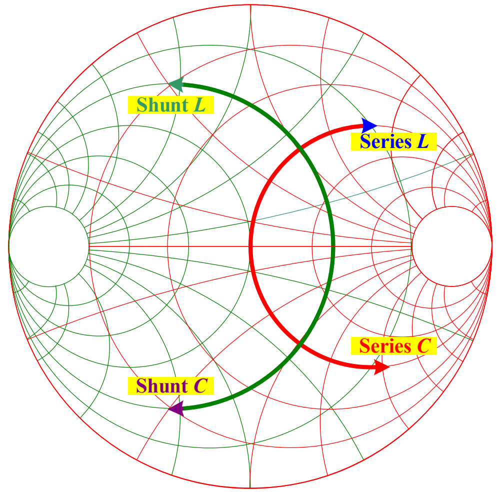
5 Review

- Matching network useful for maximum power transmission, improvement of signal-to-noise ratio(SNR), reduction of amplitude and phase errors spatially, etc.

- L-section matching network: simplest matching network including two reactive elements







- Microwave Lumped Elements