Chapter 5
Impedance Matching

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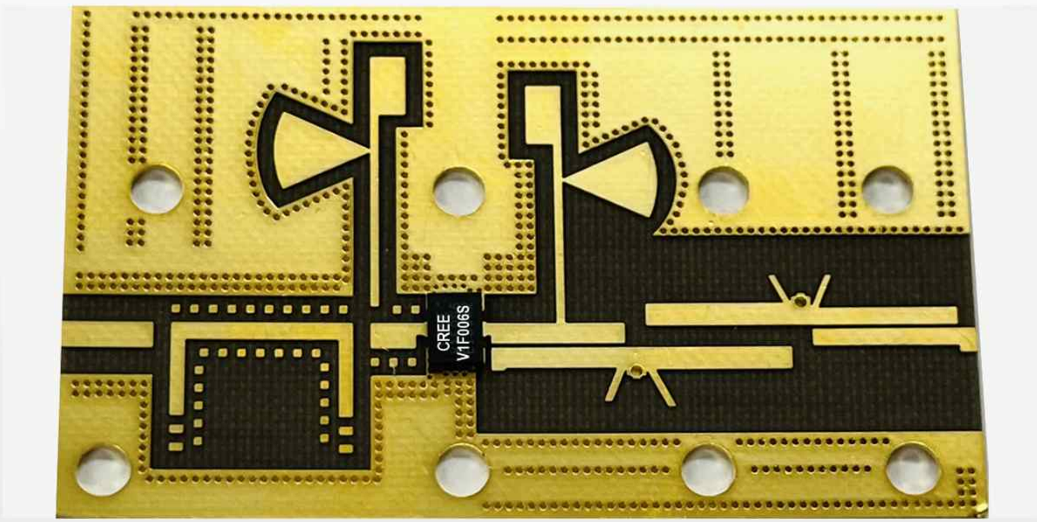


Learning Objectives

- Understanding about double stub matching
- Learn on how to use double stub matching by Smith chart
- Learn on how to use double stub matching by mathematical solution
- Practice double stub matching with example

Learning contents

- Introduction about Double Stub Matching
- Double Stub Matching by using Graphical Solution
- Double Stub Matching by using Mathematical Solution
- Impedance Matching Example using Smitch Chart and Calculation

1 Introduction about Double Stub Matching

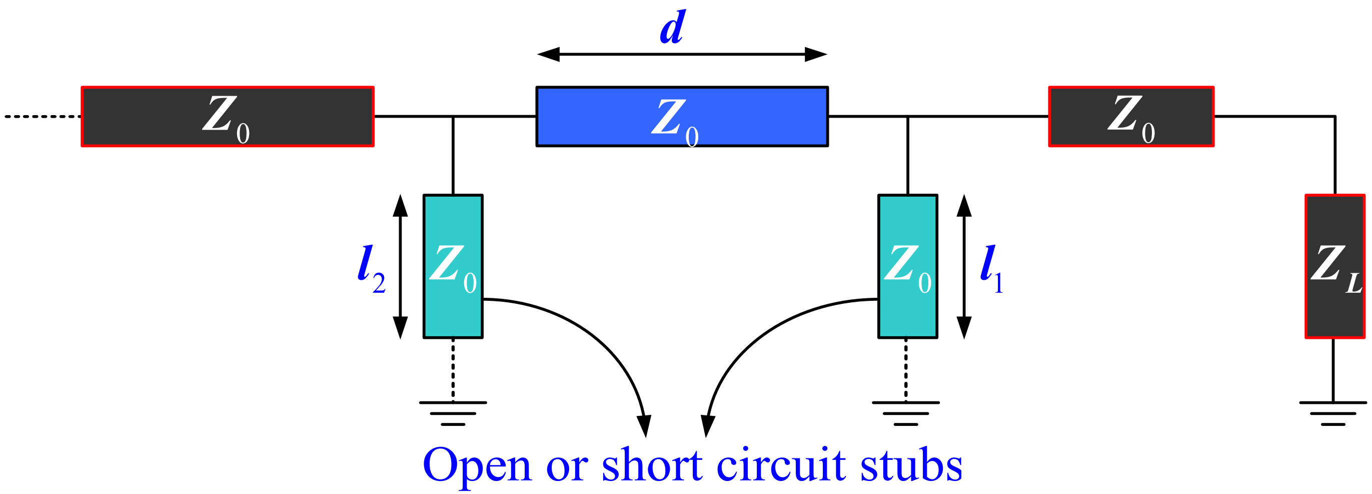
- Disadvantage of single-stub tuning: requiring a variable length (*d*) of series line between load and stub.

  - Difficult if an adjustable tuner is desired

- Double-stub: parallel connected two open- or short-circuited shunt stubs with main transmission line on a fixed length (***d***).

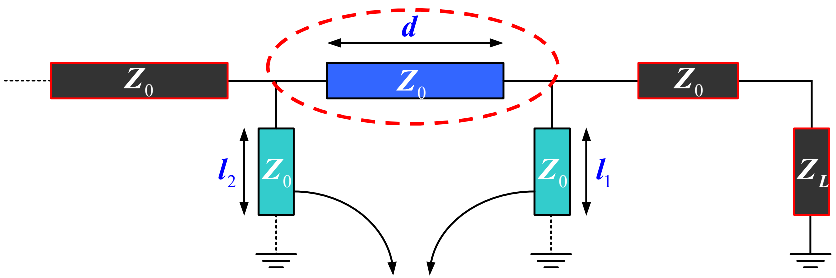
  - This matching structure is more favorable from a practical viewpoint because of adjustable to any arbitrary length from load.

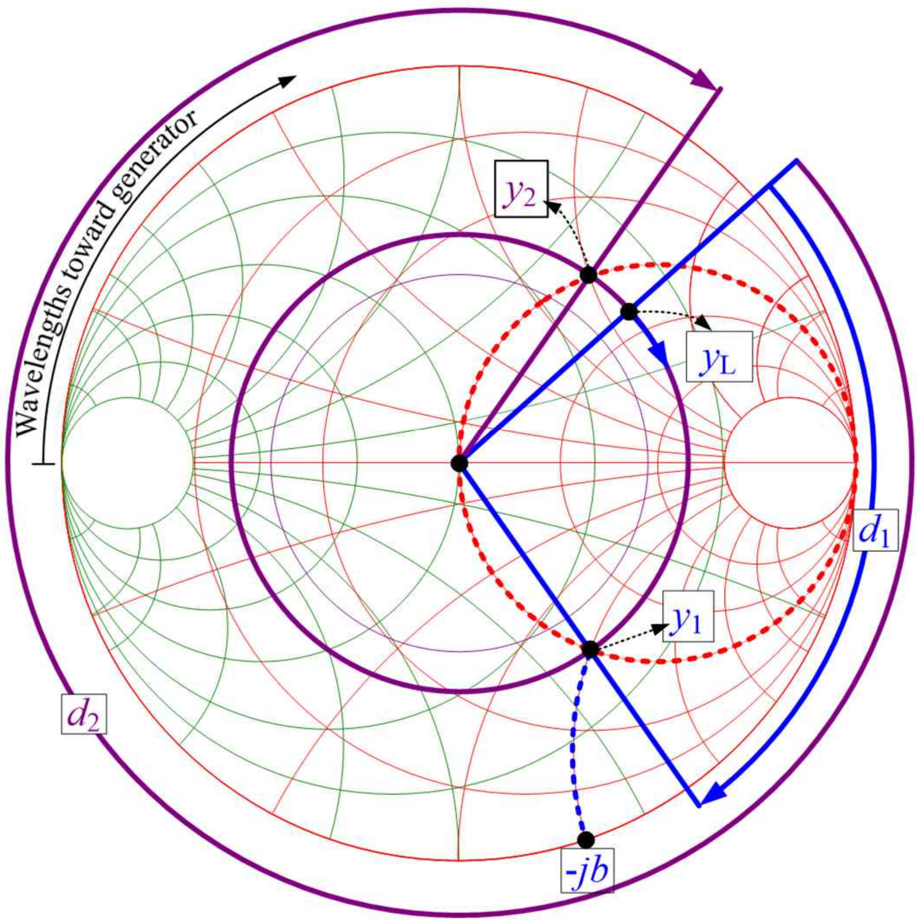
  - Two series Z_0 transmission lines can be removed.



2 Double Stub Matching by using Graphical Solution

- There are two possible solutions for one load admittance (or impedance) point.
  - By moving load admittance (or impedance) along reflection coefficient circle (d_1, d_2), those two intersection points (y_1, y_2) with unit conductance circle (g = 1) are selected as the solutions.
  - In general, the shortest moving length is chosen as first solution y_1 and another one chose as second solution y_2.





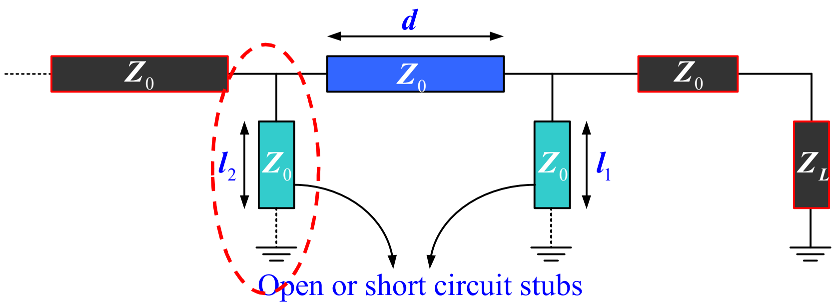
2 Double Stub Matching by using Graphical Solution

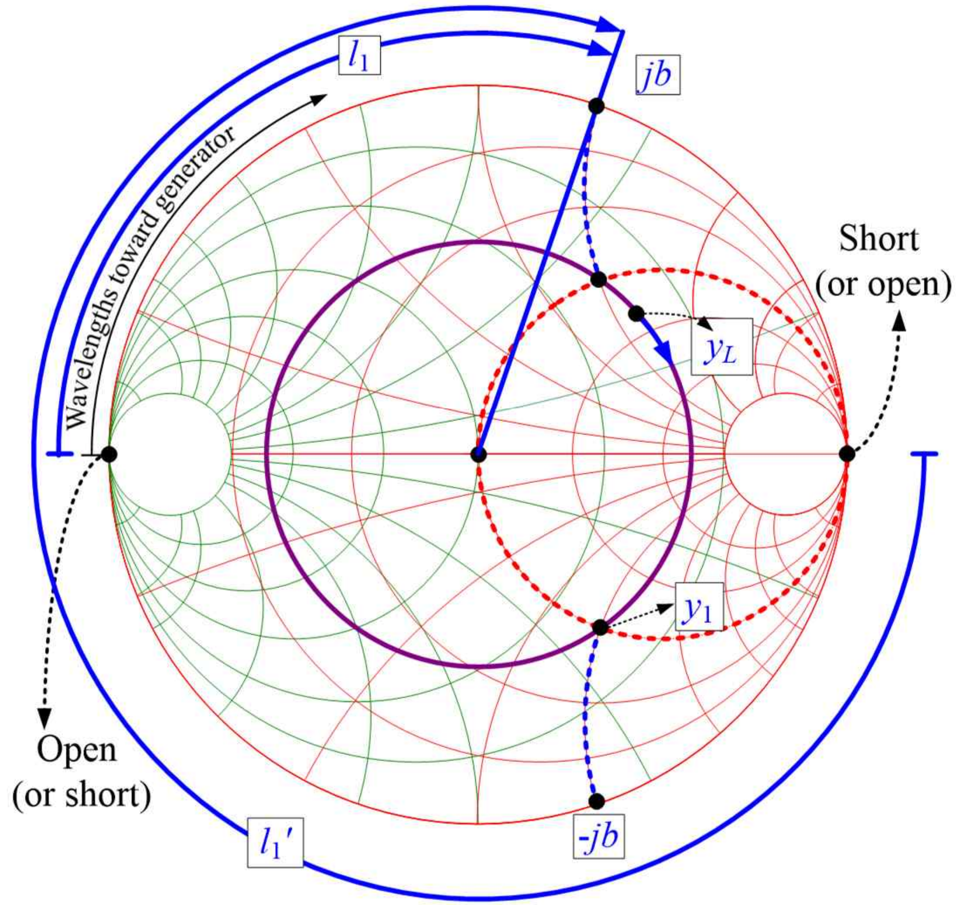
- Choosing stub(s) depends on following factors.

  - Aiming to cancel imaginary part of admittance (or impedance)

  - Select short- or open-circuited stub according to required condition.

  - If designer can design with either short- or open-circuited stub, then shorter length stub is preferable.





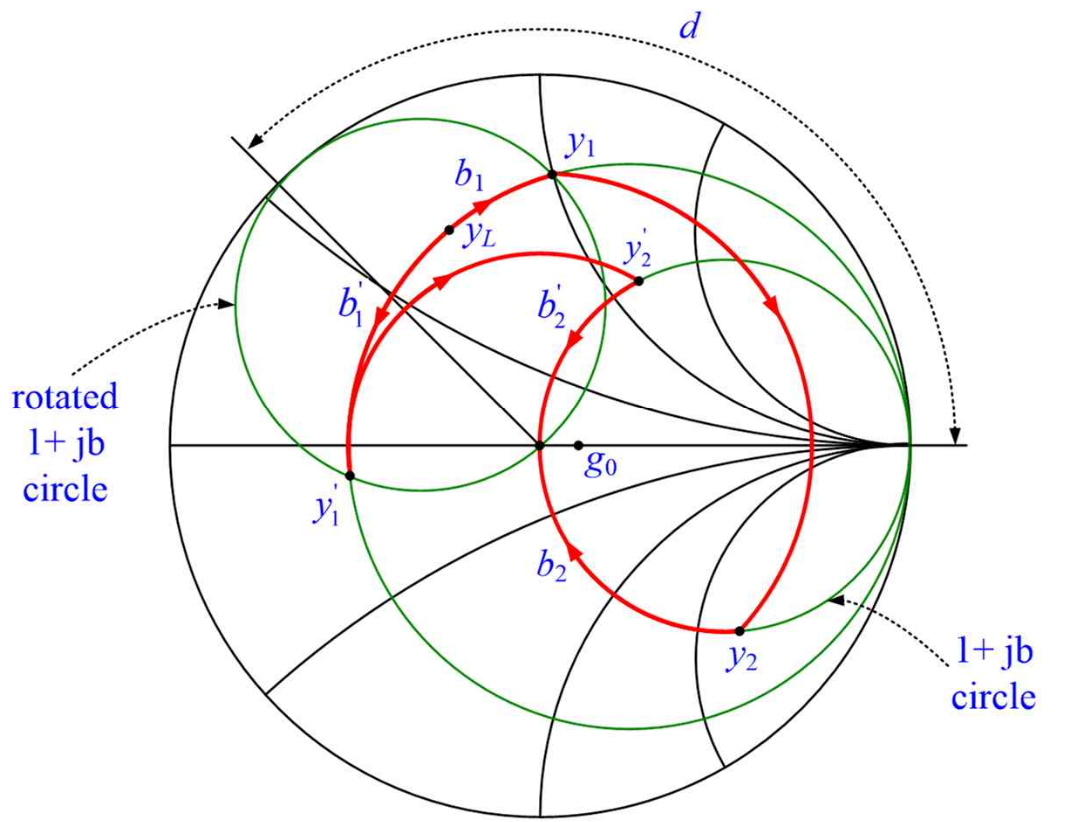
2 Double Stub Matching by using Graphical Solution

- The susceptance of the first stub, b_1 (or b_1' for the second solution), moves the load admittance ($y_L = g_0 + jb$) to y_1 (or y_1').

- The amount of rotation is d wavelengths toward the generator.

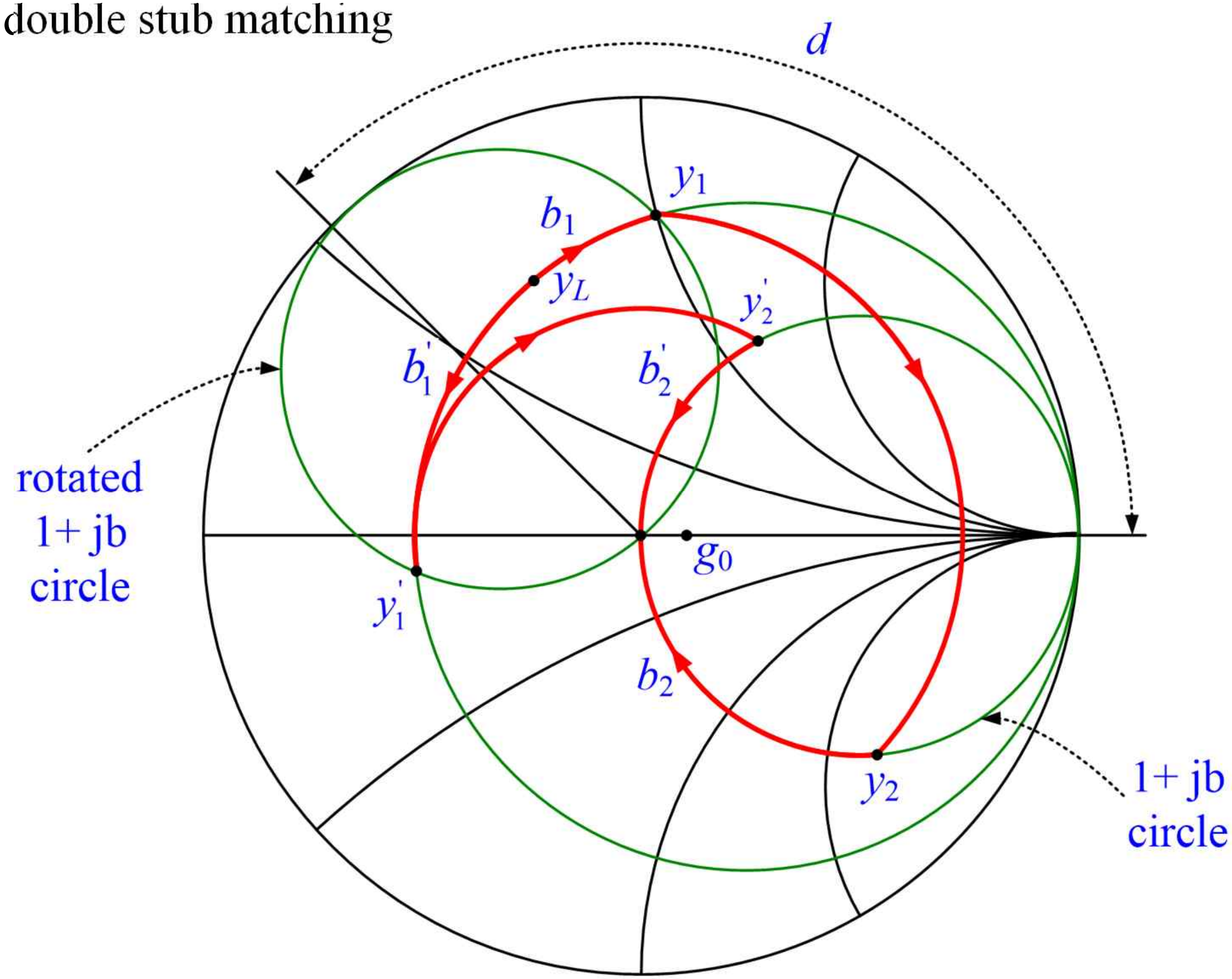
  - Unit resistance circle is rotated clockwisely.
  - Transforming y_1 (or y_1') to y_2 (or y_2'): $1 + jb$ circle

- The second stub then adds a susceptance b_2 (or b_2'), which brings us to the center of the chart, and completes the match.



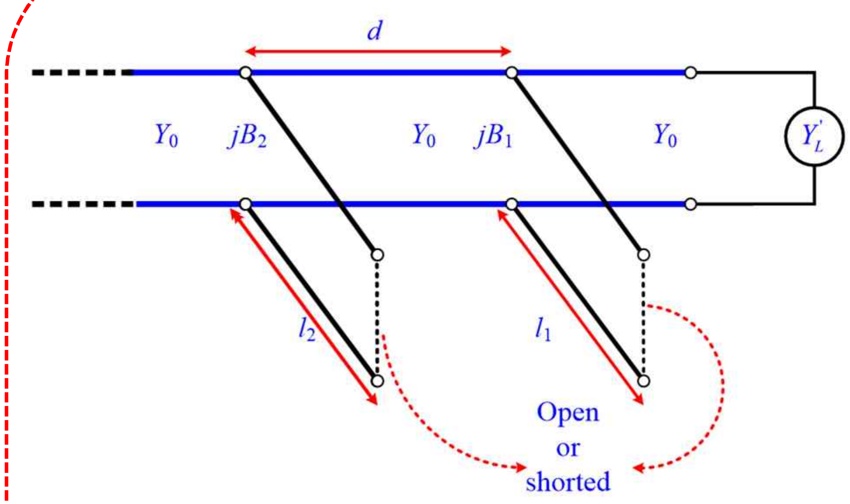
2 Double Stub Matching by using Graphical Solution

- Graphical solution for double stub matching

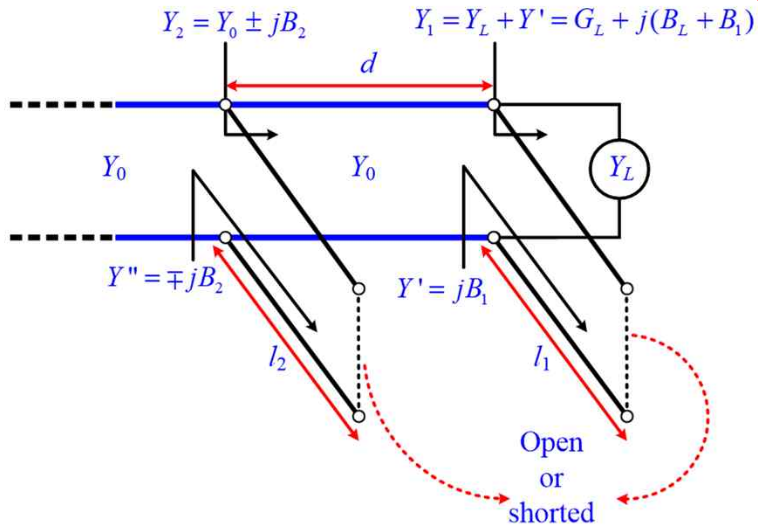


3 Double Stub Matching using Theorical Solution

- Double stub matching network



Double-stub tuning: original circuit with load an arbitrary distance from the first stub.



Double-stub tuning: equivalent circuit with load at the firs stub.

3 Double Stub Matching using Theorical Solution

- Mathematical evaluation of double stub matching

- Derivation of formulas for d, l_1, and l_2

- Load admittance with first stub:

$$Y_1 = Y_L + Y' = G_L + jB_L + jB_1 = G_L + j(B_L + B_1)$$

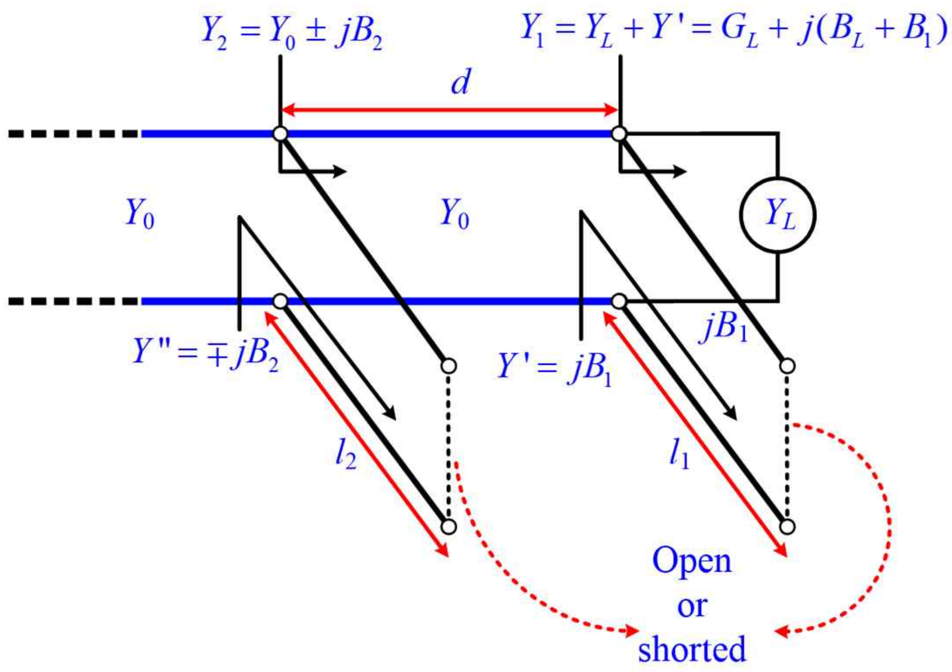
- Admittance just to right of second stub:

$$Y_2 = Y_0 \frac{Y_1 + jY_0 t}{Y_0 + jY_1 t}$$

$$= Y_0 \frac{[G_L + j(B_L + B_1)] + jY_0 t}{Y_0 + j[G_L + j(B_L + B_1)]t}$$

$$= Y_0 \frac{G_L + j(B_L + B_1 + Y_0 t)}{Y_0 + j(G_L + jB_L + jB_1)t}$$

where $t = \tan \beta d$, $Y_0 = 1/Z_0$



3 Double Stub Matching using Theorical Solution

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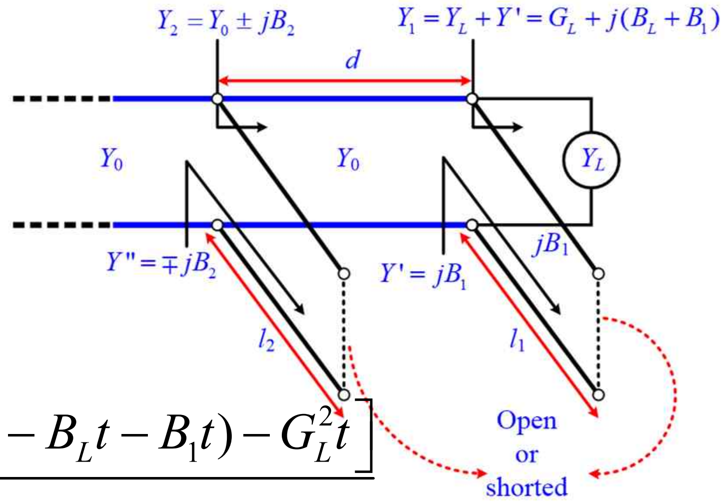


$$Y_2 = Y_0 \frac{G_L + j(B_L + B_1 + Y_0 t)}{Y_0 + j(G_L + jB_L + jB_1)t} = Y_0 \frac{G_L + j(B_L + B_1 + Y_0 t)}{(Y_0 - B_L t - B_1 t) + jG_L t}$$

$$= Y_0 \frac{\left[G_L + j(B_L + B_1 + Y_0 t)\right]\left[(Y_0 - B_L t - B_1 t) - jG_L t\right]}{\left[(Y_0 - B_L t - B_1 t) + jG_L t\right]\left[(Y_0 - B_L t - B_1 t) - jG_L t\right]}$$

$$= Y_0 \frac{\left[G_L (Y_0 - B_L t - B_1 t) + (B_L + B_1 + Y_0 t) G_L t\right] + j\left[(B_L + B_1 + Y_0 t)(Y_0 - B_L t - B_1 t) - G_L^2 t\right]}{(Y_0 - B_L t - B_1 t)^2 + (G_L t)^2}$$

$$= G_2 + jB_2$$

$$\Rightarrow \mathrm{Re}(Y_2) = G_2 = Y_0 \frac{G_L (Y_0 - B_L t - B_1 t) + (B_L + B_1 + Y_0 t) G_L t}{(Y_0 - B_L t - B_1 t)^2 + (G_L t)^2}$$

$$\Rightarrow \mathrm{Im}(Y_2) = B_2 = Y_0 \frac{(B_L + B_1 + Y_0 t)(Y_0 - B_L t - B_1 t) - G_L^2 t}{(Y_0 - B_L t - B_1 t)^2 + (G_L t)^2}$$

3 Double Stub Matching using Theorical Solution

- To match network, $\mathrm{Re}(Y_2) = Y_0$

$$\mathrm{Re}(Y_2) = G_2 = Y_0$$

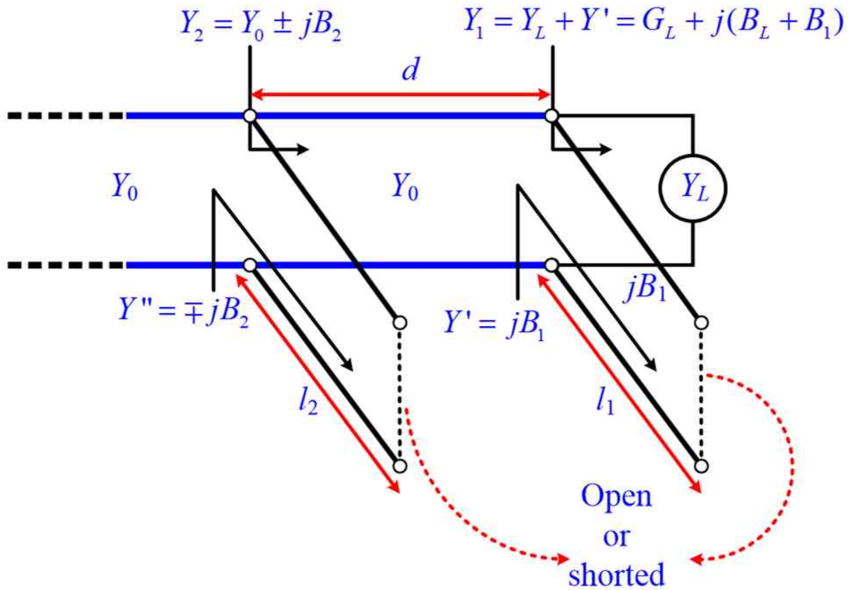
$$\Leftrightarrow Y_0 \frac{G_L(Y_0 - B_L t - B_1 t) + (B_L + B_1 + Y_0 t)G_L t}{(Y_0 - B_L t - B_1 t)^2 + (G_L t)^2} = Y_0$$

$$G_L(Y_0 - B_L t - B_1 t) + (B_L + B_1 + Y_0 t)G_L t = (Y_0 - B_L t - B_1 t)^2 + (G_L t)^2$$

$$G_L Y_0 + G_L Y_0 t^2 = (Y_0 - B_L t - B_1 t)^2 + (G_L t)^2$$

$$(G_L t)^2 + (Y_0 - B_L t - B_1 t)^2 - G_L Y_0 - G_L Y_0 t^2 = 0$$

$$G_L{}^2 - G_L Y_0 \frac{1+t^2}{t^2} + \frac{(Y_0 - B_L t - B_1 t)^2}{t^2} = 0 \qquad (1)$$



- By using quadratic formula,

$$G_L = Y_0 \frac{1+t^2}{2t^2}\left[1 \pm \sqrt{1 - \frac{4t^2(Y_0 - B_L t - B_1 t)^2}{Y_0^2(1+t^2)^2}}\right]$$

3 Double Stub Matching using Theorical Solution

- Since G_L must be real,

$$0 \le \frac{4t^2(Y_0 - B_L t - B_1 t)^2}{Y_0^2(1+t^2)^2} \le 1$$

$$\Rightarrow \quad 0 \le G_L \le Y_0 \frac{1+t^2}{t^2} = \frac{Y_0}{\sin^2 \beta d}$$

$$G_L = Y_0 \frac{1+t^2}{2t^2}\left[1 \pm \sqrt{1 - \frac{4t^2(Y_0 - B_L t - B_1 t)^2}{Y_0^2(1+t^2)^2}}\right]$$

$$t = \tan \beta d$$

$$\frac{1+\tan^2 \beta d}{\tan^2 \beta d} = \frac{1}{\sin^2 \beta d}$$

- After d has been fixed (t can be found), the first stub (B_1) susceptance can be determined from (1) as:

$$G_L{}^2 - G_L Y_0 \frac{(1+t^2)}{t^2} + \frac{(Y_0 - B_L t - B_1 t)^2}{t^2} = 0 \quad (1)$$

$$(Y_0 - B_L t - B_1 t)^2 = G_L Y_0 (1+t^2) - G_L{}^2 t^2$$

$$-B_1 t = -Y_0 + B_L t \pm \sqrt{G_L Y_0 (1+t^2) - G_L{}^2 t^2}$$

$$B_1 = -B_L + \frac{Y_0 \pm \sqrt{G_L Y_0 (1+t^2) - G_L{}^2 t^2}}{t} \quad (2) \quad \leftarrow B_1 = f(G_L, B_L, Y_0, t)\text{: known value}$$

3 Double Stub Matching using Theorical Solution

- B_2 = -Im(Y_2):

$$B_2 = Y_0 \frac{G_L^2 t - (B_L + B_1 + Y_0 t)(Y_0 - B_L t - B_1 t)}{(Y_0 - B_L t - B_1 t)^2 + (G_L t)^2} \quad (3) \quad \leftarrow B_2 = f(G_L, Y_0, t)\text{: known value}$$

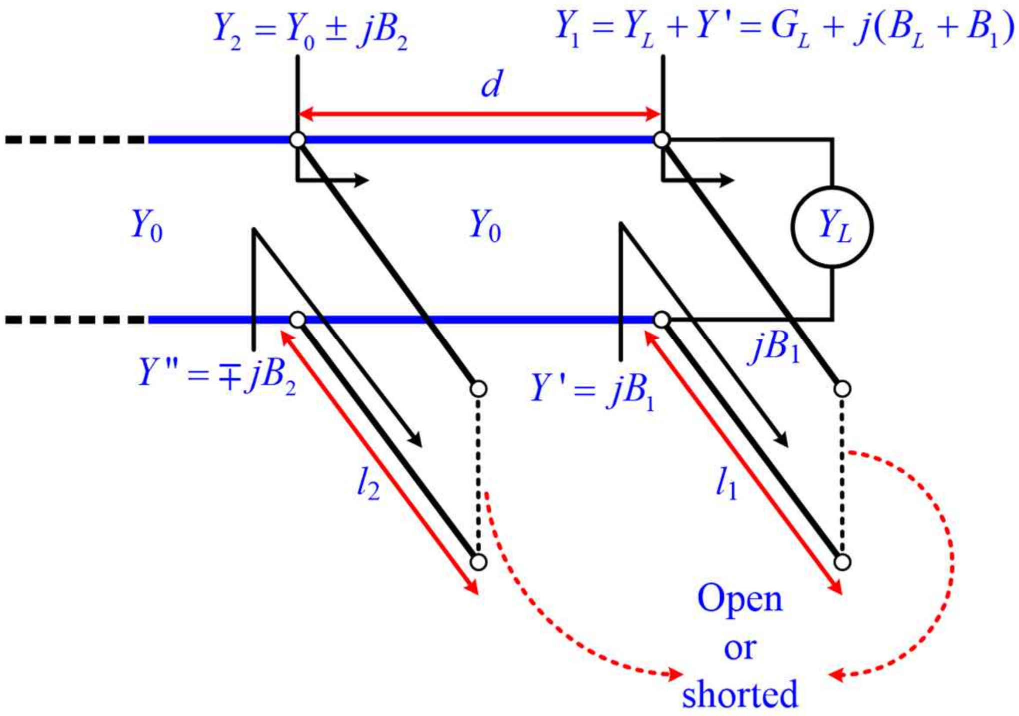
- Open-circuited stub length:

$$\frac{l_o}{\lambda} = \frac{1}{2\pi}\tan^{-1}\left(\frac{B}{Y_0}\right)$$

- Short-circuited stub length:

$$\frac{l_s}{\lambda} = -\frac{1}{2\pi}\tan^{-1}\left(\frac{Y_0}{B}\right)$$

where $B = B_1, B_2$



4 Double Stub Impedance Matching Example using Calculation and Smith Chart

- Design a double stub shunt tuner network to match a load impedance $Z_L = 25 - j75$ [Ω] to 50 [Ω] line at a frequency of 3 GHz. The stubs are open-circuit and spaced $\lambda/8$ apart.

  a) Using theorical solution to determine double stubs.

  b) Using graphical solution to determine double stubs.

  c) Plot return loss of all solution in dB from 2 GHz to 4 GHz.

Solution

a) Determine double open-circuit stubs by theorical solution

$$Y_L = G_L + jB_L = \frac{1}{Z_L} = \frac{1}{25 - j75} = 0.004 + j0.012$$

$$Y_0 = \frac{1}{Z_0} = 0.02$$

- Set $d = \lambda/8$

4 Double Stub Impedance Matching Example using Calculation and Smith Chart

$$t = 1, \qquad Y_0 = 0.02\ [S]$$
$$G_L = 0.004[S], \qquad B_L = 0.012\ [S]$$

- By using equation (2) and (3),

$$B_1 = -B_L + \frac{Y_0 \pm \sqrt{(1+t^2)G_L Y_0 - G_L^2 t^2}}{t} = -0.012 + \frac{0.02 \pm \sqrt{(1+1^2)(0.004)(0.02) - 0.004^2 \times 1^2}}{1}$$

$$\therefore \begin{cases} B_1 = 0.02 & \text{for "+" sign} \\ B_1^{'} = -0.004 & \text{for "-" sign} \end{cases}$$

$$B_2 = \frac{\pm Y_0 \sqrt{Y_0 G_L (1+t^2) - G_L^2 t^2} + G_L Y_0}{G_L t} = \frac{\pm 0.02\sqrt{0.02 \times 0.004(1+1^2) - 0.004^2 \times 1^2} + 0.004 \times 0.02}{0.004 \times 1}$$

$$\therefore \begin{cases} B_2 = 0.08 & \text{for "+" sign} \\ B_2^{'} = -0.04 & \text{for "-" sign} \end{cases}$$

4 Double Stub Impedance Matching Example using Calculation and Smith Chart

- For open circuit stub:

⇒ 1st solution

$$\frac{l_{o1}}{\lambda} = \frac{1}{2\pi}\tan^{-1}\left(\frac{B_1}{Y_0}\right) = \frac{1}{2\pi}\tan^{-1}\left(\frac{0.02}{0.02}\right) = 0.125 \rightarrow l_{o1} = 0.125\lambda \rightarrow 45^{o}$$

$$\frac{l_{o2}}{\lambda} = \frac{1}{2\pi}\tan^{-1}\left(\frac{B_2}{Y_0}\right) = \frac{1}{2\pi}\tan^{-1}\left(\frac{0.08}{0.02}\right) = 0.211 \rightarrow l_{o2} = 0.211\lambda \rightarrow 75.96^{o}$$

⇒ 2nd solution

$$\frac{l_{o1}^{'}}{\lambda} = \frac{1}{2\pi}\tan^{-1}\left(\frac{B_{o1}^{'}}{Y_0}\right) = \frac{1}{2\pi}\tan^{-1}\left(\frac{-0.004}{0.02}\right) = -0.031 \rightarrow l_{o1} = (-0.031 + 0.5)\lambda = 0.469\lambda \rightarrow 168.84^{o}$$

$$\frac{l_{02}^{'}}{\lambda} = \frac{1}{2\pi}\tan^{-1}\left(\frac{B_2^{'}}{Y_0}\right) = \frac{1}{2\pi}\tan^{-1}\left(\frac{-0.04}{0.02}\right) - 0.176 \rightarrow l_{o2}^{'} = (-0.176 + 0.5)\lambda = 0.324\lambda \rightarrow 116.64^{o}$$

4 Double Stub Impedance Matching Example using Calculation and Smith Chart

b) Determine double open-circuit stubs by graphical solution

- Normalize admittance

$$y_L = \frac{1}{z_L} = \frac{Z_0}{Z_L} = \frac{50}{25 - j75} = 0.2 + j0.6$$

- By moving every point of $g = 1$ circle about $\lambda/8$ toward the load, the susceptance of the first stub are one of two possible values:

$$b_1 = 1.6 - 0.6 = 1 \text{ or } b_1' = 0.39 - 0.6 = -0.21$$

- Transform through the $\lambda/8$ section of line by rotating along a constant-radius (SWR) circle toward generator. Selected two solutions

$$y_2 = 1 + j2,\ y_2' = 1 - j4$$

- Susceptance of second stub

$$b_2 = -2,\ b_2' = 4$$

4 Double Stub Impedance Matching Example using Calculation and Smith Chart

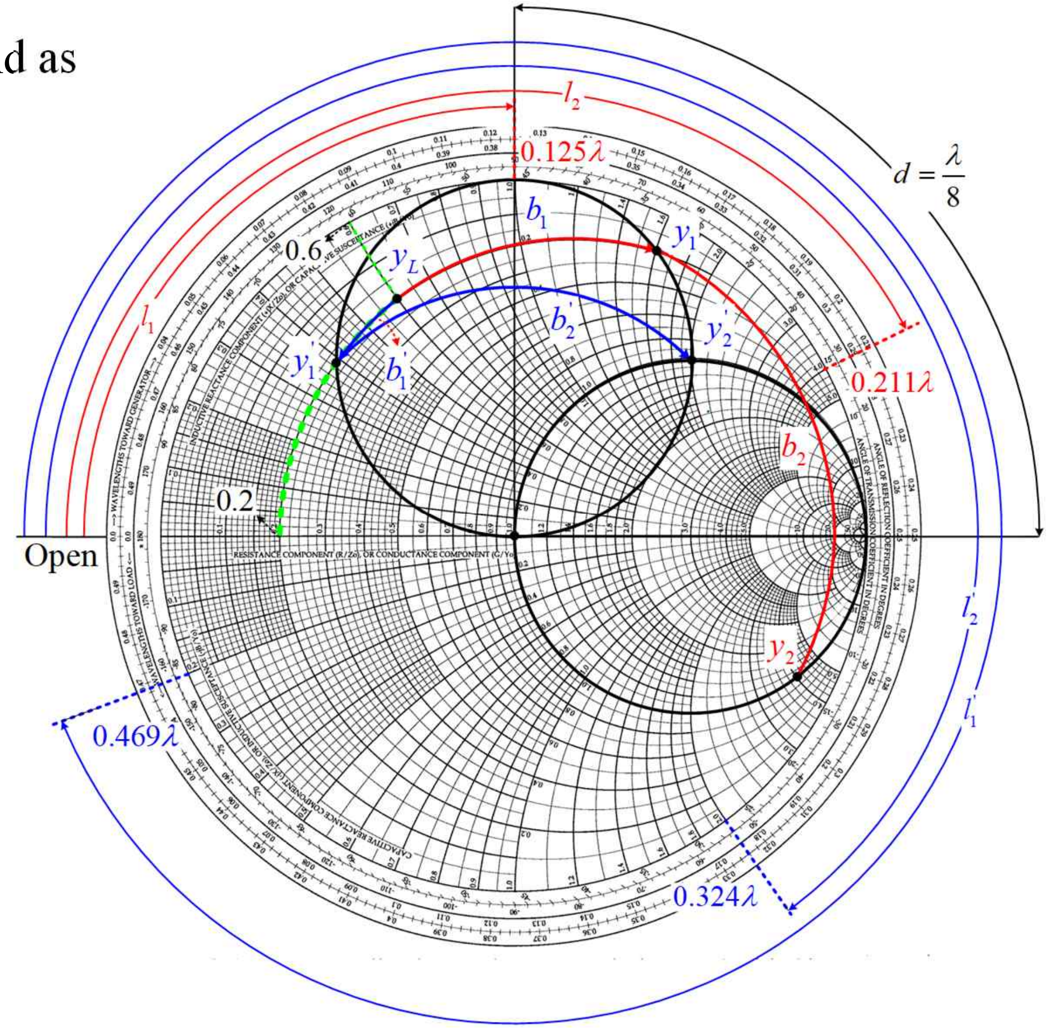
- The lengths of the open-circuited stubs are then found as

⇒ 1st solution

$l_1 = 0.125\lambda \rightarrow 45^{\circ}, \quad l_2 = 0.211\lambda \rightarrow 75.96^{\circ}$

⇒ 2nd solution

$l_1^{'} = 0.469\lambda \rightarrow 168.84^{\circ}, \quad l_2^{'} = 0.324\lambda \rightarrow 116.64^{\circ}$

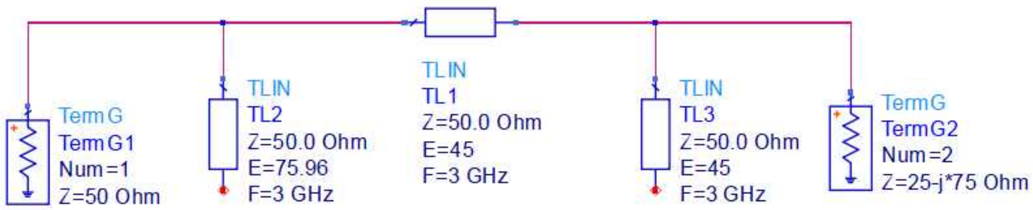


4 Double Stub Impedance Matching Example using Calculation and Smith Chart

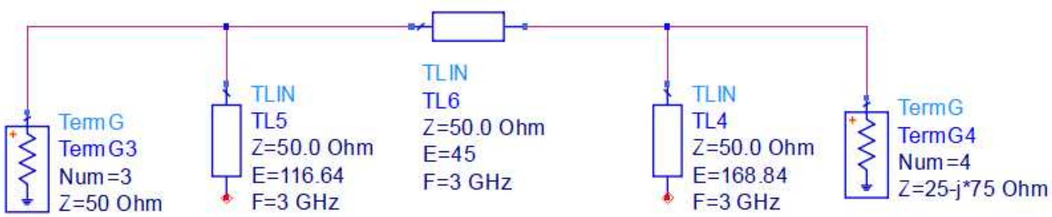
c) Plot the return loss of all solution in dB from 2 GHz to 4 GHz.

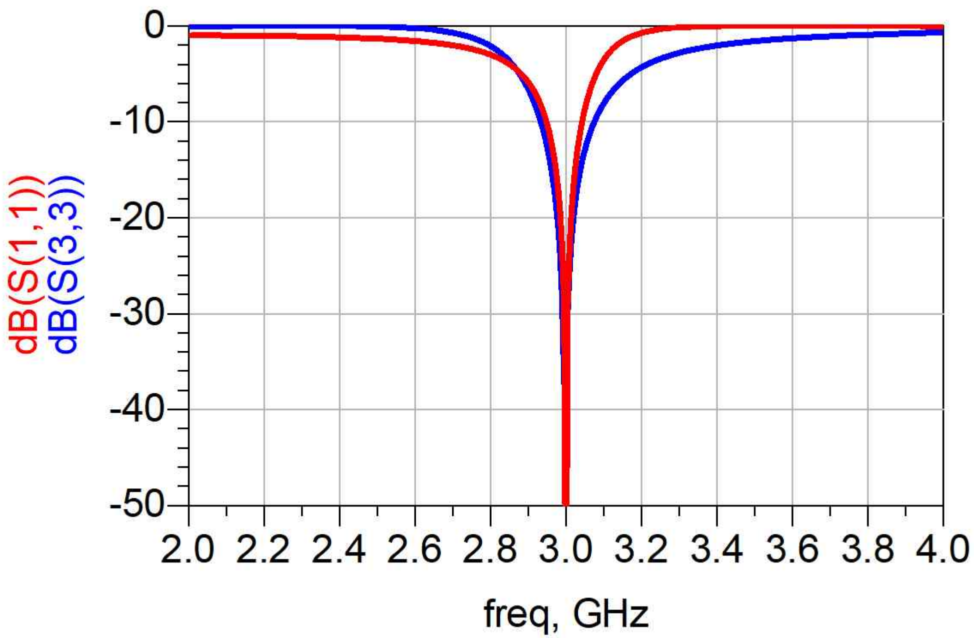
- Double open-circuited stubs matchings

- Solution #01



- Solution #02





5 Review

- Double stub impedance matching circuit
- Open-circuited stub length:

$$\frac{l_o}{\lambda} = \frac{1}{2\pi}\tan^{-1}\left(\frac{B}{Y_0}\right)$$

- Short-circuited stub length:

$$\frac{l_s}{\lambda} = \frac{-1}{2\pi}\tan^{-1}\left(\frac{Y_0}{B}\right)$$

where $B = B_1, B_2$ and $B_1 = -B_L + \dfrac{Y_0 \pm \sqrt{G_L Y_0 (1+t^2) - G_L{}^2 t^2}}{t}$, $B_2 = \dfrac{\pm Y_0 \sqrt{Y_0 G_L (1+t^2) - G_L^2 t^2} + G_L Y_0}{G_L t}$, $t = \tan \beta d$