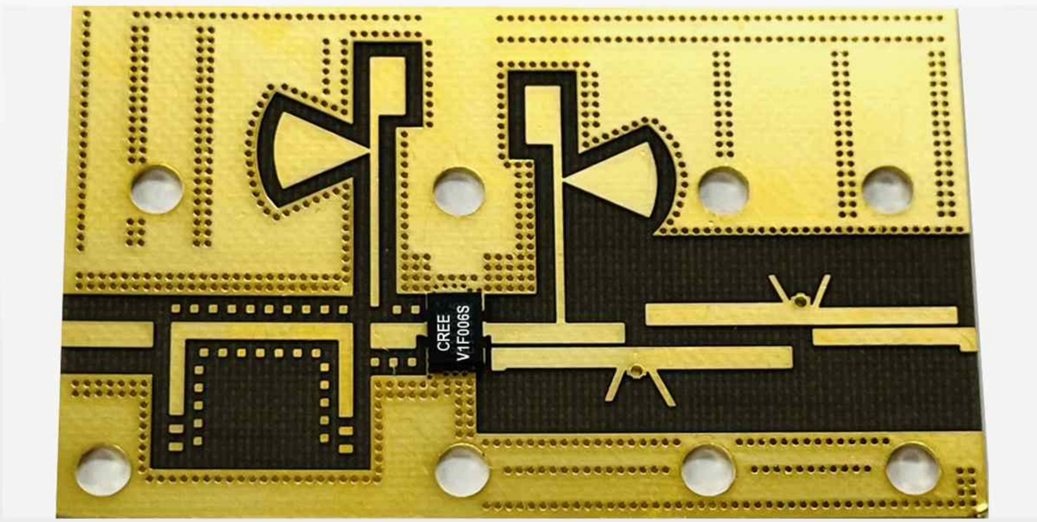
Chapter 5
Impedance Matching

Prof. Jeong, Yongchae



Learning Objectives

- Understanding an overview of quarter-wave transformer
- Learn how to match impedance by using quarter-wave transformer.
- Practice impedance matching by quarter-wave transformer

Learning contents

- Introduction about Quarter-wave Transformer
- Impedance Matching using Quarter-wave Transformer
- Quarter-wave Transformer Examples

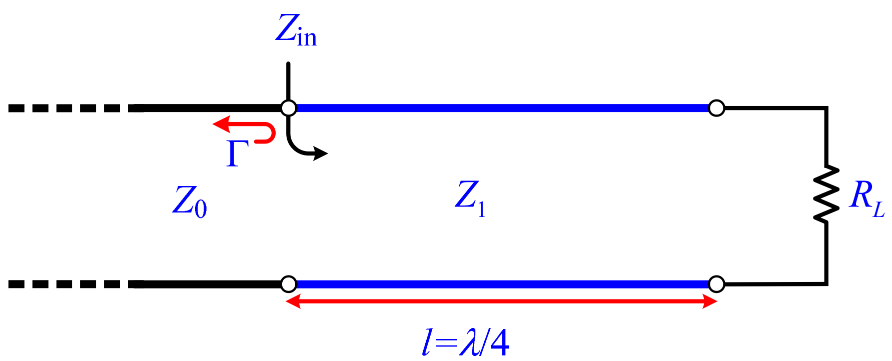
1 Introduction about Quarter-wave Transformer

- The quarter-wave impedance transformer having a transmission line segment with a length equal to one-quarter of the wavelength ($\lambda/4$) at the operating frequency can transform impedances in a predictable manner.

  - By carefully choosing the characteristic impedance of the quarter-wave section, it is possible to match the load impedance (R_L) to desirable impedance (Z_0). → ***Only real-to-real impedance matching***

  - Quarter-wave transformers target a particular frequency, and the length of the transformer is equal to $\lambda_0/4$ only at designed frequency (f_0). → ***Limited frequency band characteristics***

  - Single-section $\lambda/4$ transformer: narrow band application
    Multi-section $\lambda/4$ transformer: broad band application



1 Introduction about Quarter-wave Transformer

- Quarter-wave transformer is used for impedance matchings

  - between a resistive load and transmission line
  - between resistive source and load.
  - between two different characteristic impedance transmission lines

- Advantages of quarter-wave transformer

  - Simple, easy implementation
  - No additional component (or circuit)
  - Low cost

- Disadvantages

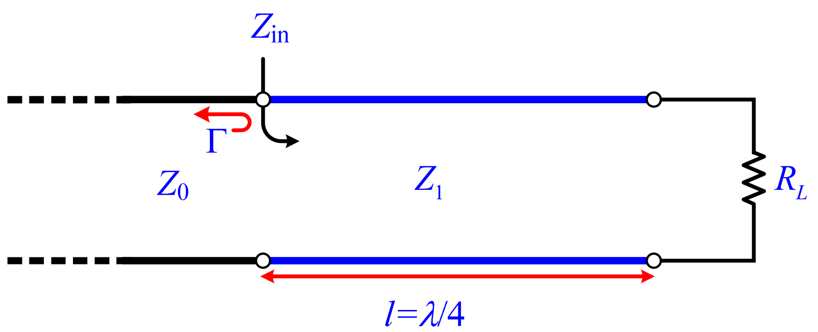
  - Frequency sensitivity
  - Much space requirement especially for low frequencies
  - Low ratio of impedance transformer

2 Impedance Matching using Quarter-wave Transformer

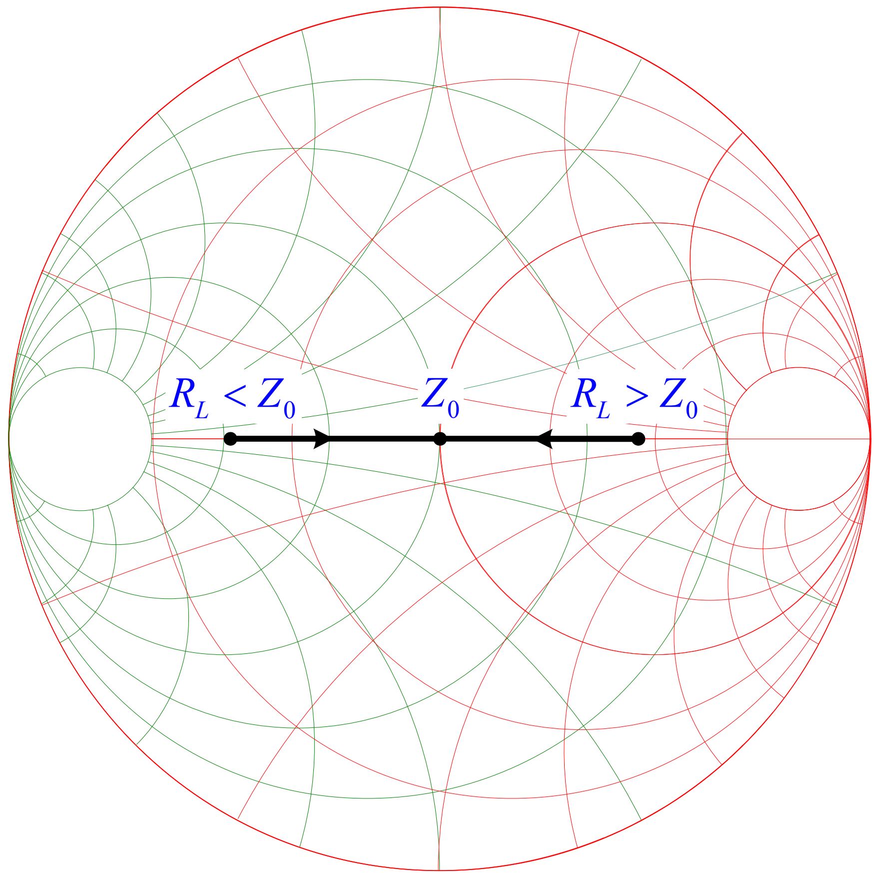
- Characteristic impedance of matching section

$$Z_1 = \sqrt{Z_0 R_L} \quad \text{or} \quad Z_1^2 = Z_0 R_L$$

where $l = \lambda_0/4$: electrical length at operating frequency, f_0.



A single-section quarter-wave matching transformer

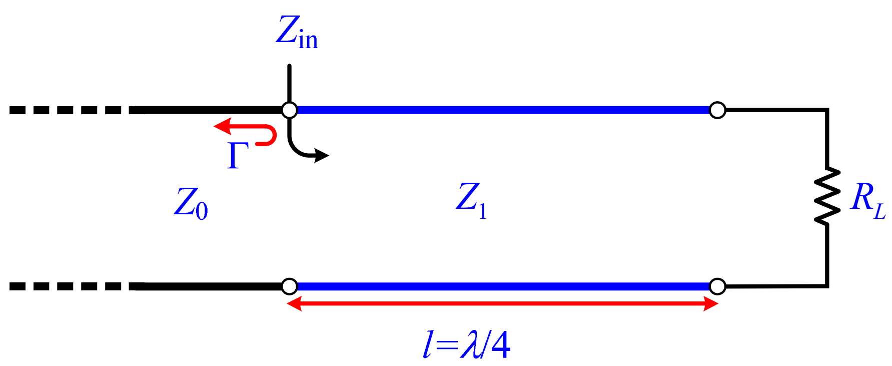


2 Impedance Matching using Quarter-wave Transformer

- Input impedance seen looking into matching section:

$$Z_{in} = Z_1 \frac{R_L + jZ_1 t}{Z_1 + jR_L t}$$

where $t = \tan\beta l = \tan\theta$ and $\beta l = \theta = \pi / 2$



- Reflection coefficient

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{Z_1 \frac{R_L + jZ_1 t}{Z_1 + jR_L t} - Z_0}{Z_1 \frac{R_L + jZ_1 t}{Z_1 + jR_L t} + Z_0}$$

$$= \frac{Z_1(R_L + jZ_1 t) - Z_0(Z_1 + jR_L t)}{Z_1(R_L + jZ_1 t) + Z_0(Z_1 + jR_L t)} = \frac{Z_1(R_L - Z_0) + jt(Z_1^2 - Z_0 R_L)}{Z_1(R_L + Z_0) + jt(Z_1^2 + Z_0 R_L)} \quad \leftarrow Z_1^2 = Z_0 R_L$$

$$= \frac{Z_1(R_L - Z_0)}{Z_1(R_L + Z_0) + j2tZ_0 R_L} = \frac{R_L - Z_0}{R_L + Z_0 + j2t\sqrt{Z_0 R_L}}$$

2 Impedance Matching using Quarter-wave Transformer

- Reflection coefficient magnitude:

$$|\Gamma| = \frac{|R_L - Z_0|}{[(R_L + Z_0)^2 + 4t^2 Z_0 R_L]^{1/2}}$$

$$= \frac{1}{\{[(R_L + Z_0)/(R_L - Z_0)]^2 + 4t^2 Z_0 R_L/(R_L - Z_0)^2]\}^{1/2}}$$

$$= \frac{1}{\{1 + [4Z_0 R_L/(R_L - Z_0)]^2 + 4t^2 Z_0 R_L/(R_L - Z_0)^2]\}^{1/2}}$$

$$= \frac{1}{\{1 + [4Z_0 R_L/(R_L - Z_0)]^2 (1 + t^2)\}^{1/2}}$$

$$= \frac{1}{\{1 + [4Z_0 R_L/(R_L - Z_0)]^2 \sec^2\theta\}^{1/2}} \quad \leftarrow \; 1 + t^2 = 1 + \tan^2\theta = \sec^2\theta$$

$$\frac{1}{|\Gamma|} = \{1 + [4Z_0 R_L/(R_L - Z_0)]^2 \sec^2\theta\}^{1/2}$$

$$\frac{1}{|\Gamma|^2} = 1 + [4Z_0 R_L/(R_L - Z_0)]^2 \sec^2\theta$$

2 Impedance Matching using Quarter-wave Transformer

- If the frequency is near the design frequency (f_0), then $l \approx \lambda_0/4$ and $\theta \approx \pi/2$.

- Since $\sec^2\theta >> 1$ (@$\theta \approx \pi/2$),

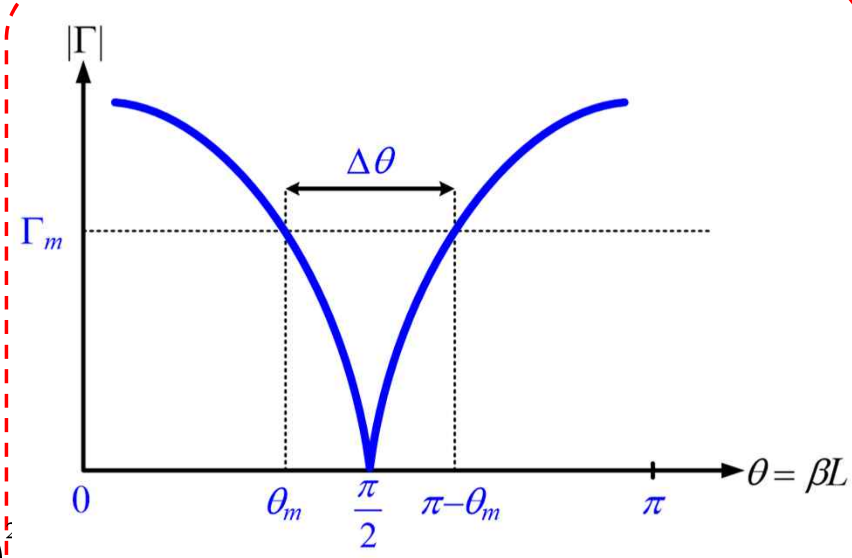
$$|\Gamma| = \frac{|R_L - Z_0|}{2\sqrt{Z_0 R_L}}|\cos\theta| \quad \text{for } \theta \text{ near } \pi/2$$

- If the maximum value (Γ_m) of the tolerable reflection coefficient magnitude is given, then the bandwidth of the matching transformer can be defined as

$$\Delta\theta = 2\left(\frac{\pi}{2} - \theta_m\right)$$

$$\frac{1}{\Gamma_m^2} = 1 + \left(\frac{2\sqrt{Z_0 R_L}}{R_L - Z_0}\sec\theta_m\right)^2, \quad \frac{1}{\Gamma_m^2} - 1 = \frac{1-\Gamma_m^2}{\Gamma_m^2} = \left(\frac{2\sqrt{Z_0 R_L}}{R_L - Z_0}\frac{1}{\cos\theta_m}\right)^2$$

$$\cos\theta_m = \frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_0 R_L}}{|R_L - Z_0|} \Rightarrow \theta_m = \cos^{-1}\left(\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_0 R_L}}{|R_L - Z_0|}\right)$$



Approximate behavior of the reflection coefficient magnitude for a single-section quarter-wave transformer

2 Impedance Matching using Quarter-wave Transformer

- For TEM transmission lines,

$$\theta = \beta l = \frac{2\pi f}{v_p}\frac{v_p}{4 f_0} = \frac{\pi f}{2 f_0}$$

$$f_m = \frac{2\theta_m f_0}{\pi} \quad @\theta = \theta_m$$

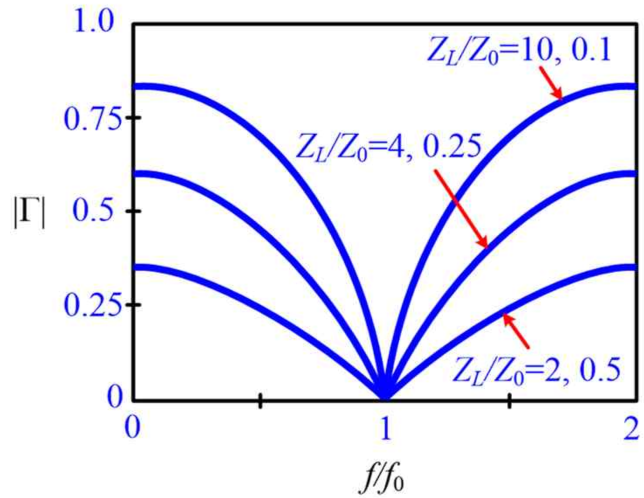
- Fractional bandwidth:

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{2 f_m}{f_0}$$

$$= 2 - \frac{2}{f_0}\left(\frac{2\theta_m f_0}{\pi}\right) = 2 - \frac{4\theta_m}{\pi}$$

$$= 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right]$$

$$\theta_m = \cos^{-1}\left(\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_0 R_L}}{|R_L - Z_0|}\right)$$



Reflection coefficient magnitude versus frequency for a single-section quarter-wave matching transformer with various load mismatches.

2 Impedance Matching using Quarter-wave Transformer

- Fractional bandwidth is usually expressed as a percentage as like $100\left(\frac{\Delta f}{f_0}\right)\%$

- Impedance transforming frequency bandwidth is increased as Z_L is closer to Z_0 (or $Z_L/Z_0 \rightarrow 1$).

▪ Practical issues

- Non-TEM transmission lines (such as waveguides): nonlinear propagation constant on frequency

⇒ Limited practice frequency bandwidth

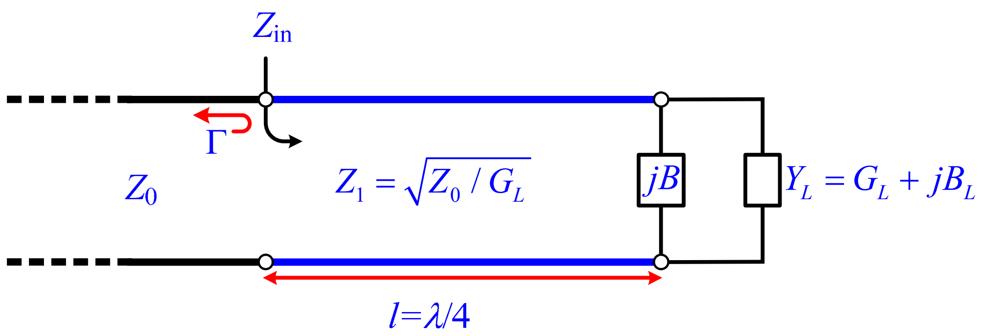
⇒ In practice, the bandwidth of the transformer is often small enough that these complications do not substantially affect the result.

- Reactance associated with discontinuities between transmission lines

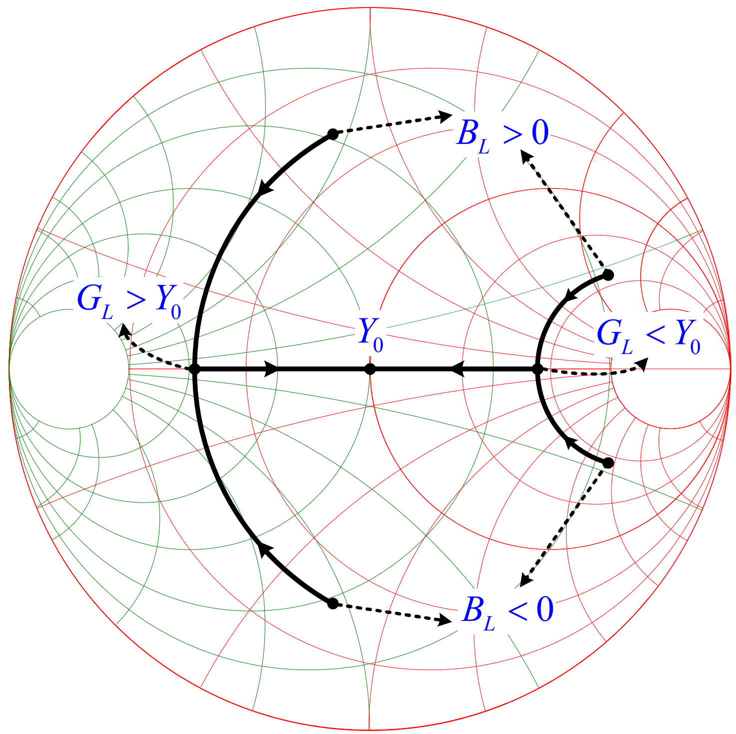
⇒ This problem often be compensated by making a small adjustment in the length of the matching section.

2 Impedance Matching using Quarter-wave Transformer

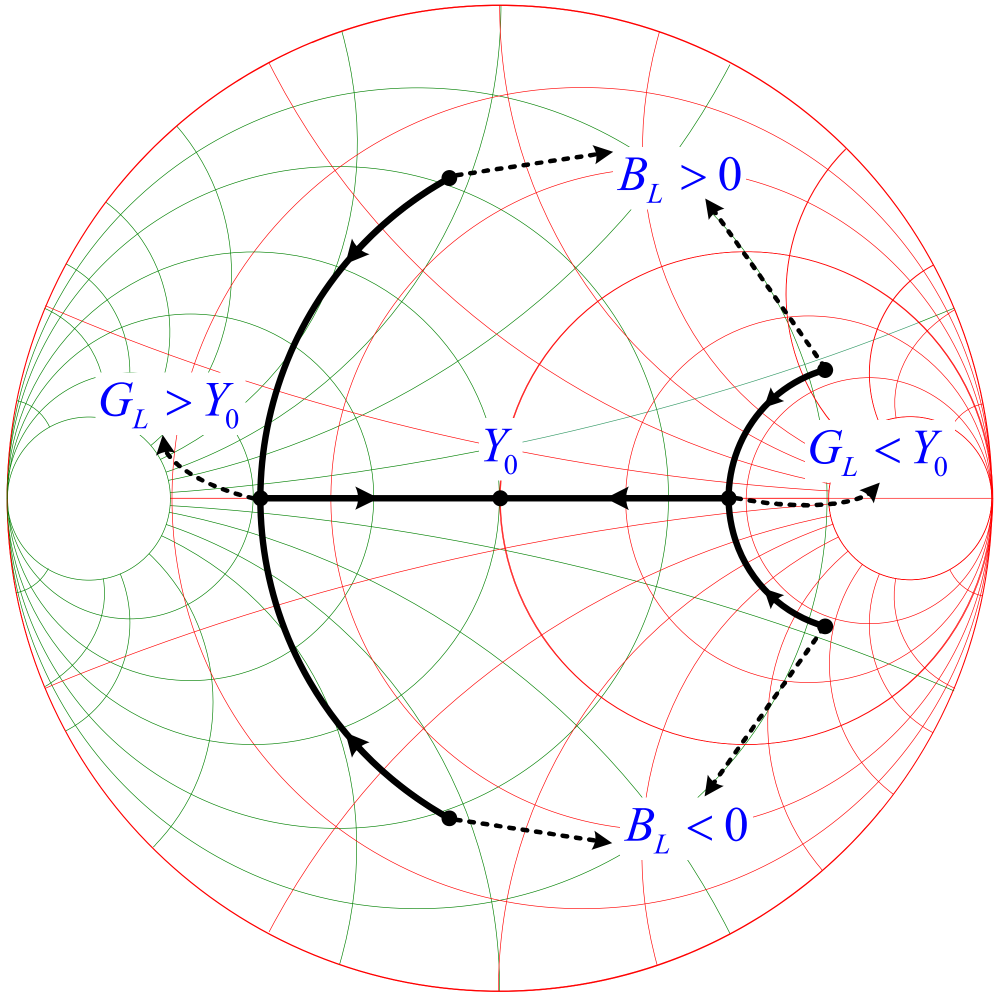
- $\lambda/4$ impedance transformer can be used for matching an arbitrary admittance $Y_L = G_L + jB_L$.
  - Firstly, parallelly connection of load with the short- (or open-circuit element having a susceptance of $B = -B_L$.
  - Transformed only G_L of load admittance can be easily matched by using $\lambda/4$ impedance transformer $\left(Z_1 = \sqrt{Z_0 / G_L}\right)$.
- A long $\lambda/4$ impedance transformer can be minimized with a meander structure.



A single-section quarter-wave matching transformer with arbitrary admittance.



❷ Impedance Matching using Quarter-wave Transformer



3 Quarter-wave Transformer Examples

- **Example 1] Matching real load impedance**

Design a single-section $\lambda/4$ impedance transformer to match a load Z_L = 20 [Ω] to 50 [Ω] line at a frequency of 2 GHz. Determine the percent bandwidth for which the SWR ≤ 1.3.

Solution:

- Matching section impedance: $Z_1 = \sqrt{Z_o Z_L} = \sqrt{50 \times 20} = 31.62[\Omega]$

- Length of matching section: $\lambda_0/4$ @ 2 GHz.

- Reflection coefficient

$$\Gamma_m = \frac{\text{SWR - 1}}{\text{SWR} + 1} = \frac{1.3 - 1}{1.3 + 1} = 0.13$$

- Fractional bandwidth:

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right]$$

$$= 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{0.13}{\sqrt{1-(0.13)^2}}\frac{2\sqrt{(50)(20)}}{|20 - 50|}\right] = 0.3565 \text{ or } 35.65\%$$

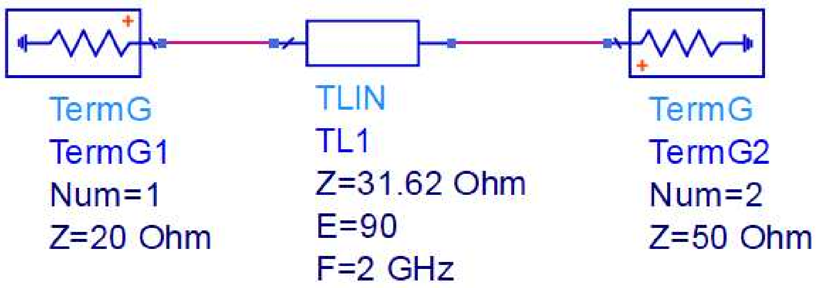
3 Quarter-wave Transformer Examples

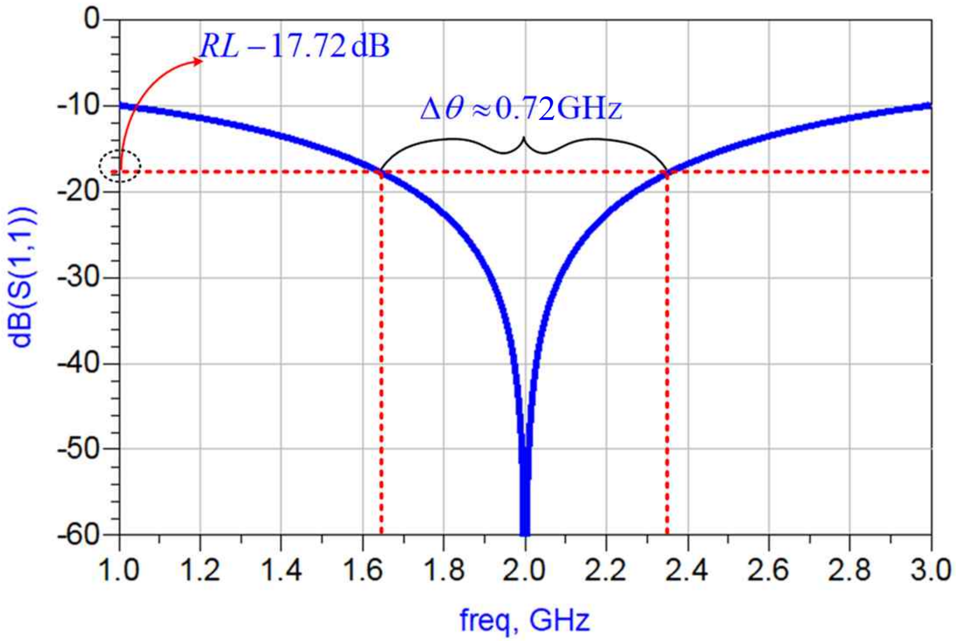
- Microwave circuit simulation results

$$\mathrm{SWR} = 1.3 \Leftrightarrow \Gamma = 0.13 \Leftrightarrow \mathrm{RL} = -20\log(\Gamma) = -17.72\,\mathrm{dB}$$

$$\Rightarrow \text{Bandwidth at RL=-17.72 dB} \approx 0.72\mathrm{GHz} \approx 100\left(\frac{0.72}{2}\right)\% \approx 35.6\%$$

Using formula: $\Delta\theta = 35.65\%$





3 Quarter-wave Transformer Examples

- **Example 2] Matching with complex load impedance**

Design a circuit with a load Z_L=100 + j200 [Ω] is to be matched Z_0 = 50 [Ω] line at 3 GHz, using a $\lambda/4$ impedance transformer and lumped element and plot return loss from 2 GHz to 4 GHz.

Solution

- Admittance:

$$Y_L = \frac{1}{Z_L} = \frac{1}{100 + j200} = 0.002 - j0.004\ [\mathrm{S}] \quad (G_L = 0.002, B_L = -0.004)$$

- Susceptance for lumped element: B = -B_L = 0.004 (to cancel imaginary part)

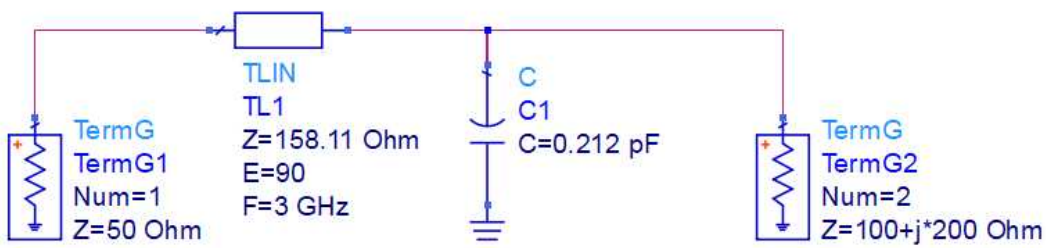
- Capacitance corresponding to B:

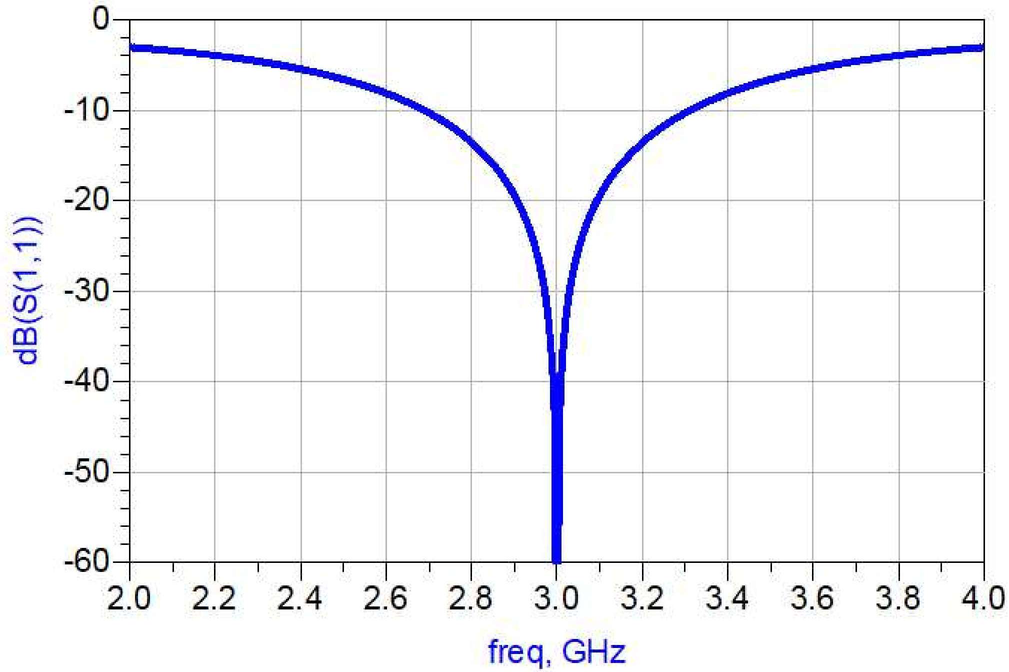
$$B = \omega C \Rightarrow C = \frac{B}{\omega} = \frac{0.004}{2 \times \pi \times 3 \times 10^9} = 0.212\ \mathrm{pF}$$

- Quarter-wave impedance: (matching real part from 500 [Ω]-to-50 [Ω])

$$Z_1 = \sqrt{Z_0 / G_L} = \sqrt{50 / 0.002} = 158.11\ [\Omega]$$

3 Quarter-wave Transformer Matching Examples





4 Review

- Impedance of matching section: $Z_1 = \sqrt{Z_0 R_L}$ or $Z_1^2 = Z_0 R_L$
- Reflection coefficient: $\Gamma = \dfrac{R_L - Z_0}{R_L + Z_0 + j2t\sqrt{Z_0 R_L}}$
- Bandwidth: $\Delta\theta = 2 - \dfrac{4}{\pi}\cos^{-1}\left[\dfrac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\dfrac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right]$