Chapter 7
Power Divider and Directional Coupler

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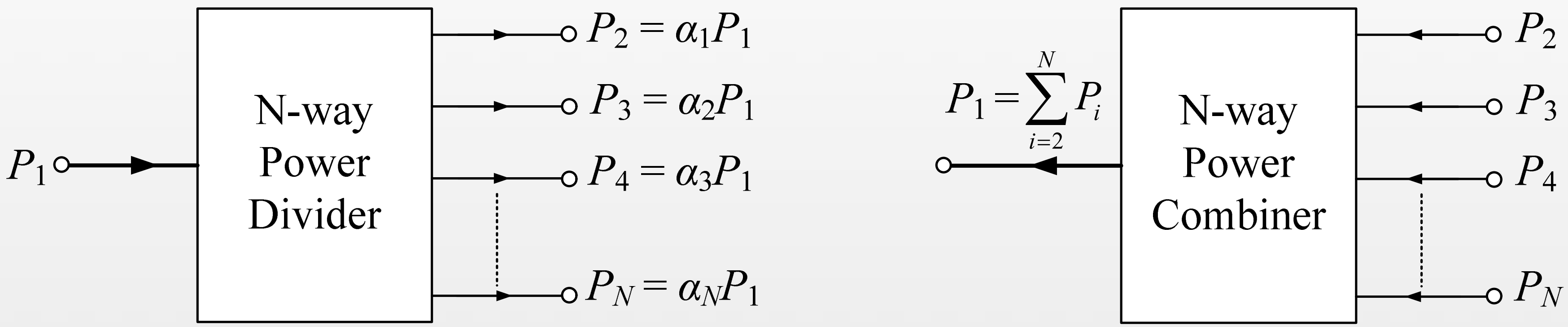


Learning Objectives

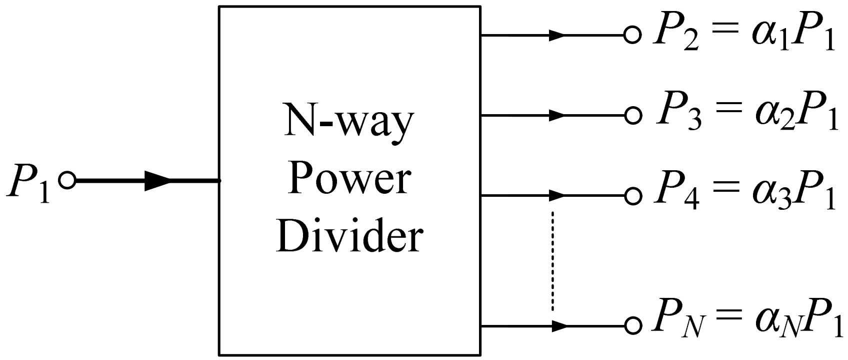
- Know about power divider and combiner
- Understand basic properties of dividers

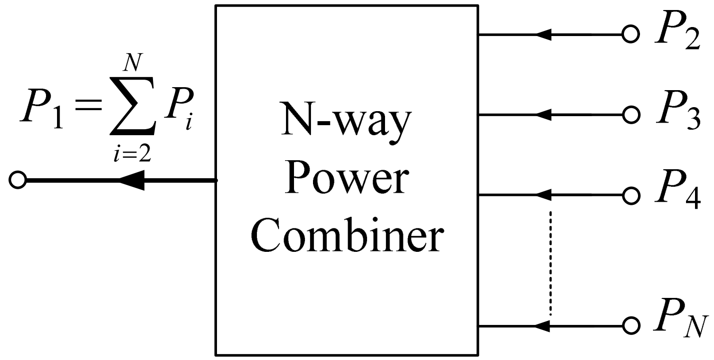
Learning contents

- Introduction of Power Dividers and Combiners
- Basic Properties of Power Dividers

1 Introduction of Power Dividers and Combiners

- **Power dividers** and **combiners** are microwave circuits for power division or combining.
- **Power divider:** circuits for power division of input signal into two or more output paths with lesser power.
- **Power combiner**: circuits for power combining the power of two or more input signals into an output path with greater power.

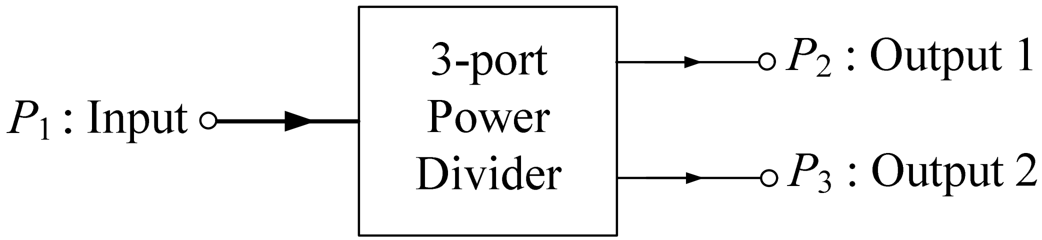




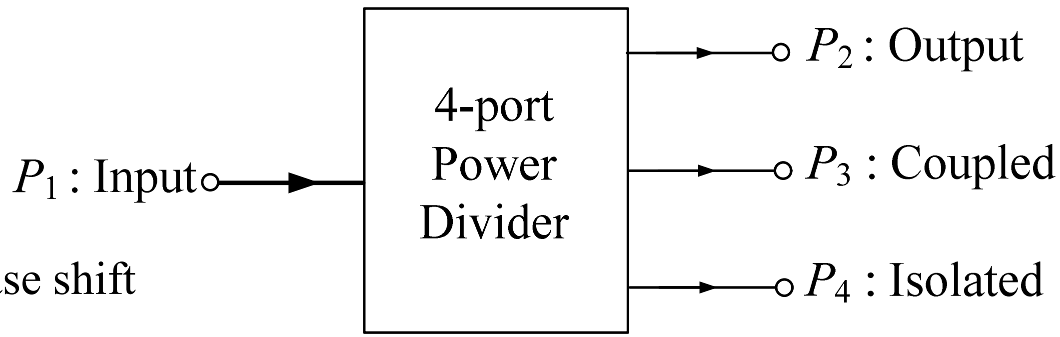
- If $\alpha_2 = \alpha_3 = \alpha_4 = ... = \alpha_N$, the output power at each output ports are equally power divided.
- If $\alpha_2 \neq \alpha_3 \neq \alpha_4 \neq .... \neq \alpha_N$, the output power at each output ports is different (unequal power division).
- Typically, the power division and power combining functions can be used reciprocally.

1 Introduction of Power Dividers and Combiners

- **Typical 3-port power divider**
  - T-junction power divider
  - Resistive power divider
  - Wilkinson power divider
  - Others



- **Typical 4-port power divider**
  - Directional coupler
  - Branch line hybrid: 90° phase shift
  - Ring hybrid (or rat-race hybrid): 180° phase shift
  - Others



2 Basic Properties of Power Dividers

- **3-port Networks (T-Junctions)**

  - Simplest type of power divider

  - 3-port network with two inputs and one output (or with one input and two outputs).

  - Scattering matrix as like an arbitrary three-port network:

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

  - In case of passive and no anisotropic materials, [S] matrix must be *reciprocal* and symmetric ($S_{ij} = S_{ji}$).

- If all ports are *matched* and *lossless* network, then $S_{ii} = 0$.

  - Revised scattering matrix:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

2 Basic Properties of Power Dividers

- By the unitary property of the scattering parameter (⇔ energy conservation: $[S]^* [S]^t = [U]$),

$$\begin{bmatrix} 0 & S_{12}^* & S_{13}^* \\ S_{12}^* & 0 & S_{23}^* \\ S_{13}^* & S_{23}^* & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|S_{12}|^2 + |S_{13}|^2 = 1 \quad (= |S_{21}|^2 + |S_{31}|^2) \qquad S_{13}^* S_{23} = 0$$

$$|S_{12}|^2 + |S_{23}|^2 = 1 \quad (= |S_{12}|^2 + |S_{32}|^2) \qquad S_{23}^* S_{12} = 0$$

$$|S_{13}|^2 + |S_{23}|^2 = 1 \qquad S_{12}^* S_{13} = 0$$

- These equations can't be satisfied simultaneously:

→ From right side equations, at least two of the three parameters (S_{12}, S_{13}, and S_{23}) must be zero.

→ Inconsistent left side equations.

→ ***Implying a three-port network cannot be lossless, reciprocal, and matched at all ports***.

2 Basic Properties of Power Dividers

- If the 3-port network is nonreciprocal, then $\boldsymbol{S_{ij} \neq S_{ji}}$ and the conditions of input matching at all ports and energy conservation can be satisfied. ⇒ **Circulator**

- [S] matrix of a nonreciprocal and matched 3-port network

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

- For lossless network (**[*S*]*[*S*]*ᵗ* =[*U*]**),

$$\begin{bmatrix} 0 & S_{12}^* & S_{13}^* \\ S_{21}^* & 0 & S_{23}^* \\ S_{31}^* & S_{32}^* & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{21} & S_{31} \\ S_{12} & 0 & S_{32} \\ S_{13} & S_{23} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{cases} S_{13}^* S_{23} = 0 \\ S_{21}^* S_{31} = 0 \\ S_{32}^* S_{12} = 0 \end{cases} \quad \begin{matrix} |S_{12}|^2 + |S_{13}|^2 = 1 \\ |S_{21}|^2 + |S_{23}|^2 = 1 \\ |S_{31}|^2 + |S_{32}|^2 = 1 \end{matrix}$$

2 Basic Properties of Power Dividers

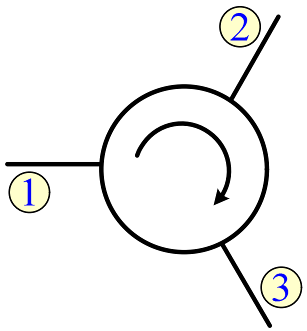
- These equations can be satisfied in one of two ways.

$$S_{12} = S_{23} = S_{31} = 0\,, \qquad |S_{21}| = |S_{32}| = |S_{13}| = 1 \qquad \text{: clockwise}$$

$$S_{21} = S_{23} = S_{31} = 0\,, \qquad |S_{12}| = |S_{23}| = |S_{31}| = 1 \qquad \text{: count-clockwise}$$

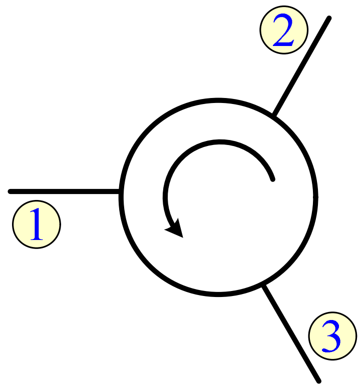
→ $\boldsymbol{S_{ij} \neq S_{ji}}$ for $\boldsymbol{i \neq j}$: **Nonreciprocal**

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



(Clockwise circulator)

$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



(Counter-clockwise circulator)

2 Basic Properties of Power Dividers

- A lossless and reciprocal 3-port network in condition of only matched two ports (ports 1 and 2)

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

- To be lossless, the following unitary conditions (**[*S*]*[*S*]*ᵗ* =[*U*]**) must be satisfied:

$$\begin{bmatrix} 0 & S_{12}^* & S_{13}^* \\ S_{12}^* & 0 & S_{23}^* \\ S_{13}^* & S_{23}^* & S_{33}^* \end{bmatrix}\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{cases} S_{13}^* S_{23} = 0 \\ S_{12}^* S_{13} + S_{23}^* S_{33} = 0 \\ S_{23}^* S_{12} + S_{33}^* S_{13} = 0 \end{cases} \quad \& \quad \begin{cases} |S_{12}|^2 + |S_{13}|^2 = 1 \\ |S_{12}|^2 + |S_{23}|^2 = 1 \\ |S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = 1 \end{cases}$$

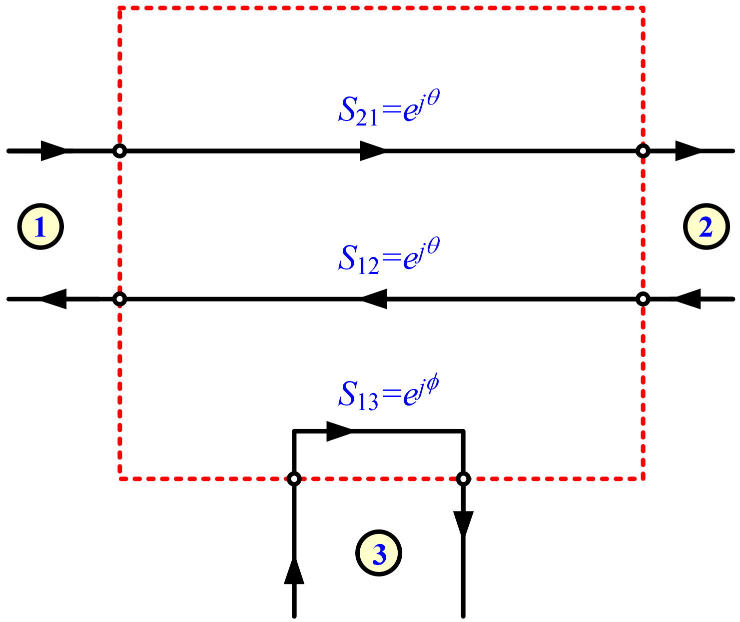
2 Basic Properties of Power Dividers

- Due to right-side two equations, $|S_{13}| = |S_{23}|$
- From left-side equations, $S_{13} = S_{23} = 0$
- Finally, $|S_{12}| = |S_{33}| = 1$

$$\Leftarrow \begin{cases} S_{13}^* S_{23} = 0 \\ S_{12}^* S_{13} + S_{23}^* S_{33} = 0 \\ S_{23}^* S_{12} + S_{33}^* S_{13} = 0 \end{cases} \quad \& \quad \begin{cases} |S_{12}|^2 + |S_{13}|^2 = 1 \\ |S_{12}|^2 + |S_{23}|^2 = 1 \\ |S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = 1 \end{cases}$$

- *S*-parameters of a reciprocal, lossless 3-port network matched at ports 1 and 2:

$$[S] = \begin{bmatrix} 0 & e^{j\theta} & 0 \\ e^{j\theta} & 0 & 0 \\ 0 & 0 & e^{j\phi} \end{bmatrix}$$



2 Basic Properties of Power Dividers

- **Four-Port Networks (Directional Couplers)**

  - [S] matrix of a reciprocal and matched four-port network:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

  - Multiplication of **row 1** (of 1st matrix) and **column 2** (of 2nd matrix), and multiplication of **row 4** (of 1st matrix) and **column 3** (of 2nd matrix) by using unitarity property (or energy conservation, **[*S*]*[*S*]*ᵗ* =[*U*]**):

$$\begin{bmatrix} 0 & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & S_{24}^* \\ S_{13}^* & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & S_{24}^* & S_{34}^* & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{cases} S_{13}^* S_{23} + S_{14}^* S_{24} = 0 & (1) \\ S_{14}^* S_{13} + S_{24}^* S_{23} = 0 & (2) \end{cases}$$

2 Basic Properties of Power Dividers

- Multiplication of **row 1** (of 1st matrix) and **column 3** (of 2nd matrix), and multiplication of **row 4** (of 1st matrix) and **column 2** (of 2nd matrix):

$$\begin{bmatrix} 0 & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & S_{24}^* \\ S_{13}^* & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & S_{24}^* & S_{34}^* & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S_{12}^* S_{23} + S_{14}^* S_{34} = 0 \qquad (3)$$

$$S_{14}^* S_{12} + S_{34}^* S_{23} = 0 \qquad (4)$$

- Obtained expressions by multiplications of matrix elements:

$$S_{13}^* S_{23} + S_{14}^* S_{24} = 0 \qquad (1)$$

$$S_{14}^* S_{13} + S_{24}^* S_{23} = 0 \qquad (2)$$

$$S_{12}^* S_{23} + S_{14}^* S_{34} = 0 \qquad (3)$$

$$S_{14}^* S_{12} + S_{34}^* S_{23} = 0 \qquad (4)$$

- For [Eq. (2) × S_{13}* – Eq. (1) × S_{24}*]:

$$S_{14}^* \left(|S_{13}|^2 - |S_{24}|^2 \right) = 0 \qquad (5)$$

- For [Eq. (3) × S_{12} – Eq. (4) × S_{34}]:

$$S_{23} \left(|S_{12}|^2 - |S_{34}|^2 \right) = 0 \qquad (6)$$

2 Basic Properties of Power Dividers

- One way for satisfying Eqs. (5) and (6) is $\boldsymbol{S_{14} = S_{23} = 0}$.

→ **Isolation of directional coupler**

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

- Self-products of the rows of the unitary [S] matrix ($\mathbf{[S]^*[S]^t = [U]}$):

$$|S_{12}|^2 + |S_{13}|^2 = 1$$
$$|S_{12}|^2 + |S_{24}|^2 = 1$$
$$\rightarrow |S_{13}| = |S_{24}|$$

$$|S_{13}|^2 + |S_{34}|^2 = 1$$
$$|S_{24}|^2 + |S_{34}|^2 = 1$$
$$\rightarrow |S_{13}| = |S_{24}|$$

→ **Couplings (S_{12}, S_{34}) and throughs (S_{13}, S_{24}) of directional coupler**

2 Basic Properties of Power Dividers

- Further simplification can be made by choosing the phase references on three of the four ports.

$$S_{12} = S_{34} = \alpha,\ S_{13} = \beta e^{j\theta},\ \text{and}\ S_{24} = \beta e^{j\phi}$$

where α, β : real number

θ, β : phase constsants to be determined

- Multiplication of **row 2** (of 1st matrix) and **column 3** (of 2nd matrix)

$$\begin{bmatrix} 0 & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & S_{24}^* \\ S_{13}^* & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & S_{24}^* & S_{34}^* & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S_{12}^* S_{13} + S_{24}^* S_{34} = 0$$

$$\rightarrow \alpha\beta e^{j\theta} + \alpha\beta e^{-j\phi} = 0$$

$$\rightarrow \theta + \phi = \pi \pm 2n\pi$$

2 Basic Properties of Power Dividers

- Ignoring integer multiples of 2π,

1) Symmetrical coupler: $\theta = \phi = \pi/2$

$\rightarrow$The phases of the terms having amplitude β are chosen equal.

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix} \quad \leftarrow [S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \alpha & \beta e^{j\theta} & 0 \\ \alpha & 0 & 0 & \beta e^{j\phi} \\ \beta e^{j\theta} & 0 & 0 & \alpha \\ 0 & \beta e^{j\phi} & \alpha & 0 \end{bmatrix}$$

2) Anti-symmetrical coupler: $\theta = 0$, $\phi = \pi$

$\rightarrow$The phases of the terms having amplitude β are chosen to be 180° apart.

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

2 Basic Properties of Power Dividers

- **Basic operations of directional coupler:**

  - Incident port: port 1

  - Coupling port: port 3 (coupling coefficient $|S_{13}|^2 = \beta^2$)

  - Through port: port 2 (delivering coefficient $|S_{12}|^2 = \alpha^2 = 1-\beta^2$)

  - Isolation port: port 4 (no power is delivered)

$$\text{Coupling} = C = 10\log\frac{P_1}{P_3} = -20\log\beta \ \text{[dB]}$$

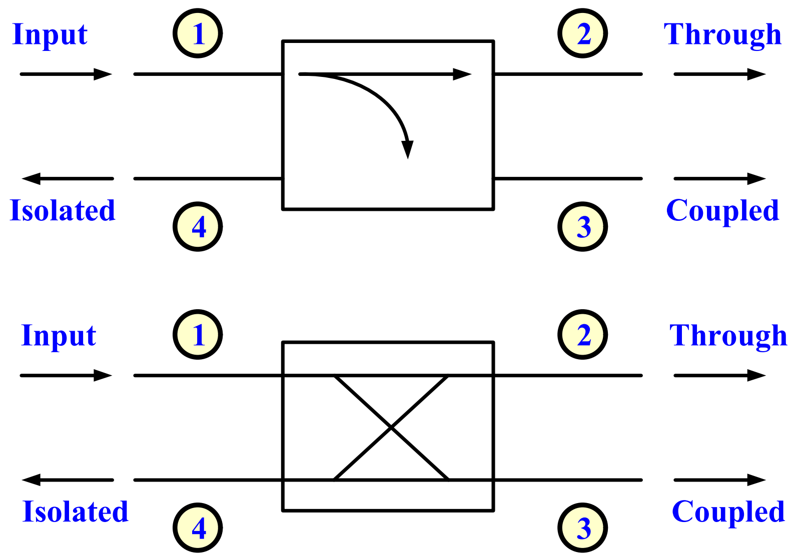
$$\text{Directivity} = D = 10\log\frac{P_3}{P_4} = -20\log\frac{\beta}{|S_{14}|} \ \text{[dB]}$$

$$\text{Isolation} = I = 10\log\frac{P_1}{P_4} = -20\log|S_{14}|$$

$$= 10\log\frac{P_1}{P_3}\frac{P_3}{P_4} = 10\log\frac{P_1}{P_3} + 10\log\frac{P_3}{P_4}$$

$$= C + D \ \text{[dB]}$$

- Commonly used directional coupler symbols



2 Basic Properties of Power Dividers

- **Hybrid couplers: special cases of directional coupler**

1) Type 1: quadrature hybrid

90° phase shift between ports 2 and 3 ($\theta - \phi = \pi/2$).

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

2) Type 2: magic-T hybrid or rat-race hybrid

180° phase difference between ports 2 and 3 when fed at port 4.

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

3 Review

- Power divider and combiner
- Properties of power divider