Microwave Engineering 5-5

Chapter 5 Impedance Matching

Prof. Jeong, Yongchae

Learning Objectives

- Understanding what is theory of small reflection
- Learn about impedance transformation using single-section
- Learn about impedance transformation using multi-Section

Learning contents

- Introduction about Theory of Small Reflections
- Single-Section Transformer
- Multi-Section Transformer

1 Introduction about Theory of Small Reflection

- § An important and useful approximation when considering multi-section matching networks is the '**Theory of Small Reflections'**.
- Theory of small reflections provides a simple mathematical form for analyzing the frequency response of many microwave circuits. \rightarrow Total reflection coefficient can be obtained from the partial reflections from several small discontinuities.
- § For applications requiring **more bandwidth** than single quarter-wave section, **multi-section transformers can be used**.
- Applications:
	- *Engineering and Telecommunications*: improving the performances on transmission lines, circuits, and antennas by reducing signal reflections.
	- *Optics*: designing anti-reflective coatings for lenses, glasses, and screens to reduce glare and increase transparency.
	- *Acoustics*: developing materials and structures for soundproofing and acoustic treatment to minimize reflected sound waves and enhance sound quality.

1.1 Single-Section Transformer 2

• Derivation an approximate expression for overall reflection coefficient Γ

1.1 Single-Section Transformer 2

- Expression of total reflection as an infinite sum of partial reflections and transmissions:

Single-Section Transformer
\npression of total reflection as an infinite sum of partial reflections and transmissions:
\n
$$
\Gamma = \Gamma_1 + T_{12}T_{21}\Gamma_3e^{-j2\theta} + T_{12}T_{21}\Gamma_3^2\Gamma_2e^{-j4\theta} + ...
$$
\n
$$
= \Gamma_1 + T_{12}T_{21}\Gamma_3e^{-j2\theta}\sum_{n=0}^{\infty} \Gamma_2^n\Gamma_3^n e^{-j2n\theta}
$$
\n
$$
\lim_{n \to \infty} \frac{\Gamma_1 \Leftrightarrow \Gamma_2 \Leftrightarrow \Gamma_2}{\Gamma_1 \Leftrightarrow \Gamma_2 \Leftrightarrow \Gamma_2}
$$
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$$
\lim_{n \to \infty} \frac{\Gamma_1 \Leftrightarrow \Gamma_2 \Leftrightarrow \Gamma_2}{\Gamma_2 \Leftrightarrow \Gamma_2}
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\frac{\Gamma_1 \Leftrightarrow \Gamma_2}{\Gamma_1 \Leftrightarrow \Gamma_2}
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\frac{\Gamma_1 \Leftrightarrow \Gamma_2}{\Gamma_2 \Leftrightarrow \Gamma_2}
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- Using the geometric series and small reflection condition,

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1.1 Single-Section Transformer 2

- (continue)

Single-Section Transformer
\n
$$
= \frac{\Gamma_1 - \Gamma_1 \Gamma_2 \Gamma_3 e^{-j2\theta} + (1 - \Gamma_1^2) \Gamma_3 e^{-j2\theta}}{1 + \Gamma_1 \Gamma_3 e^{-j2\theta}}
$$
\n
$$
= \frac{\Gamma_1 + \Gamma_2 e^{-j2\theta}}{1 + \Gamma_1 \Gamma_3 e^{-j2\theta}} \qquad (1)
$$
\n
$$
\Gamma \cong \Gamma_1 + \Gamma_3 e^{-j2\theta} \qquad (2)
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\Gamma \cong \Gamma_1 + \Gamma_3 e^{-j2\theta} \qquad (2)
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\Gamma \cong \Gamma_1 + \Gamma_3 e^{-j2\theta} \qquad (2)
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$$
\Gamma + \Gamma_1 \Gamma_3 e^{-2j\theta} \approx 1 \text{ because } \Gamma_1 \Gamma_3 \approx 0
$$
\n
$$
\Gamma
$$

- Intuitive ideas:
	- 1) The total reflection is dominated by the reflection from the initial discontinuity between Z_1 and Z_2 (Γ_1), and the first reflection from the discontinuity between Z_2 and Z_L (Γ_3).
	- 2) The $e^{-j2\theta}$ term accounts for the round-trip phase delay when the incident wave travels up and down the line.

Consider a sequence of N transmission line sections and each section has **equal length** θ , but **dissimilar characteristic impedances**: uence of *N* transmission line sections and each section has equal length θ , but dissing
impedances:
 Z_0 Z_1 Z_2 Z_3 Z_4
 Z_5 Z_6

Partial reflection coefficients for a

multisection matching transformer *x* a sequence of *N* transmission line sections and each section has **equal length** *θ*, but dissimilar erristic impedances:
 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{$ *La* sequence of *N* transmission line sections and each section has equal length θ , but dissimilar erristic impedances:
 Z_0 Z_1 Z_2 Z_3 Z_4 Z_5 Z_6 Z_7 Z_8 Z_9 Z_1 Z_2 Z_3 Z_4 Z_5 Z_6 ⁺ - - - G = G = G = **Example 11 FallS10 FINET**

a sequence of *N* transmission line sections and each section has equal length *θ*, but dissimilar
 $\frac{1}{\sqrt{2}}$
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Partial reflection coefficients for a

multisection matching transformer

§ Derivation an approximate expression for the total reflection coefficient Γ. $\begin{array}{c}\n\begin{array}{c}\n\hline\n\end{array}\n\end{array}$

n an approximate expression for the total reflection
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 $\begin{array}{c}\n\hline\n\end{array}$
 $\begin{array}{c}\n\hline\n\end{array}$
 Partial reflection

m an approximate expression for the total reflection
 $\frac{1}{1} - \frac{Z_0}{1 + Z_0}, \dots, \Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$ (@1<n<N),..., $\Gamma_N = \frac{Z_1}{Z_1}$

dons: Partial reflection coefficients for a

multisection matching transformer

imate expression for the total reflection coefficient Γ .
 $n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$ (@1<n<N),..., $\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}$ Partial reflection coefficients for a

Partial reflection coefficients for a

expression for the total reflection coefficient Γ .
 $\frac{n+1}{n+1} - \frac{Z_n}{Z_n}$ (@1<n<N),..., $\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}$

$$
T_0 = \frac{Z_1 - Z_0}{Z_1 + Z_0}, \dots, \Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} \quad (Q_1 < n < N), \dots, \Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}
$$

Assumptions:

1) All Z_n increase or decrease monotonically across the transformer. 2) *R^L* is real*.*

• If $R_L < Z_0$,

• If $R_L > Z_0$,

- Since R_L is real and lossless transmission lines are assumed, all Γ_n will be real.
	- Due to gradually impedance transition from one section to another, '*Zn+*¹ *Zⁿ* ' will be **small.**
		- \Rightarrow Each marginal reflection coefficient Γ_n will be **real** and have a **small** magnitude.
		- Þ **Theory of Small Reflections**
	- Input reflection coefficient of multi-section impedance transformer

Multi-Section Transformer
\n
$$
\text{ncc } R_L
$$
 is real and lossless transmission lines are assumed, all Γ_n will be real.
\n \Rightarrow Each marginal reflection coefficient Γ_n will be real and have a small magnitude.
\n \Rightarrow **Theory of Small Reflections**
\nput reflection coefficient of multi-section impedance transformer
\n $\frac{b_0}{a_0} = \Gamma(\theta)$
\n $\Gamma_{in}(\theta) \cong \Gamma_0 + \Gamma_1 e^{-j2\theta} + \Gamma_2 e^{-j4\theta} + \dots + \Gamma_N e^{-j2N\theta}$ (3)
\n $= \sum_{n=0}^{N} \Gamma_n e^{-j2N\theta}$
\n $\Gamma_{in}(\theta) \equiv \frac{\sum_{n=0}^{N} \Gamma_n e^{-j2N\theta}}{\sum_{n=0}^{N} \Gamma_n e^{-j2N\theta}} = \frac{\sum_{n=0}^{N} \Gamma_n e^{-j2N\theta}}{\sum$

- To simplify this process, transformer is **symmetrical**,

$$
\Gamma_0 = \Gamma_N, \Gamma_1 = \Gamma_{N-1}, \Gamma_2 = \Gamma_{N-2}, \cdots
$$

 \rightarrow This does not mean that $Z_0 = Z_N$, $Z_1 = Z_{N-1}$, $Z_2 = Z_{N-2}$, \cdots **Multi-Section Transformer**
simplify this process, transformer is **symmetrical**,
 $\Gamma_0 = \Gamma_N$, $\Gamma_1 = \Gamma_{N-1}$, $\Gamma_2 = \Gamma_{N-2}$,
 \rightarrow This does not mean that $Z_0 = Z_N$, $Z_1 = Z_{N-1}$, $Z_2 = Z_{N-2}$, ...
can be re-written as

- (3) can be re-written as:

Multi-Section Transformer
\n
$$
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\rightarrow
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\n3) can be re-written as:
\n
$$
\Gamma_{in}(\theta) \cong \Gamma_0 + \Gamma_1 e^{-j2\theta} + \Gamma_2 e^{-j4\theta} + \cdots + \Gamma_N e^{-j2N\theta}
$$
\n
$$
= (\Gamma_0 + \Gamma_N e^{-j2N\theta}) + (\Gamma_1 e^{-j2\theta} + \Gamma_{N-1} e^{-j2(N-1)\theta}) + (\Gamma_2 e^{-j4\theta} + \Gamma_{N-2} e^{-j2(N-2)\theta}) + \cdots +
$$
\n
$$
= e^{-jN\theta} {\Gamma_0 [\cos N\theta + e^{-jN\theta}] + \Gamma_1 [e^{j(N-2)} + e^{-j(N-2)\theta}] + \cdots}
$$
\n
$$
= 2e^{-jN\theta} {\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2)\theta + \cdots} \leftarrow e^{j\pi} + e^{-j\pi} = 2\cos(x) \qquad \boxed{\Gamma_0 = \Gamma_N, \Gamma_1 = \Gamma_{N-1}, \Gamma_2 = \Gamma_{N-2}, \text{ etc.}}
$$

\nFor odd *N*, the last term is $\Gamma_{(N-1)/2}(\theta^{i\theta} + e^{j\theta})$ as
\n
$$
\Gamma(\theta) = 2e^{-jN\theta} [\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2)\theta + \cdots + \Gamma_N \cos(N-2n)\theta + \cdots + \Gamma_{(N-1)/2} \cos \theta]
$$
\n
$$
= \frac{10}{\pi}
$$

- For odd *N*, the last term is $\Gamma_{(N-1)/2}(e^{j\theta} + e^{-j\theta})$ as $\Gamma(\theta) = 2e^{-jN\theta}[\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2)\theta + ... + \Gamma_n \cos(N-2n)\theta + ... + \Gamma_{(N-1)/2} \cos \theta]$

- For even *N*, the last term is $\Gamma_{N/2}$ as

 $\Gamma(\theta) = 2e^{-jN\theta}[\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2)\theta + ... + \Gamma_n \cos(N-2n)\theta + ... + \frac{1}{2} \Gamma_{N/2}]$ 2 $N/2$

- è *Above result imply that the desired reflection coefficient responds a function of* ^q *by using enough sections* N *and properly choosing* $\varGamma_n.$ **ction Transformer**

ust term is $\Gamma_{N/2}$ as
 α $\cos N\theta + \Gamma_1 \cos(N-2)\theta + ... + \Gamma_n \cos(N-2n)\theta + ... + \frac{1}{2}\Gamma_{N/2}$

imply that the desired reflection coefficient responds a function of θ by using enough

ad properly choosing Γ_n . **Multi-Section Transformer**

even N, the last term is $\Gamma_{N/2}$ as
 $(\theta) = 2e^{-jN\theta} [\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2)\theta + ... + \Gamma_n \cos(N-2n)\theta + ... + \frac{1}{2} \Gamma_{N/2}]$

Above result imply that the desired reflection coefficient responds a function of $\$ **Multi-Section Transformer**
or even N, the last term is Γ_{N2} as
 $\Gamma(\theta) = 2e^{-jN\theta}[\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2)\theta + ... + \Gamma_n \cos(N-2n)\theta + ... + \frac{1}{2}\Gamma_{N/2}]$
A *dove result imply that the desired reflection coefficient responds a function of* $\sum_{n} \cos(N-2n)\theta + \dots + \frac{1}{2} \Gamma_{N/2}$]
 on coefficient responds a function of θ *by using enough*
- § *Questions*
	- How to determine the necessary number of sections *N* ?
	- How do determine the values of all reflection coefficients Γ_n ?
	- *Answer*: Answers will be clear if the design multi-section transformers for a small reflection is designed with two of the most commonly used passband responses:
		- Binomial (maximally flat) response
		- Chebyshev (equal-ripple) response.

4 Example

Ex.] Consider the 2-stage quarter-wave transformer of below figure with $Z_1 = 75 \Omega$, $Z_2 = 125 \Omega$, and $R_L = 200$ $Ω$. Evaluate the worst-case percent error in computing $|Γ|$ from the approximate small reflection theory and the almost exact result. **Example**

Consider the 2-stage quarter-wave transformer of below figure with $Z_1 = 75 \Omega$, $Z_2 = 125 \Omega$,

Evaluate the worst-case percent error in computing $|\Gamma|$ from the approximate small reflection the

sole txact resu **IMPILe**

sider the 2-stage quarter-wave transformer of below figure with $Z_1 = 75 \Omega$, $Z_2 = 125 \Omega$,

ate the worst-case percent error in computing $|\Gamma|$ from the approximate small reflection that

cast result.

on coeffi **IMPle**

sider the 2-stage quarter-wave transformer of below figure with $Z_1 = 75 \Omega$, $Z_2 = 125 \Omega$,

ate the worst-case percent error in computing | Γ | from the approximate small reflection the

cast result.

on coeffic 125 75 200 125 0.25, 0.23 2. 2-stage quarter-wave transformer of below figure with $Z_1 = 75 \Omega$, $Z_2 = 125 \Omega$, and $R_L = 200$

vorst-ease percent error in computing [Γ] from the approximate small reflection theory and the

titudent between transmi *Z Z R Z* **EXECUTE:**
 *Z Z*₂ = 125 Ω, and *R_t* = 200

and the worst-case percent error in computing || | from the approximate small lefelction theory and the
 Z z + *Z*₂ + *Z*₂ = 125 Ω, and *R_t* = 200

(ion coefficien **Example**

1.] Consider the 2-stage quarter-wave transformer of below figure with $Z_1 = 75 \Omega$, $Z_2 = 125 \Omega$, and $R_L = 200$

Evaluate the worst-case percent error in computing [1] from the approximate small reflection theo **Imple**

der the 2-stage quarter-wave transformer of below figure with $Z_1 = 75 \Omega$, $Z_2 = 125 \Omega$, and $R_L = 200$

e the worst-case percent error in computing ||1| from the approximate small reflection theory and the

ct re Γ_1 \longrightarrow Γ_2 $\qquad \qquad$ Γ_3 $\qquad \qquad$
 Z_1 =75 Z_2 =125 R_L =200 Let us with $Z_1 = 75$ Ω, $Z_2 = 125$ Ω, and $R_L = 200$

is approximate small reflection theory and the
 $\frac{\Gamma_1 \leftarrow \sum_{i=1}^{n} z_i}{\sum_{i=1}^{n} z_i}$
 $Z_2 = 125$
 $Z_3 = 125$
 $Z_4 = 200$
 $\beta = 0$

ECLE depend on denominator.

Secu w figure with $Z_1 = 75 \Omega$, $Z_2 = 125 \Omega$, and $R_L = 200$

om the approximate small reflection theory and the
 $\begin{array}{r}\nT_1 \leftarrow T_2 \leftarrow T_2 \qquad T_3 \leftarrow T_4 \\
\hline\nZ_2 = 125\n\end{array}\n\end{array}\n\begin{array}{r}\nZ_2 = 125 \\
Z_3 = 125\n\end{array}\n\begin{array}{r}\nZ_2 = 125 \\
R_$ ge quarter-wave transformer of below figure with $Z_1 = 75 \Omega$, $Z_2 = 125 \Omega$, and $R_1 = 200$

sec percent error in computing $|\Gamma|$ from the approximate small reflection theory and the

between transmission lines
 $\frac{R_1 - Z_$ **ample**

msider the 2-stage quarter-wave transformer of below figure with $Z_1 = 75 \Omega$, $Z_2 = 125 \Omega$, and $R_L = 200$

uste the worst-ease percent error in computing $|\Gamma|$ from the approximate small reflection theory and the **i.e.**

the 2-stage quarter-wave transformer of below figure with $Z_1 = 75 \Omega$, $Z_2 = 125 \Omega$, and $R_L = 200$

worst-ease percent error in computing $|\Gamma|$ from the approximate small reflection theory and the

entity. The set **Example**

Consider the 2-stage quarter-wave transformer of below figure with $Z_1 = 75 \Omega$, $Z_2 = 125 \Omega$, and $R_L = 200$

cost exact result.

cost exact result.

downthese the considerate means in computing [1] from the app **EXECUTE:**

Sider the 2-stage quarter-wave transformer of below figure with $Z_1 = 75 \Omega$, $Z_2 = 125 \Omega$, and $R_L = 200$

ate the worst-case percent error in computing [1] from the approximate small reflection theory and the
 IMPIE

sider the 2-stage quarter-wave transformer of below figure

the worst-case percent error in computing $|\Gamma|$ from the

act result.

no coefficient between transmission lines
 $\frac{1}{1+Z_1} = \frac{125-75}{125+75} = 0.25$, 2-stage quarter-wave transformer of below figure with $Z_1 = 75$

orst-case percent error in computing $|\Gamma|$ from the approximate studies

t.
 $\frac{\Gamma_1 \leftrightarrow \Gamma_2}{25 + 75} = 0.25, \Gamma_3 = \frac{R_L - Z_2}{R_L + Z_2} = \frac{200 - 125}{200 + 125} = 0.23$ msider the 2-stage quarter-wave transformer of below figure

uate the worst-case percent error in computing $|\Gamma|$ from the

exact result.

tion coefficient between transmission lines
 $\frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{125 - 75}{1$ **Example**

Consider the 2-stage quarter-wave transformer of below figure with $Z_1 = 75 \Omega$, $Z_2 = 125 \Omega$, and $R_2 = 200$

Cyalutate the vorst-case percent error in computing |F| from the approximate small reflection theory **IMPILe**

sider the 2-stage quarter-wave transformer of below figure with $Z_1 = 75 \Omega$, $Z_2 = 125 \Omega$, and K

ate the worst-case percent error in computing $|\Gamma|$ from the approximate small reflection theory.

ate tresult.

Solution

- Reflection coefficient between transmission lines

$$
\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{125 - 75}{125 + 75} = 0.25, \ \Gamma_3 = \frac{R_L - Z_2}{R_L + Z_2} = \frac{200 - 125}{200 + 125} = 0.23
$$

- Compare Γ the approximate and exact expressions

The worst error will be occurred in conditions of $\theta = 0^{\circ}$ or $\theta = 180^{\circ}$. at the worst-case percent error in computing $|\Gamma|$ from the

act result.

on coefficient between transmission lines
 $\frac{1}{2} - \frac{Z_1}{Z_1} = \frac{125 - 75}{125 + 75} = 0.25$, $\Gamma_3 = \frac{R_L - Z_2}{R_L + Z_2} = \frac{200 - 125}{200 + 125} = 0.23$

e 1. $\frac{\Gamma_1 \rightarrow \frac{1}{20} + \Gamma_2}{25 + 75}$

cient between transmission lines
 $\frac{25 - 75}{25 + 75} = 0.25$, $\Gamma_3 = \frac{R_L - Z_2}{R_L + Z_2} = \frac{200 - 125}{200 + 125} = 0.23$

opproximate and exact expressions
 $\frac{20}{20} \approx \Gamma_1 + \Gamma_3 e^{-j2\theta}$ \rightarrow coefficient between transmission lines
 $\frac{Z_1}{Z_1} = \frac{125-75}{125+75} = 0.25$, $\Gamma_3 = \frac{R_L - Z_2}{R_L + Z_2} = \frac{200-125}{200+125} = 0.23$

the approximate and exact expressions
 $\frac{\Gamma_3 e^{-j2\theta}}{i\Gamma_3 e^{-j2\theta}} \equiv \Gamma_1 + \Gamma_3 e^{-j2\theta}$ \longrightarrow It.

icient between transmission lines

<u>125-75</u>
 $\frac{125-75}{125+75} = 0.25, \Gamma_3 = \frac{R_L - Z_2}{R_L + Z_2} = \frac{200-125}{200+125} = 0.23$

pproximate and exact expressions
 $\frac{2\theta}{r^{2\theta}} \approx \Gamma_1 + \Gamma_3 e^{-j2\theta}$ → The greatest error will o 2θ $j2\theta$ $j2\theta$ \qquad $j2\theta$ – 1 \pm 1 \pm 3 $e^{-j2\theta}$ $\sim \Gamma + \Gamma$ $e^{-j2\theta}$ $e^{-j2\theta}$ \longrightarrow The greatest error will occur $e^{-j2\theta}$ ¹ ³ \rightarrow 1 IIC greatest en θ θ $\frac{-j2\theta}{-i2\theta} \approx \Gamma_1 + \Gamma_3 e^{-j2\theta} \longrightarrow$ The g $-j2\theta$

- Almost exact Γ - G using small reflection theory

$$
\Gamma \approx \frac{\Gamma_1 + \Gamma_3 e^{-j2\theta}}{1 + \Gamma_1 \Gamma_3 e^{-j2\theta}} = \frac{0.25 + 0.23e^{j0}}{1 + 0.25 \times 0.23e^{j0}} = 0.454
$$
\n
$$
\Gamma \approx \Gamma_1 + \Gamma_3 e^{-j2\theta} = 0.25 + 0.23e^{j0} = 0.48
$$

- Thus, the worst-case percent error is about 0.026 or 5.7% .

5 Review

■ Single-section transformer ■ Multi-section transformer

$$
\Gamma \cong \Gamma_1 + \Gamma_3 e^{-2j\theta}
$$

$$
\Gamma_{\text{in}}(\theta) \cong \Gamma_0 + \Gamma_1 e^{-2j\theta} + \Gamma_2 e^{-4j\theta} + \dots + \Gamma_N e^{-2jN\theta}
$$

