Chapter 7
Power Divider and Directional Coupler

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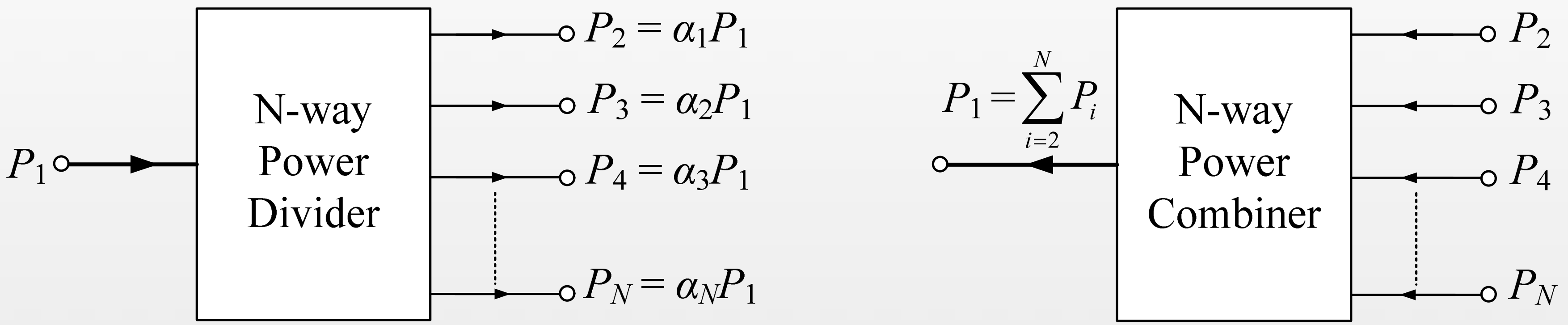


Learning Objectives

- Know about Wilkinson power divider
- Analyze the Wilkinson power divider

Learning contents

- Wilkinson power divider
- Analysis of Wilkinson power divider

1 Wilkinson Power Divider

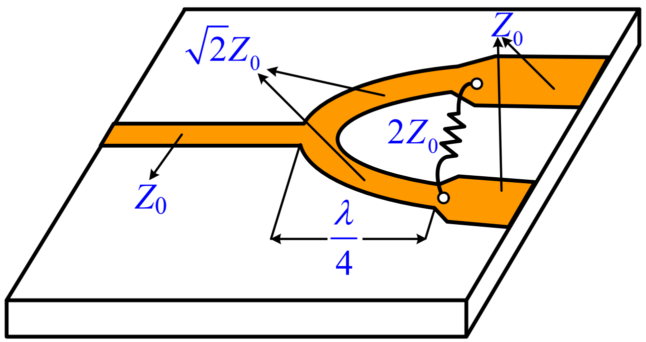
- What is Wilkinson power divider?

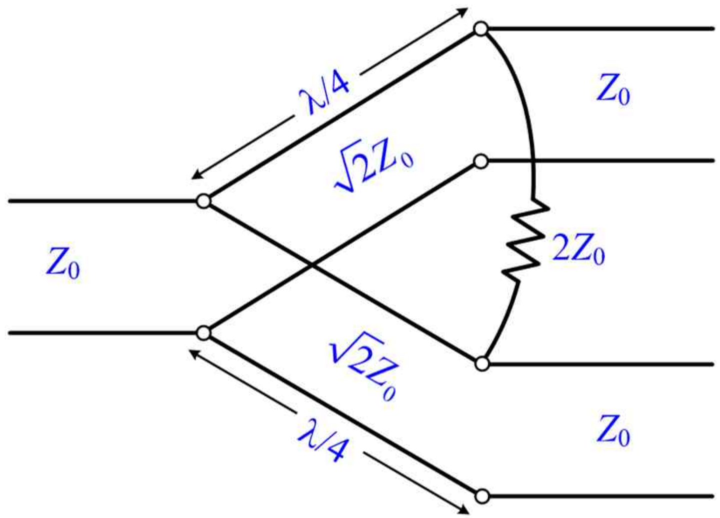
  - Named after its inventor, Ernest J. Wilkinson

  - In-phase power splitter widely used in radio frequency (RF) and microwave circuits and applications.

  - Splitting an input signal into two equal-phased and equal/unequal-amplitude output signals while maintaining impedance matching and high isolation between output ports.

  - Realizable with several kinds of transmission line





1 Wilkinson Power Divider

- Important properties of Wilkinson power divider

  - *High isolation*: providing excellent isolation between output ports than other kinds of power divider

  - *Matched output ports*: both matched output ports to port impedance in case of equal power division

  - *Low insertion loss*: very little signal power loss in power splitting

  - *Phase coherence*: in-phase output port transmitted signals

  - *Broadband operation*: versatile for various applications due to broadband characteristics.

  - *High power handling*: suitable for high-power RF applications.

1 Wilkinson Power Divider

- Input signal splitting

  - The input signal at port 1 is split up equally into two output signals by the two transmission lines.

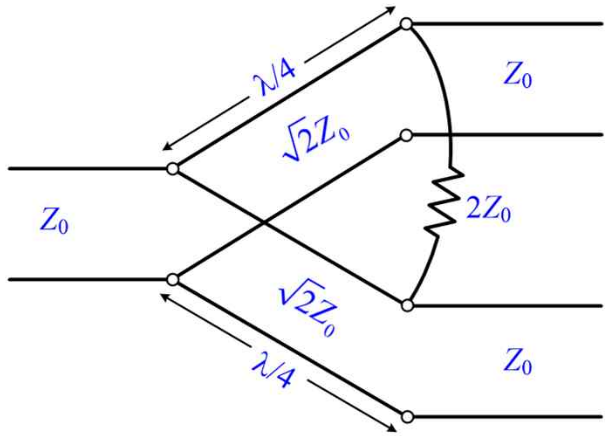
  - These transmission lines are typically quarter-wavelength ($\lambda/4$) at the operating center frequency.

- Resistor for isolation

  - A resistor (referred to as an isolation resistor) is connected between the two output ports (ports 2 and 3).

  - This resistor provides the high isolation between the output ports. If signals with equal amplitude and out-of-phase enter both output ports, they will be cancelled out each other across the resistor and prevent signal transfer between them.

  - Power handling capability of power divider depends on power capability of isolation resistor.

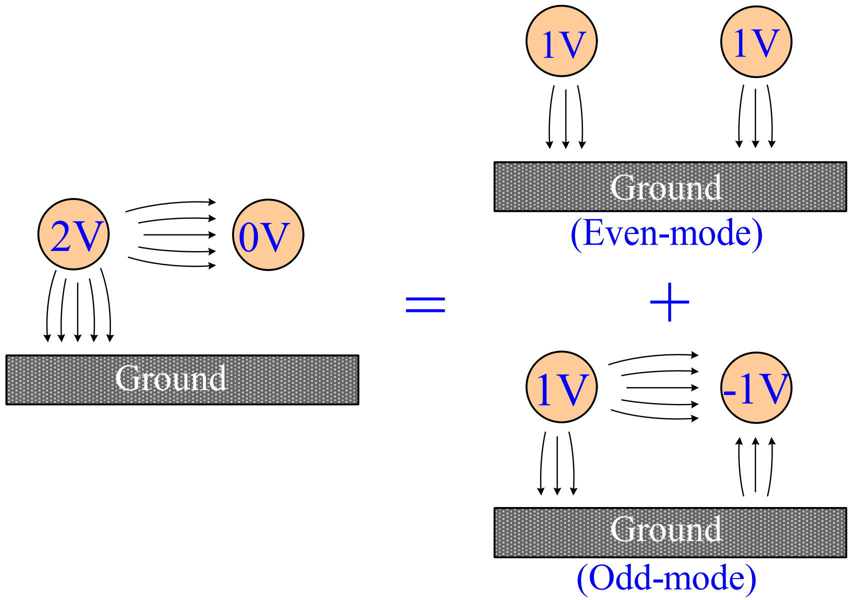


2 Analysis of Wilkinson Power Divider

- **Equal power division Wilkinson power divider**

  - Even though an arbitrary power division is possible, consider the **equal-split** (3 dB) for convenient explanation.

  - **"Even/odd-mode analysis "** technique is useful to analyze Wilkinson power divider as well as other networks to be considered in later sections.



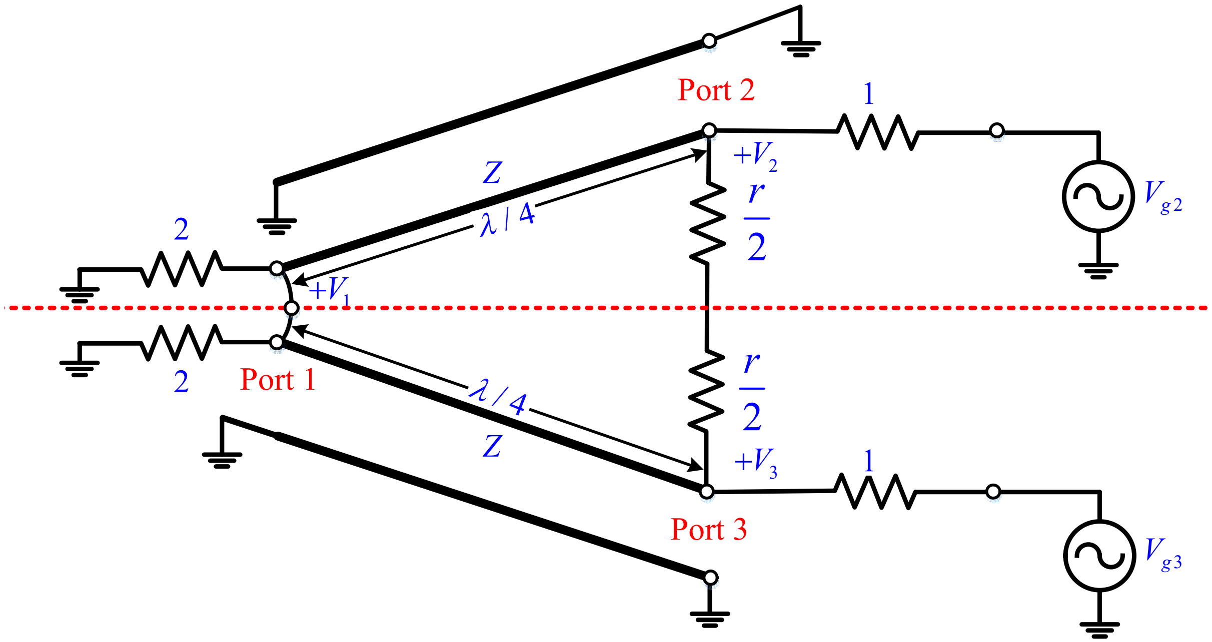
2 Analysis of Wilkinson Power Divider

- **Even-odd mode analysis:**

  - Normalize all impedances to the characteristic impedance Z_0.

  - Symmetric across the mid-plane

  - Wilkinson power divider can be normalized and expressed symmetrically.



2 Analysis of Wilkinson Power Divider

- **Even mode analysis**

  - $V_{g2} = V_{g3} = 2\text{V}$

  - $V_2^e = V_3^e$

    $\rightarrow$ No current flow through the ***r*/2** resistor

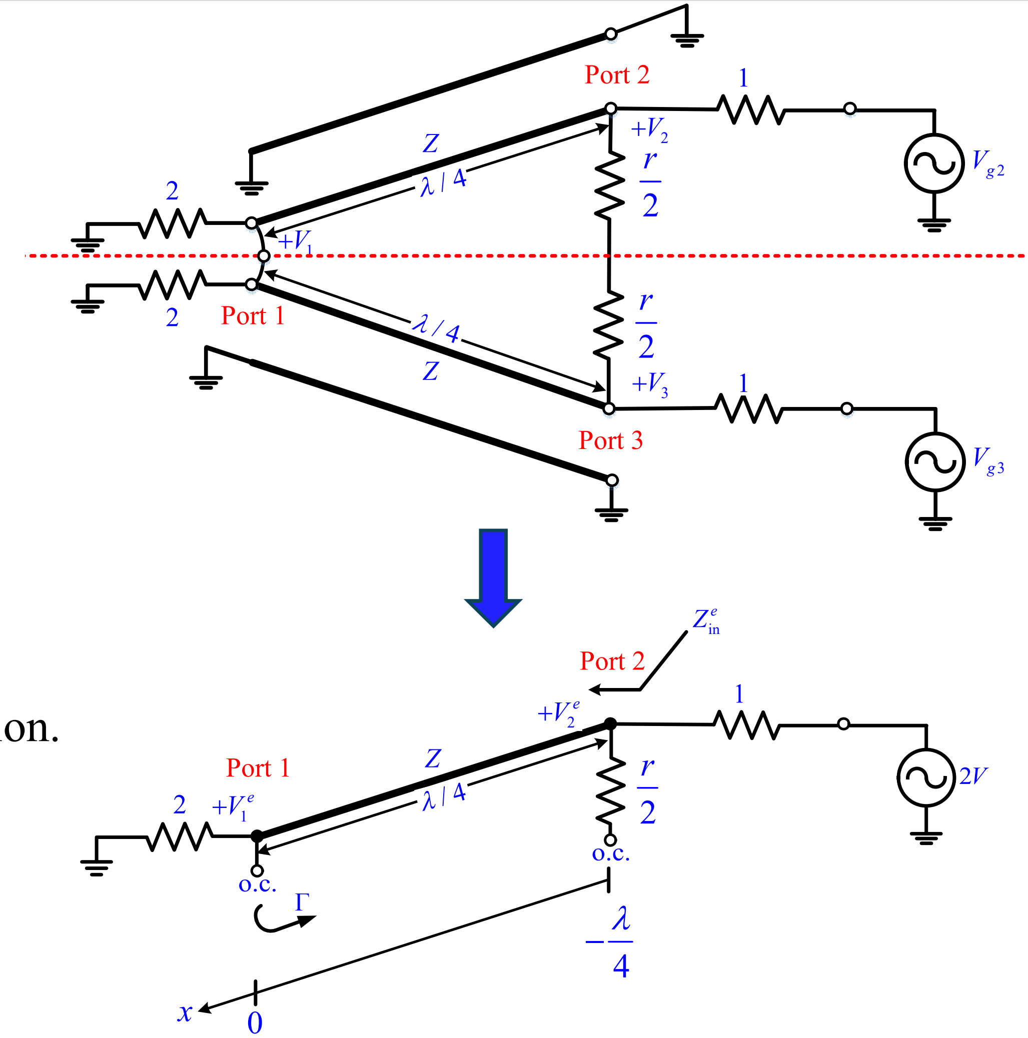
    $\rightarrow$ Bisect the equivalent network with open circuits

  - Impedance looking into port 2:

$$Z_{\text{in}}^e = \frac{Z^2}{2} \qquad \leftarrow \frac{\lambda}{4} \text{ impedance transformer}$$

  - If $Z = \sqrt{2}$, port 2 will be matched for even mode excitation.

$$\rightarrow \quad V_2^e = \text{V} \; (\because \; Z_{\text{in}}^e = 1 \text{ and } V_{g2} = 2\text{V})$$



2 Analysis of Wilkinson Power Divider

- If set $x = 0$ at port 1 and $x = -\lambda/4$ at port 2, the voltage on the transmission line section can be written as

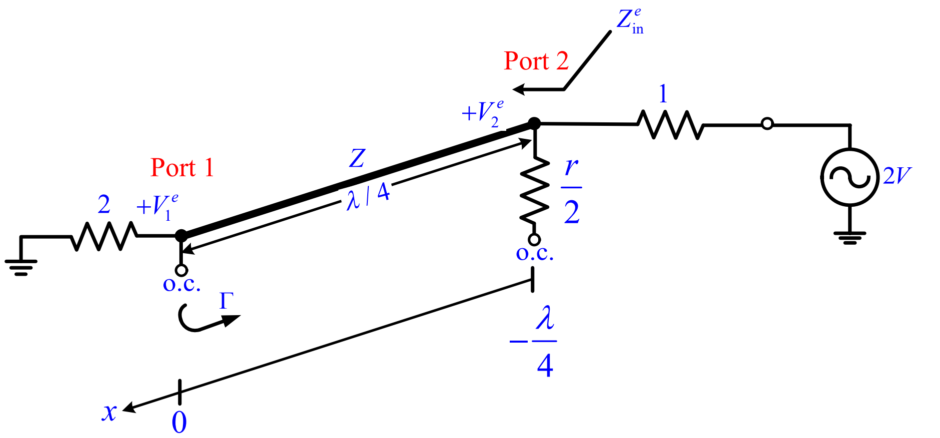
$$V(x) = V^+(e^{-j\beta x} + \Gamma e^{j\beta x})$$

- Then,

$$V_2^e = V(-\lambda/4) = V^+(e^{j\frac{2\pi}{\lambda}\frac{\lambda}{4}} + \Gamma e^{-j\frac{2\pi}{\lambda}\frac{\lambda}{4}}) = jV^+(1-\Gamma) = \mathrm{V}$$

$$\Rightarrow V^+ = j\frac{\mathrm{V}}{\Gamma - 1}$$

$$V_1^e = V(0) = V^+(1+\Gamma) = j\mathrm{V}\frac{\Gamma+1}{\Gamma-1}$$



- Reflection coefficient (Γ) seen at port 1, looking toward termination resistor of normalized value 2:

$$\Gamma = \frac{2-\sqrt{2}}{2+\sqrt{2}}$$

$$V_1^e = j\mathrm{V}\frac{\frac{2-\sqrt{2}}{2+\sqrt{2}}+1}{\frac{2-\sqrt{2}}{2+\sqrt{2}}-1} = j\mathrm{V}\frac{4}{-2\sqrt{2}} = -j\sqrt{2}\mathrm{V}$$

2 Analysis of Wilkinson Power Divider

- **Odd mode analysis:**

  - $V_{g2} = -V_{g3} = 2\text{V}$

  - $V_2{}^o = -V_3{}^o$

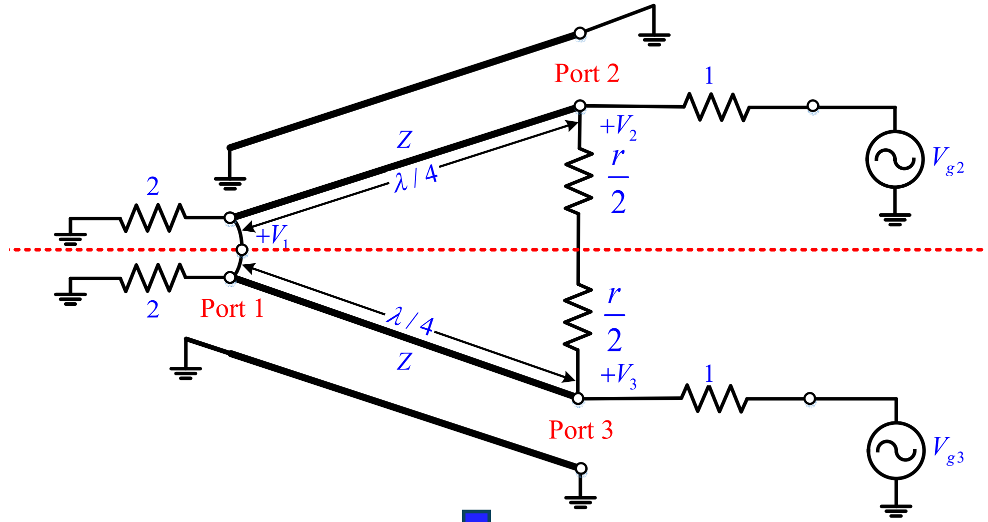
  - Bisect the network with short circuit.

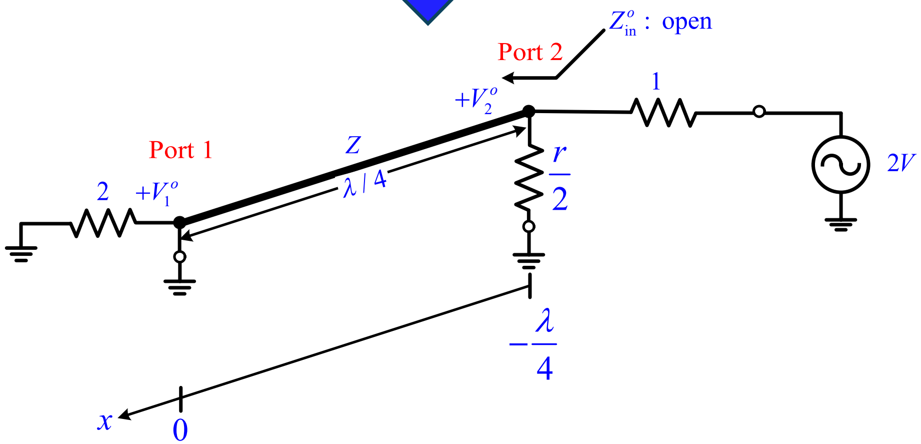
  - Impedance seen from port 2 to port 1 is like an open circuit.

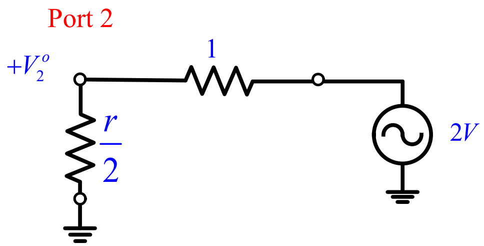
  - For port 2 matching in condition of odd mode excitation,

$$r = 2 \quad \rightarrow V_2^o = \text{V}$$

$$V_1^o = 0$$



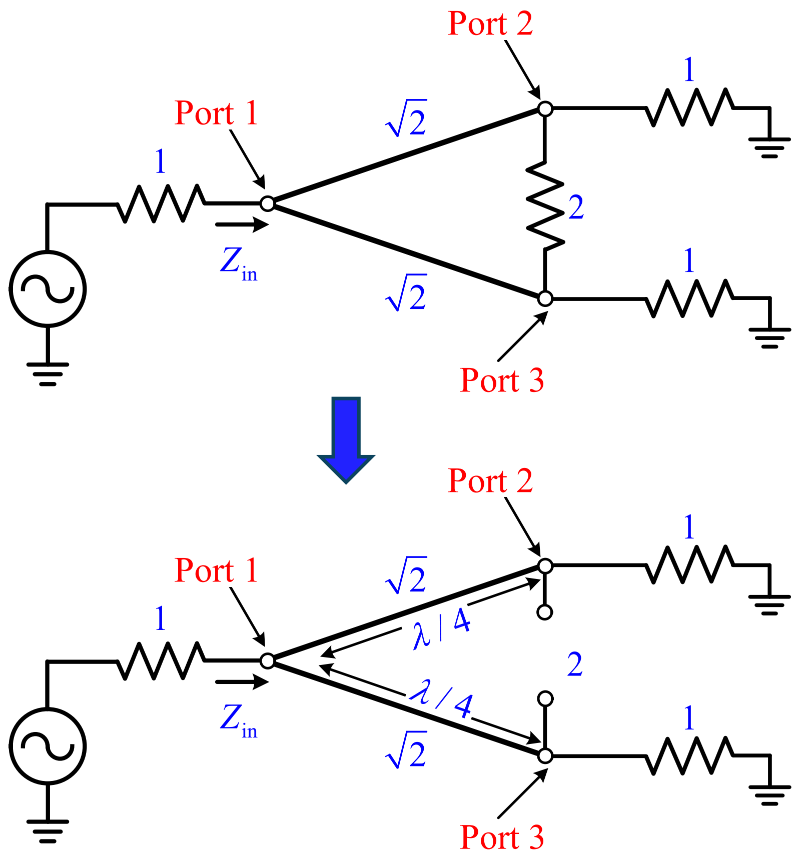




2 Analysis of Wilkinson Power Divider

- Input impedance at port 1 of the Wilkinson divider in case ports 2 and 3 are terminated in matched loads.

$$Z_{in} = \left\{ \frac{(\sqrt{2})^2}{1} \right\} // \left\{ \frac{(\sqrt{2})^2}{1} \right\} = \frac{1}{2}(\sqrt{2})^2 = 1$$



- *S*-parameters characteristics:

$S_{11} = 0$ ($Z_{in} = 1$ at port 1)

$S_{22} = S_{33} = 0$ (Ports 2 and 3 are matched for even and odd modes)

$$S_{12} = S_{21} = \frac{V_1^e + V_1^o}{V_2^e + V_2^o} = \frac{-jV\sqrt{2} + 0}{V+V} = \frac{-j}{\sqrt{2}}$$

: phase delayed & equal power division

$S_{13} = S_{31} = \frac{-j}{\sqrt{2}}$ (Symmetry between ports 2 and 3)

$S_{23} = S_{32} = 0$ (Due to short or open at bisection)

2 Analysis of Wilkinson Power Divider

- **Example:** Design an equal-split Wilkinson power divider for a 50 Ω system impedance at frequency $f_0 = 2$ GHz, and plot the return loss (S_{11}), insertion loss ($S_{21} = S_{31}$), and isolation ($S_{23} = S_{32}$) according to frequency range from 0 GHz to 4 GHz.

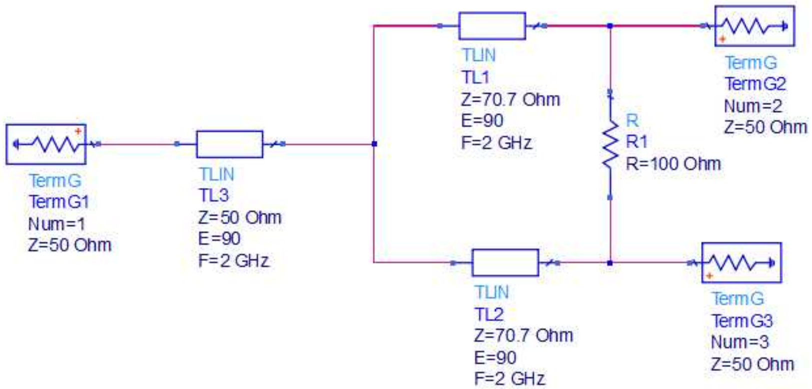
Solution:

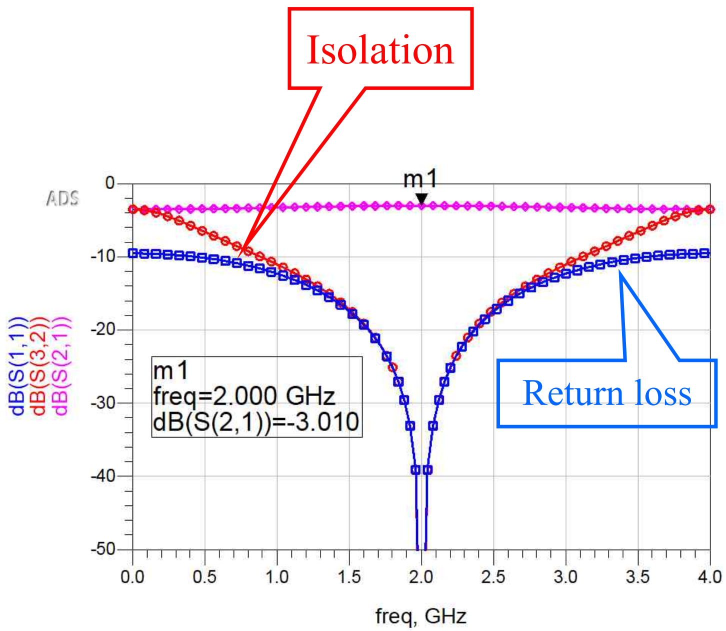
- Characteristic impedance of $\lambda/4$ transmission lines:

$$Z = \sqrt{2}Z_0 = 70.7[\Omega]$$

- Isolation resistance:

$$R = 2Z_0 = 100$$





2 Analysis of Wilkinson Power Divider

- **Unequal power division Wilkinson power divider**

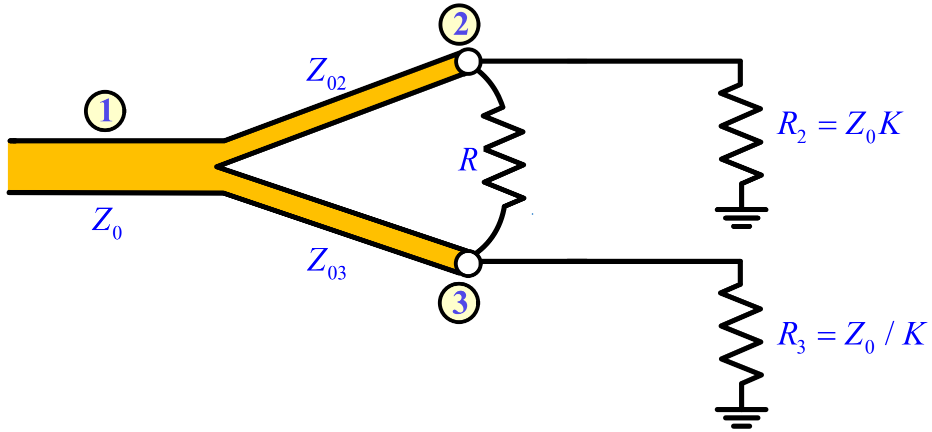
  - Unequal power split

  - Power division ratio between ports 2 and 3: $K^2 = P_3 / P_2$,

$$Z_{03} = Z_0 \sqrt{\frac{1+K^2}{K^3}}$$

$$Z_{02} = K^2 Z_{03} = Z_0 \sqrt{K(1+K^2)}$$

$$R = Z_0 \left(K + \frac{1}{K} \right)$$



  - The termination impedances of $R_2 = Z_0 K$ and $R_3 = Z_0 / K$ at ports 2 and 3 must be matched with Z_0 for proper circuit operation.

2 Analysis of Wilkinson Power Divider

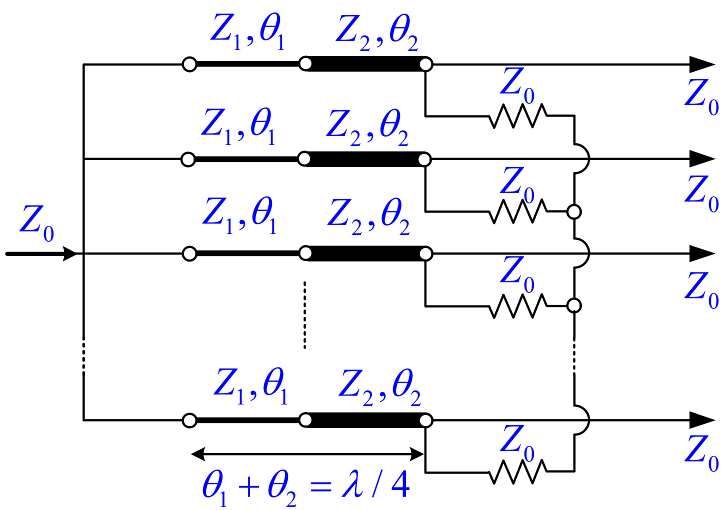
- ***N*-Way Wilkinson Divider:**

  - The Wilkinson divider can also be generalized to an N-way divider or combiner.

  - This circuit can be matched at all ports with isolation between all ports.

  - A disadvantage, however, is the fact that the divider requires crossovers for the resistors for $N \geq 3$, which makes fabrication difficult in planar form.

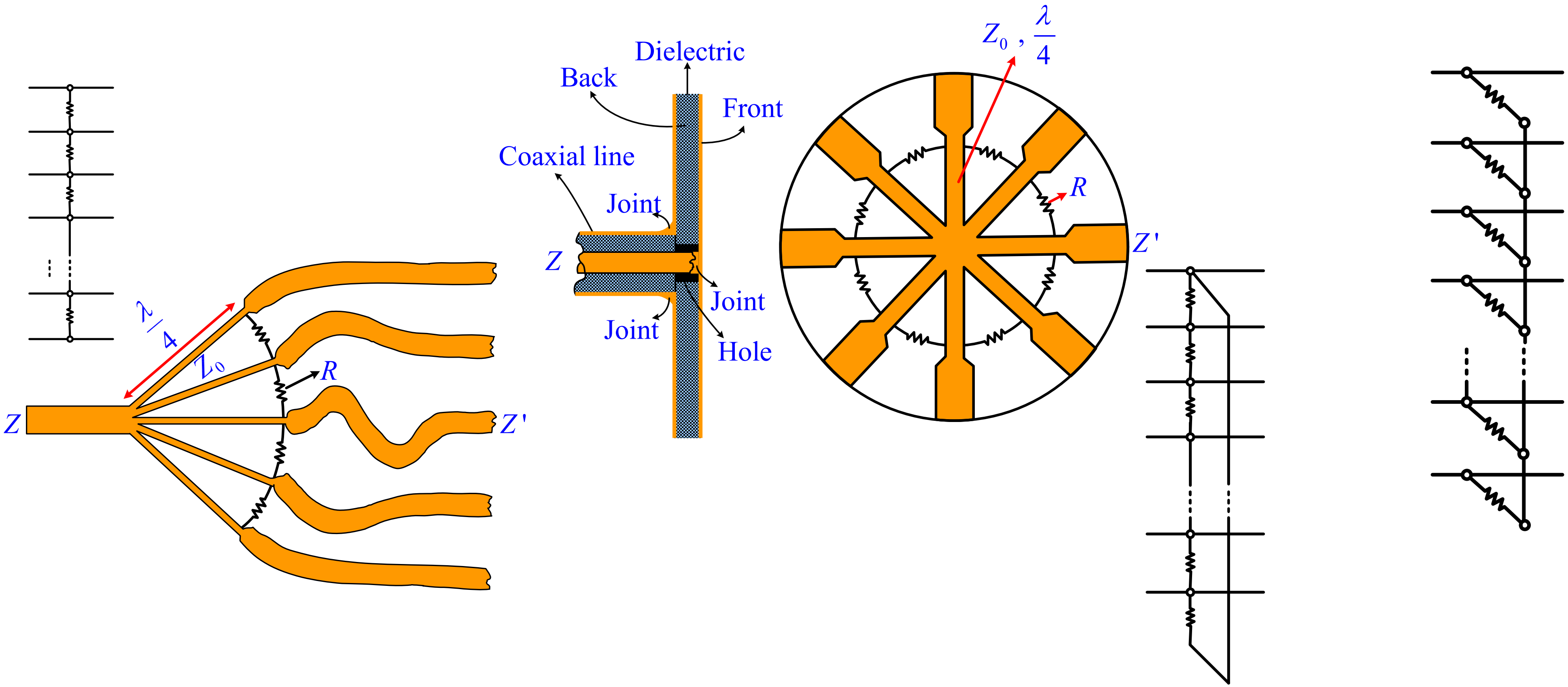
  - The Wilkinson divider can also be made with stepped multiple sections for increasing bandwidth.



2 Analysis of Wilkinson Power Divider

- Realizations of N-way Wilkinson divider:

  - n-way hybrid type
  - Fork type
  - Radial type



3 Review

- Wilkinson power divider
- Even- and odd-mode analysis
- Equal and unequal power division ratios