Chapter 5
Impedance Matching

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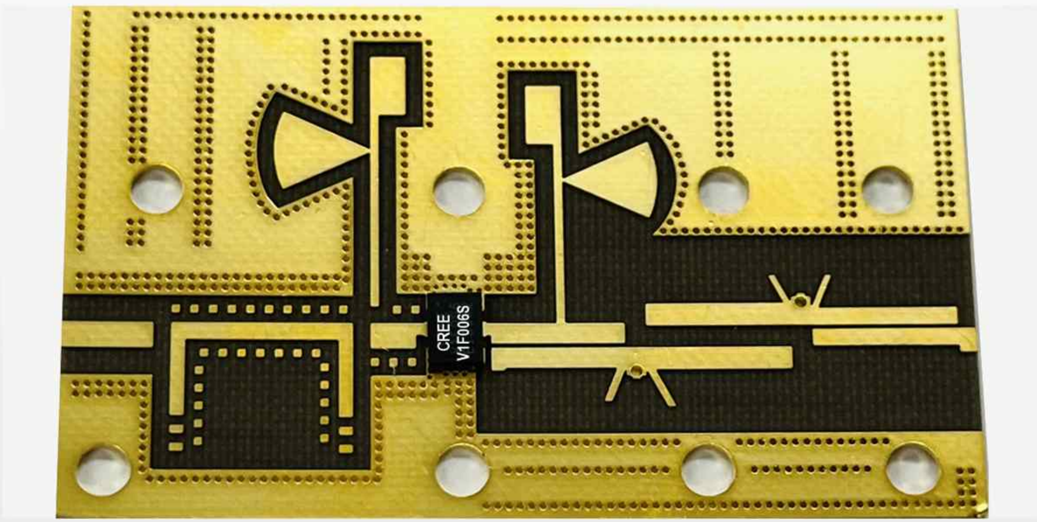


Learning Objectives

- Understanding about single stub matching
- Learn how to match impedance using Smith chart
- Learn how to match impedance using mathematical equations
- Practice impedance matching with examples

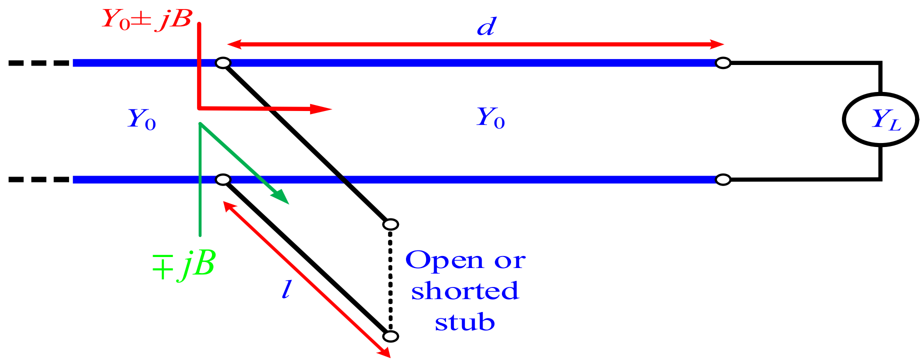
Learning contents

- Introduction about Single Stub Matching
- Shunt- and Series-Stub Matching using Graphical Solution
- Shunt- and Series-Stub Matching using Mathematical Solution
- Impedance Matching Example using Calculation and Smith Chart

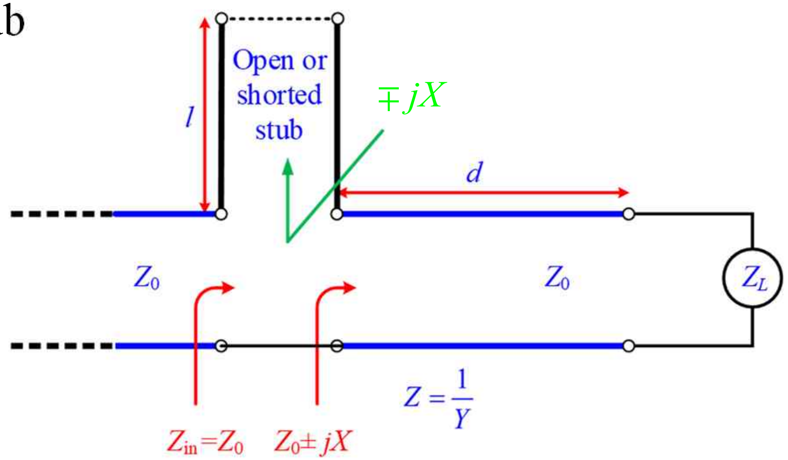
1 Introduction about Single Stub Matching

- Single stub: single open- or short-circuited length of transmission line (a 'stub'), connected either in parallel or in series with the transmission line at a certain distance from load.

- Shunt stub



- Series stub



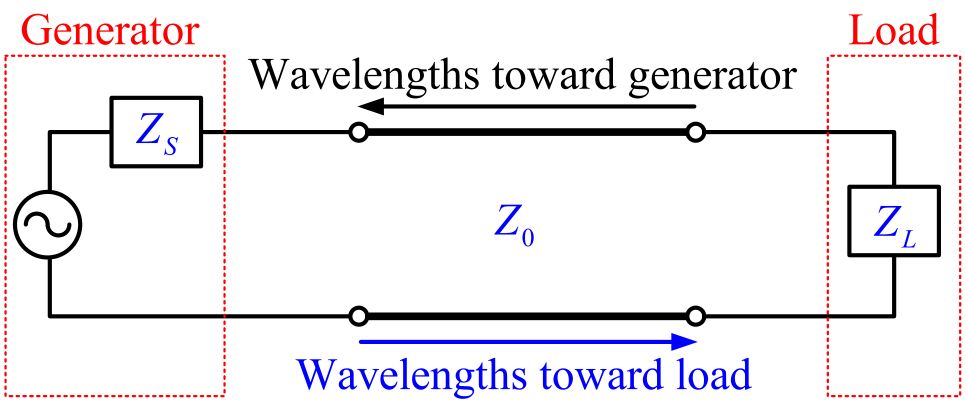
- Shunt stub matching is easy relatively than series stub in realization of microstrip line or strip line forms.
- Open-circuited stubs are easier to fabricate since a via hole through the substrate to ground plane is not needed

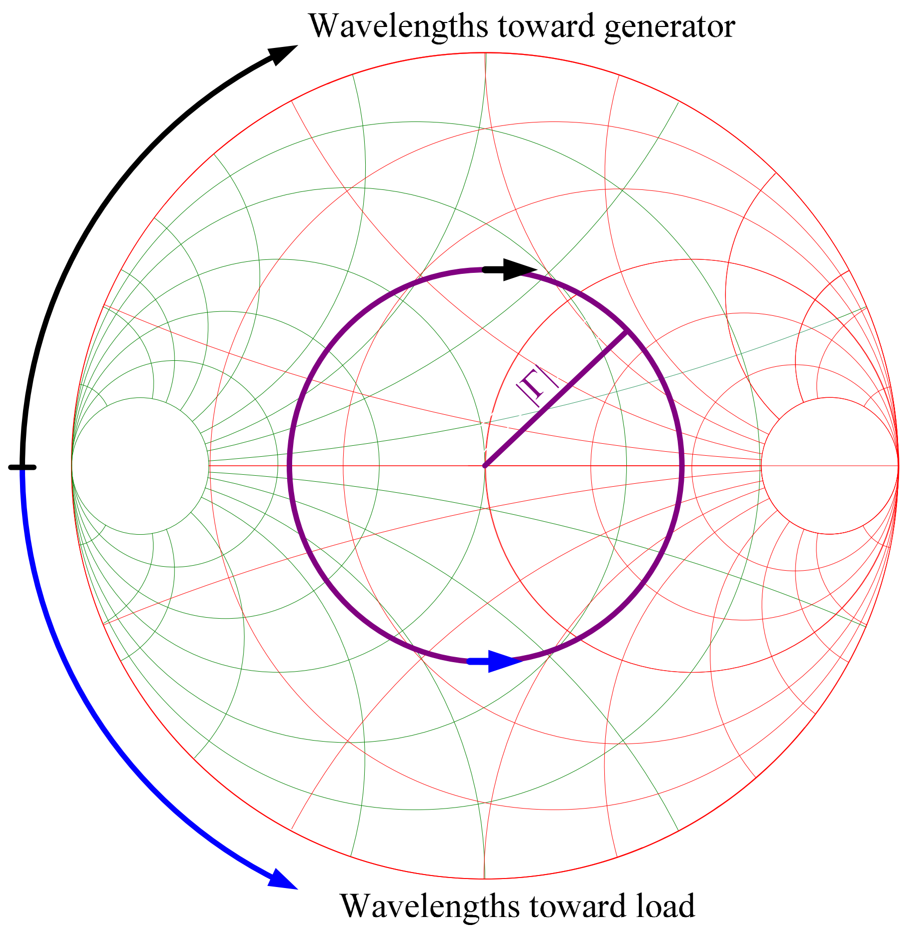
- Shunt stub case matching
  - $Y_L \rightarrow Y_0 + jB$ (by using transmission line with length d)
  - $\rightarrow Y_0$ (by using transmission line having length l)

- Series stub case matching
  - $Z_L \rightarrow Z_0 + jX$
  - $\rightarrow Z_0$

2 Shunt- and Series-Stub Matching using Graphical Solution

- Depending on the length and direction of transmission line, load reflection coefficient rotates along WTG or WTL with same radius on Smith chart.



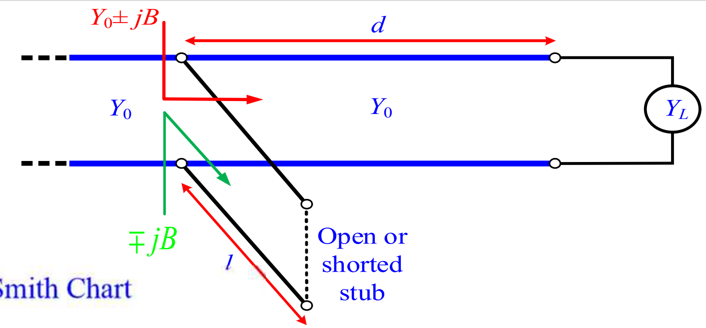


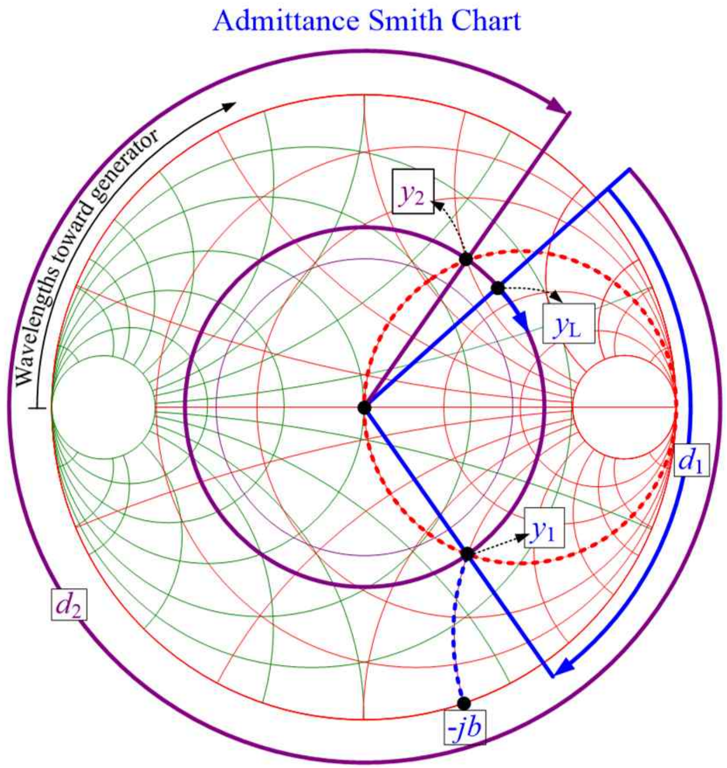
2 Shunt- and Series-Stub Matching using Graphical Solution

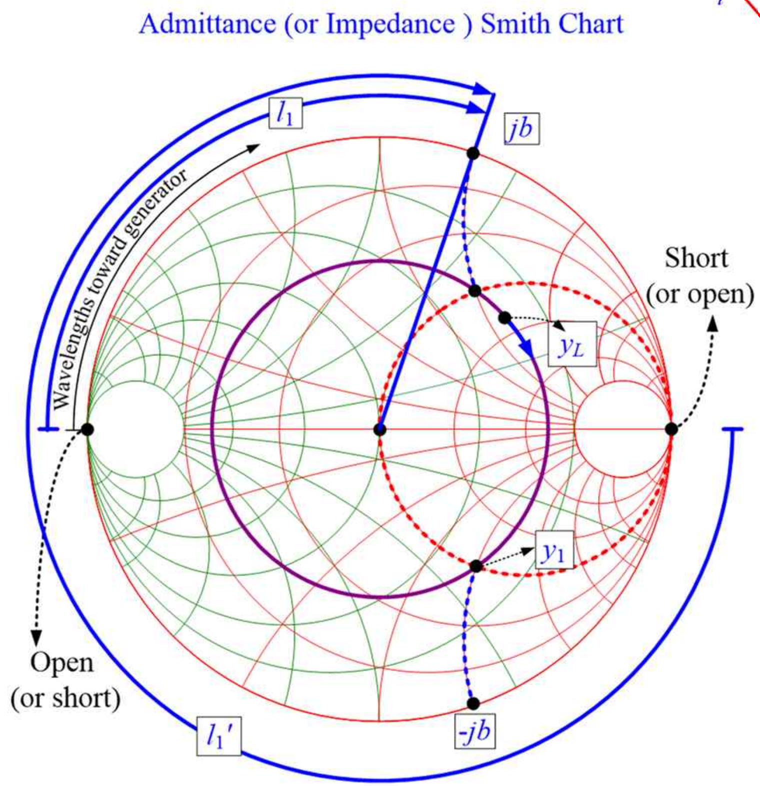
- Graphical explanation of shunt-stub matching

  - $Y_L \rightarrow Y_0 \pm jB$ (by using transmission line with length d)

    $\rightarrow Y_0$ (by using transmission line having length l)





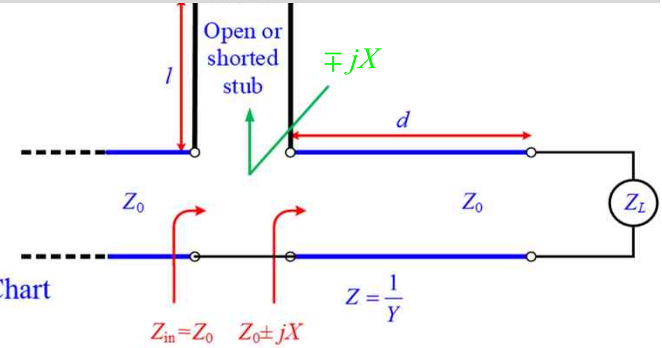


2 Shunt- and Series-Stub Matching using Graphical Solution

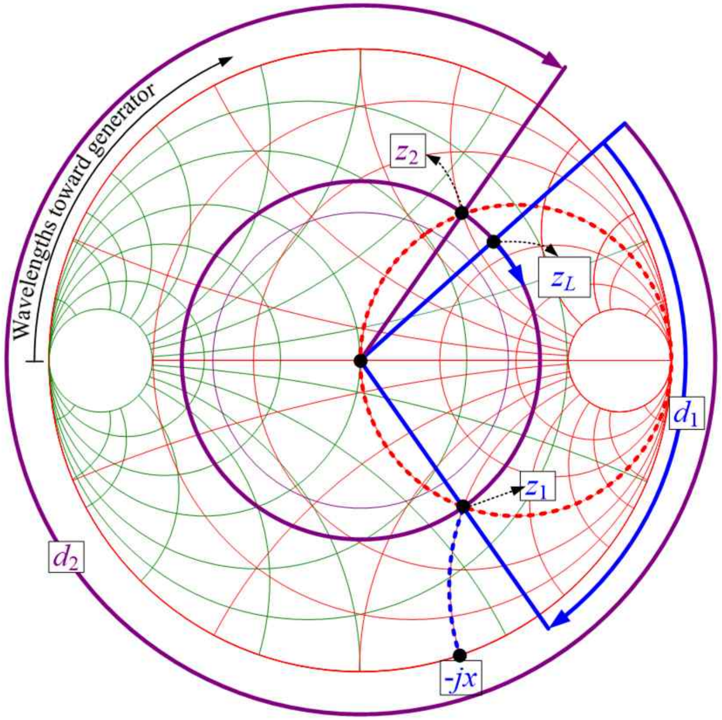
- Graphical explanation of series-stub matching

  - $Y_L \rightarrow Z_0 \pm jX$ (by using transmission line with length d)

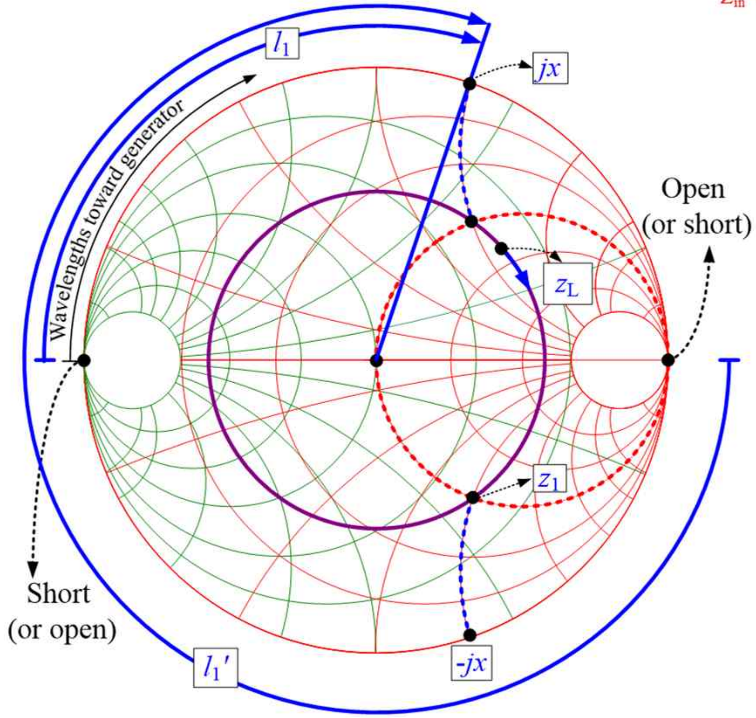
    $\rightarrow Z_0$ (by using transmission line having length l)



Impedance Smith Chart



Impedance (or Admittance) Smith Chart



3 Shunt- and Series-Stub Matching using Mathematical Solution

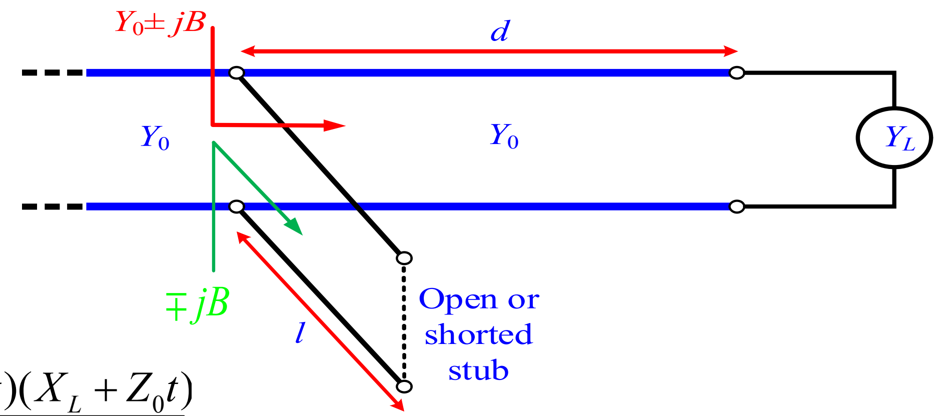
- Mathematical evaluation of **shunt-stub matching**

  - Derivation of formulas for d and l

  - Load impedance (or admittance)

$$Z_L = 1/Y_L = R_L + jX_L$$

  - Impedance Z down a length, d, of transmission line from load



$$Z = Z_0 \frac{Z_L + jZ_0 \tan\beta d}{Z_0 + jZ_L \tan\beta d} = Z_0 \frac{(R_L + jX_L) + jZ_0 t}{Z_0 + j(R_L + jX_L)t}$$

where $t = \tan\beta d$

$$Y = G + jB = Y_0 + jB = \frac{1}{Z}$$

where $G = \dfrac{R_L(1+t^2)}{R_L^2 + (X_L + Z_0 t)^2} = Y_0 = \dfrac{1}{Z_0}$ and $B = \dfrac{R_L^2 t - (Z_0 - X_L t)(X_L + Z_0 t)}{Z_0[R_L^2 + (X_L + Z_0 t)^2]}$

$$Z_0 R_L (1+t^2) = R_L^2 + (X_L + Z_0 t)^2$$

$$Z_0(R_L - Z_0)t^2 - 2X_L Z_0 t + (R_L Z_0 - R_L^2 - X_L^2) = 0$$

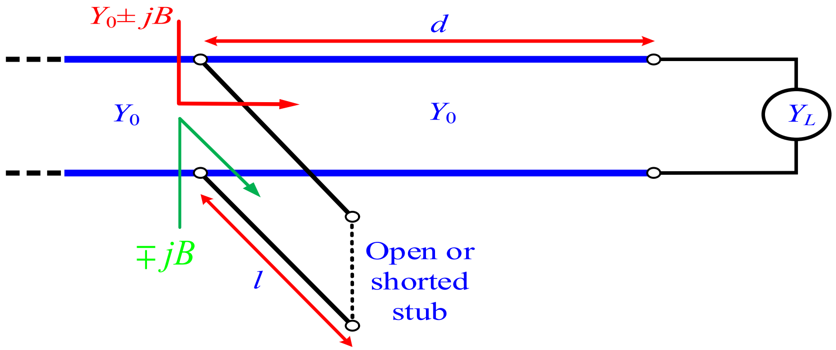
3 Shunt- and Series-Stub Matching using Mathematical Solution

- (continued)

$$Z_0(R_L - Z_0)t^2 - 2X_L Z_0 t + (R_L Z_0 - R_L^2 - X_L^2) = 0$$

$$t = \tan\beta d = \tan\frac{2\pi}{\lambda}d = \frac{X_L \pm \sqrt{R_L[(Z_0 - R_L)^2 + X_L^2]/Z_0}}{R_L - Z_0} = f(Z_0,\ R_L,\ X_L) : \text{known values} \quad @\, R_L \neq Z_0$$

$$\frac{d}{\lambda} = \begin{cases} \dfrac{1}{2\pi}\tan^{-1} t, & \text{for } t \geq 0 \\ \dfrac{1}{2\pi}(\pi + \tan^{-1} t), & \text{for } t < 0. \end{cases}$$



- If $R_L = Z_0$, then $t = -X_L / 2Z_0$.
- To find required stub lengths, $B_S = -B$.

1) For an open-circuited stub,

$$jB_S = jY_0\tan\beta l_o = jY_0\tan(2\pi l_o/\lambda) = -jB$$

$$\frac{l_o}{\lambda} = \frac{1}{2\pi}\tan^{-1}\left(\frac{B_S}{Y_0}\right) = \frac{-1}{2\pi}\tan^{-1}\left(\frac{B}{Y_0}\right)$$

2) For a short-circuited stub,

$$jB_S = -jY_0\cot\beta l_s = -jY_0\cot(2\pi l_s/\lambda) = -jB$$

$$\frac{l_s}{\lambda} = \frac{-1}{2\pi}\tan^{-1}\left(\frac{Y_0}{B_S}\right) = \frac{1}{2\pi}\tan^{-1}\left(\frac{Y_0}{B}\right)$$

➔ ***If the lengths (l_o, l_s) are negative, $\lambda/2$ can be added to give positive results.***

3 Shunt- and Series-Stub Matching using Mathematical Solution

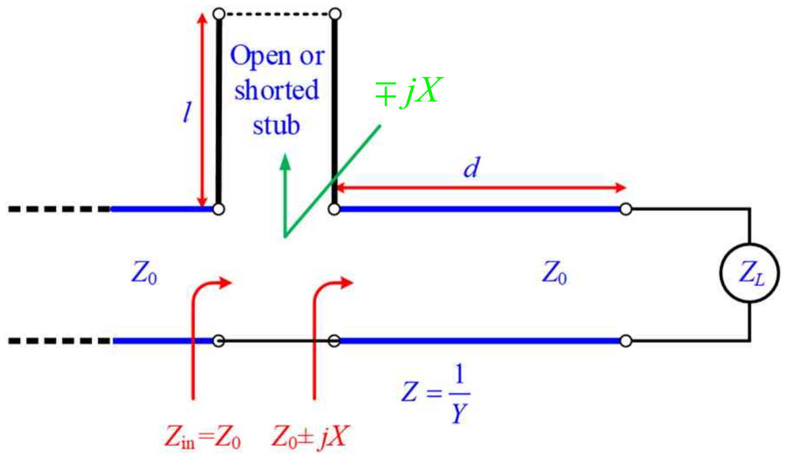
- Mathematical evaluation of **series-stub matching**

- Derivation of formulas for d and l

- Load admittance (or impedance)

$$Y_L = 1/Z_L = G_L + jB_L$$

- Admittance Y down a length, d, of transmission line from load



$$Y = Y_0 \frac{Y_L + jY_0 \tan \beta d}{Y_0 + jY_L \tan \beta d} = Y_0 \frac{(G_L + jB_L) + jY_0 t}{Y_0 + j(G_L + jB_L)t}$$

where $t = \tan \beta d\,, Y_0 = 1/Z_0$

$$Z = \frac{1}{Y} = R + jX = Z_0 + jX$$

where $R = \dfrac{G_L(1+t^2)}{G_L^2 + (B_L + Y_0 t)^2} = Z_0 = \dfrac{1}{Y_0}$ and $X = \dfrac{G_L^2 t - (Y_0 - tB_L)(B_L + tY_0)}{Y_0[G_L^2 + (B_L + Y_0 t)^2]}$

$$Y_0 G_L (1+t^2) = G_L^2 + (B_L + Y_0 t)^2$$

$$Y_0 (G_L - Y_0) t^2 - 2B_L Y_0 t + (G_L Y_0 - G_L^2 - B_L^2) = 0$$

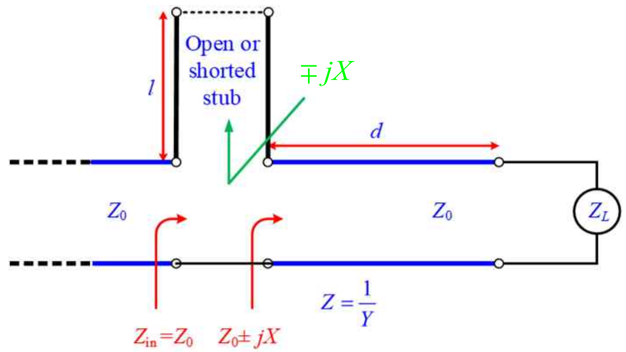
3 Shunt- and Series-Stub Matching using Mathematical Solution

- (continued)

$$Y_0(G_L - Y_0)t^2 - 2B_L Y_0 t + (G_L Y_0 - G_L^2 - B_L^2) = 0$$

$$t = \tan\beta d = \tan\frac{2\pi}{\lambda}d = \frac{B_L \pm \sqrt{G_L[(Y_0 - G_L)^2 + B_L^2]/Y_0}}{G_L - Y_0} = f(Y_0,\, G_L,\, B_L) \text{ : known values} \quad @\, G_L \neq Y_0$$

$$\frac{d}{\lambda} = \begin{cases} \dfrac{1}{2\pi}\tan^{-1} t, & \text{for } t \geq 0 \\ \dfrac{1}{2\pi}(\pi + \tan^{-1} t), & \text{for } t < 0. \end{cases}$$



- If $G_L = Y_0$, then $t = -B_L / 2Y_0$.
- To find the required stub lengths, $X_S = -X$.

1) For a short-circuited stub,

$$jX_S = jZ_0 \tan\beta l_s = jZ_0 \tan(2\pi l_s/\lambda) = -jX$$

$$\frac{l_s}{\lambda} = \frac{1}{2\pi}\tan^{-1}\left(\frac{X_S}{Z_0}\right) = \frac{-1}{2\pi}\tan^{-1}\left(\frac{X}{Z_0}\right)$$

2) For an open-circuited stub,

$$jX_S = -jZ_0 \cot\beta l_o = -jZ_0 \cot(2\pi l_o/\lambda) = -jX$$

$$\frac{l_o}{\lambda} = \frac{-1}{2\pi}\tan^{-1}\left(\frac{Z_0}{X_S}\right) = \frac{1}{2\pi}\tan^{-1}\left(\frac{Z_0}{X}\right)$$

➔ ***If the lengths (l_o, l_s) are negative, $\lambda/2$ can be added to give positive results.***

4 Impedance Matching Example using Calculation and Smith Chart

- Ex. 1] Shunt single stub matching example

Design two single-stub (short circuit) shunt tuning networks to match a load impedance $Z_L = 80 + j100$ [Ω] to 50 [Ω] line at a frequency of 3 GHz. Plot the return loss in dB from 2 to 4 GHz for each solution.

1) Theoretical solutions

- $t = \dfrac{X_L \pm \sqrt{R_L[(Z_0 - R_L)^2 + X_L^2]/Z_0}}{R_L - Z_0} = \dfrac{100 \pm \sqrt{80[(50-80)^2 + 100^2]/50}}{80-50} = \begin{cases} 7.74 & \text{for "+" sign} \\ -1.07 & \text{for "-" sign} \end{cases}$

- $B = \dfrac{R_L^2 t - (Z_0 - X_L t)(X_L + Z_0 t)}{Z_0[R_L^2 + (X_L + Z_0 t)^2]} = \begin{cases} 0.033 & \text{for } t = 7.74 \\ -0.033 & \text{for } t = -1.07 \end{cases}$

- $d = \begin{cases} [\dfrac{1}{2\pi}\tan^{-1}(t)]\lambda = \dfrac{1}{2\pi}\tan^{-1}(7.74)\lambda = 0.23\lambda = 82.8^\circ & @\ t = 7.74 \\ \dfrac{1}{2\pi}[\pi + \tan^{-1}(t)]\lambda = \dfrac{1}{2\pi}[\pi + \tan^{-1}(-1.07)]\lambda = 0.37\lambda = 133.2^\circ & @\ t = -1.07 \end{cases}$

- For short-circuited stub: $l_s = \dfrac{1}{2\pi}\tan^{-1}\left(\dfrac{Y_0}{B}\right)\lambda = \begin{cases} 0.0867\lambda = 31.21^\circ & @\ B = 0.033 \\ -0.0867\lambda & @\ B = -0.033 \text{ (negative, by adding } \lambda/2 \\ & \rightarrow l_s = 0.413\lambda = 148.68^\circ) \end{cases}$

4 Impedance Matching Example using Calculation and Smith Chart

2) Smith chart solutions

- Z_L =80 + j100
- Normalized impedance: $z_L = Z_L/Z_0$ = 1.6+j2
- Mark z_L and draw a reflection coefficient circle
- Convert to z_L to load admittance, y_L
- |Γ| circle intersects the 1 ± jb circle at two points denoted as y_1 and y_2 → d_1 or d_2

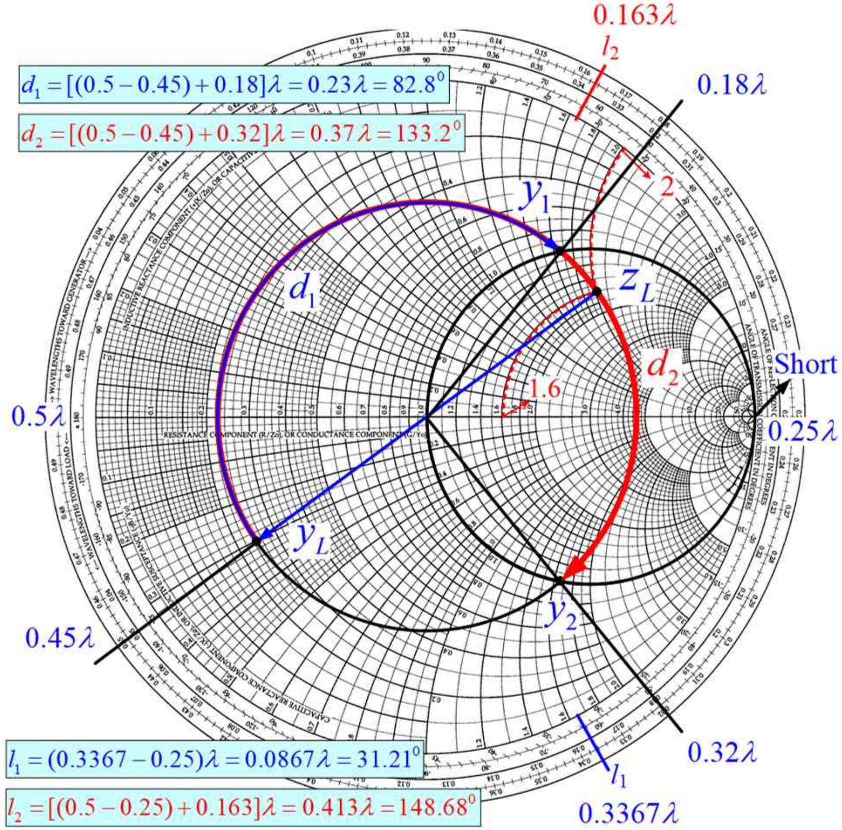
$$d_1 = [(0.5-0.45)+0.18]\lambda = 0.23\lambda \rightarrow 0.23\times 360^{\circ} = 82.8^{\circ}$$

$$d_2 = [(0.5-0.45)+0.32]\lambda = 0.37\lambda \rightarrow 0.37\times 360^{\circ} = 133.2^{\circ}$$

- At two intersections point,

$$y_1 = 1 + j1.65, \quad y_2 = 1 - j1.65$$

- By moving from $y = \infty$ (short-circuited) toward generator,

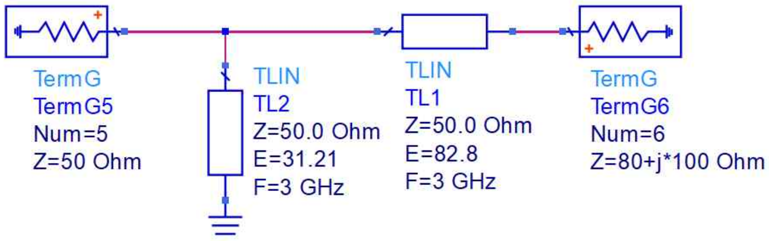


$$l_1 = (0.3367-0.25)\lambda = 0.0867\lambda \rightarrow 31.21^{\circ}, \quad l_2 = [(0.5-0.25)+0.163]\lambda = 0.413\lambda \rightarrow 148.68^{\circ}$$

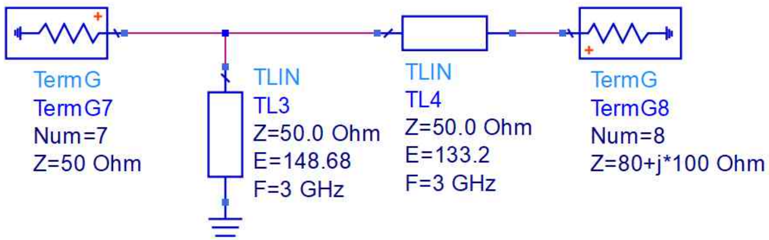
➔ ***The matching stub is kept as short as possible to improve the matching bandwidth and reduce losses caused by two lossy transmission lines.***

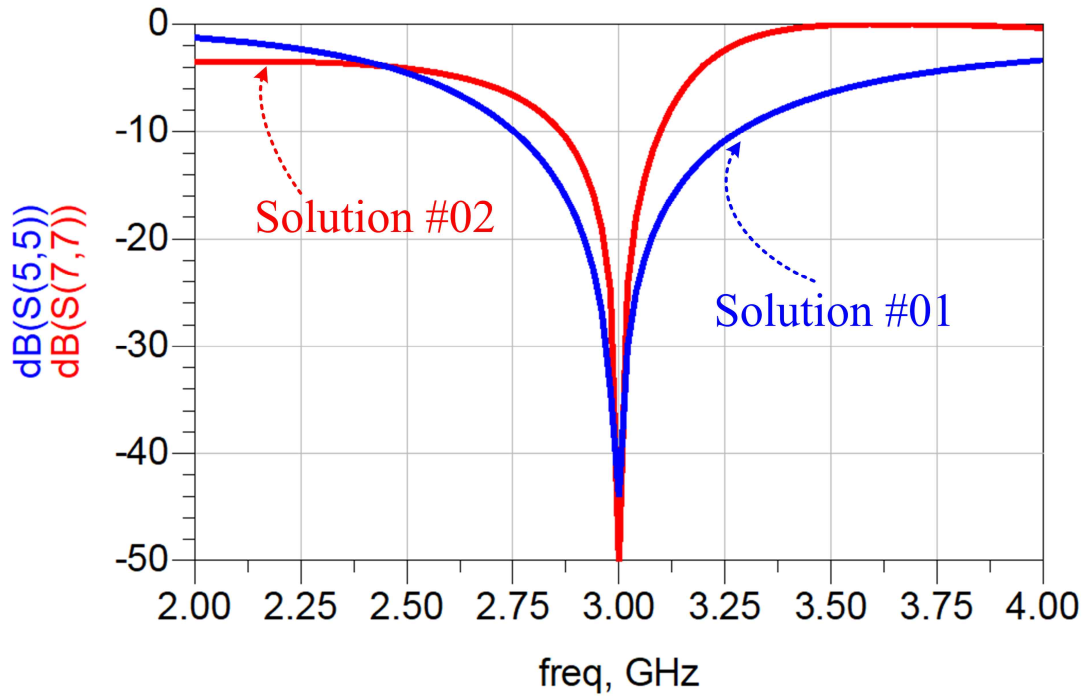
4 Impedance Matching Example using Calculation and Smith Chart

- Solution #01



- Solution #02





- Ex. 2] Series single stub matching example

Design a single series open-circuited stub network to match a load impedance Z_L = 75 + j100 [Ω] to 50 [Ω] line at a frequency of 3 GHz. Plot the return loss in dB from 2 to 4 GHz for each solution.

1) Theoretical solutions

- Load admittance

$$Y_L = G_L + jB_L = \frac{1}{Z_L} = \frac{1}{75 + j100} = 0.0048 - j0.0064$$

- $Y_0 = \frac{1}{Z_0} = 0.02$

- $t = \frac{B_L \pm \sqrt{G_L[(Y_0 - G_L)^2 + B_L^2]/Y_0}}{G_L - Y_0} = \frac{-0.0064 \pm \sqrt{0.0048[(0.02-0.0048)^2 + (-0.0064)^2]/0.02}}{0.0048 - 0.02} = \begin{cases} -0.111 & \text{for "+" sign} \\ 0.953 & \text{for "-" sign} \end{cases}$

- $X = \frac{G_L^2 t - (Y_0 - B_L t)(B_L + Y_0 t)}{Y_0[G_L^2 + (B_L + Y_0 t)^2]} = \begin{cases} 84.10 & @\ t = -0.111 \\ -84.10 & @\ t = 0.953 \end{cases}$

4 Impedance Matching Example using Calculation and Smith Chart

- Length from load to stub:

$$d = \begin{cases} \frac{1}{2\pi}[\pi + \tan^{-1}(t)]\lambda = \frac{1}{2\pi}[\pi + \tan^{-1}(-0.111)]\lambda = 0.482\lambda = 173.52^{o} & @\ t = -0.111 \\ [\frac{1}{2\pi}\tan^{-1}(t)]\lambda = \frac{1}{2\pi}\tan^{-1}(0.953)\lambda = 0.121\lambda = 43.56^{o} & @\ t = 0.935 \end{cases}$$

- For short-circuited stub

$$l_s = \frac{1}{2\pi}\tan^{-1}\left(\frac{Y_0}{B}\right)\lambda = \begin{cases} 0.085\lambda \rightarrow 30.6^{o} & @\ X = 84.10 \\ -0.085\lambda & @\ X = -84.10 \text{ (negative, by adding } \lambda/2 \rightarrow l_s = 0.415\lambda \rightarrow 149.4^{o}) \end{cases}$$

- Therefore,

$$d_1 = 0.121\lambda \rightarrow 43.56^{o},\ l_{s1} = 0.415\lambda \rightarrow 149.4^{o}$$

$$d_2 = 0.482\lambda \rightarrow 173.52^{o},\ l_{s2} = 0.085\lambda \rightarrow 30.6^{o}$$

4 Impedance Matching Example using Calculation and Smith Chart

2) Smith chart solutions

- $Z_L = 75 + j100$
- Normalized impedance: $z_L = Z_L/Z_0 = 1.5 + j2$
- Mark z_L and draw a reflection coefficient circle
- $|\Gamma|$ circle intersects the $1 + jx$ circle at two points denoted as z_1 and z_2 $\rightarrow$ d_1 or d_2

$$d_1 = (0.319 - 0.198)\lambda = 0.121\lambda \rightarrow 43.56^{\circ}$$

$$d_2 = [(0.5 - 0.198) + 0.18)]\lambda = 0.482\lambda \rightarrow 173.52^{\circ}$$

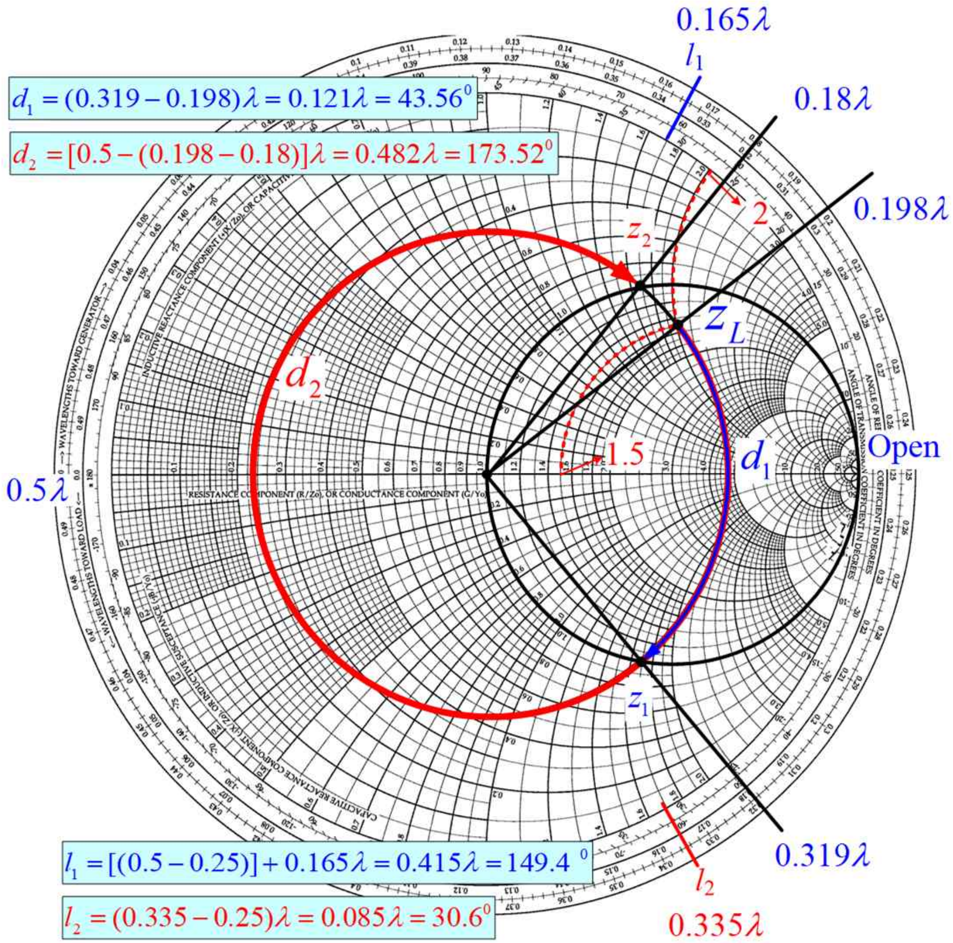
- At the two intersections point,

$$z_1 = 1 - j1.69, \quad z_2 = 1 + j1.69$$

- By moving from $z = \infty$ (open-circuited) toward generator

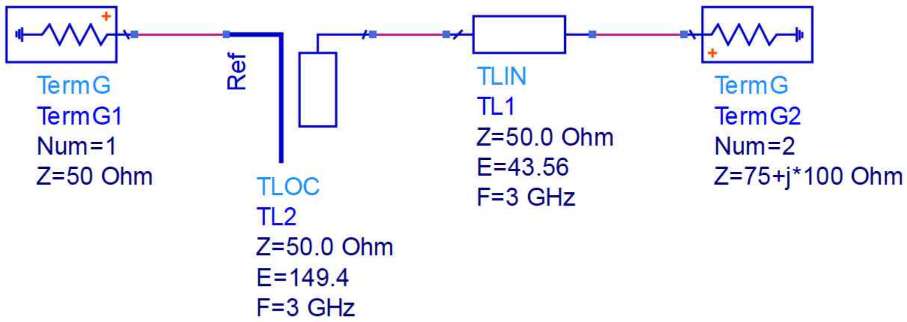
$$l_1 = [(0.5 - 0.25)] + 0.165\lambda = 0.415\lambda \rightarrow 149.4^{\circ}$$

$$l_2 = (0.335 - 0.25)\lambda = 0.085\lambda \rightarrow 30.6^{\circ}$$

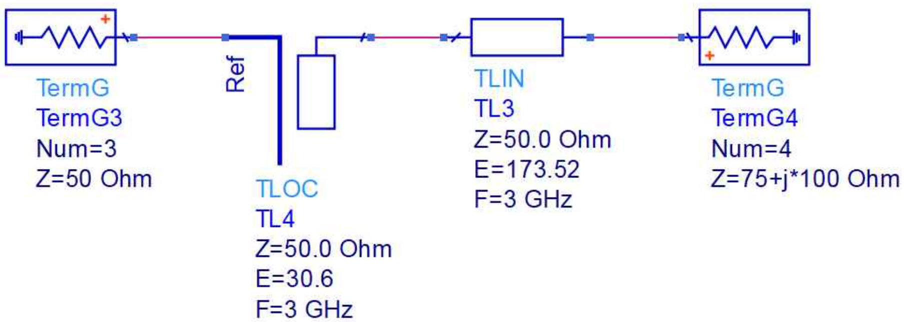


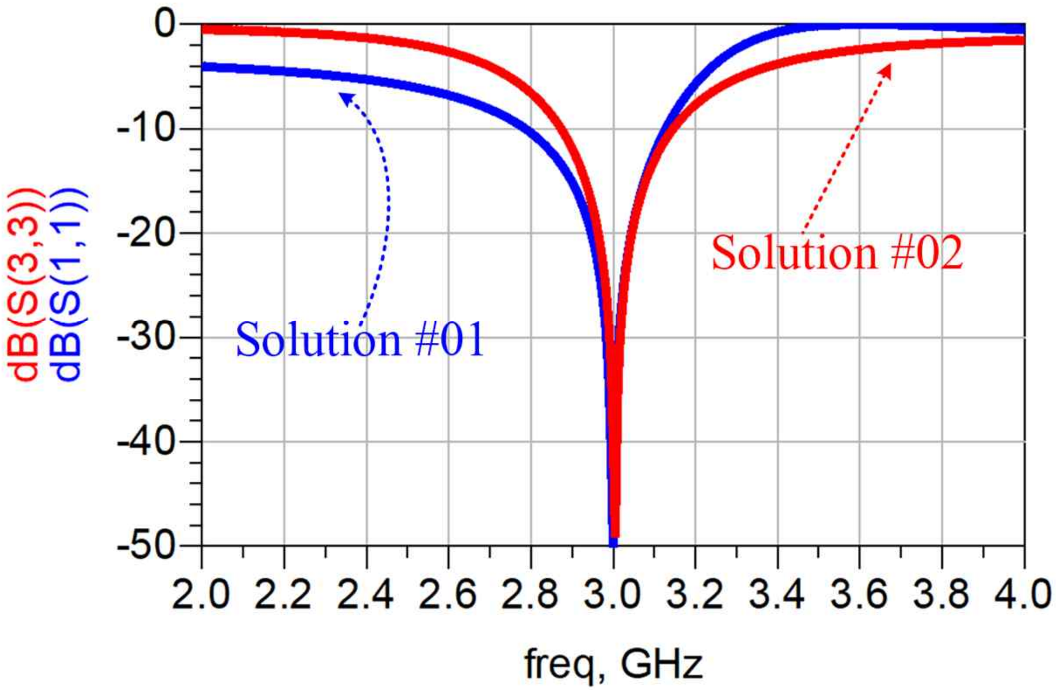
4 Impedance Matching Example using Calculation and Smith Chart

- Solution #01



- Solution #02





5 Review

- Single stub: single open- or short-circuited length of transmission line (a ‘stub’), connected either in parallel or in series with the transmission line at a certain distance from the load.
- Shunt stub is easier to realization than series stub and open stub is easier to fabrication than short stub.