Chapter 7
Power Dividers and Couplers

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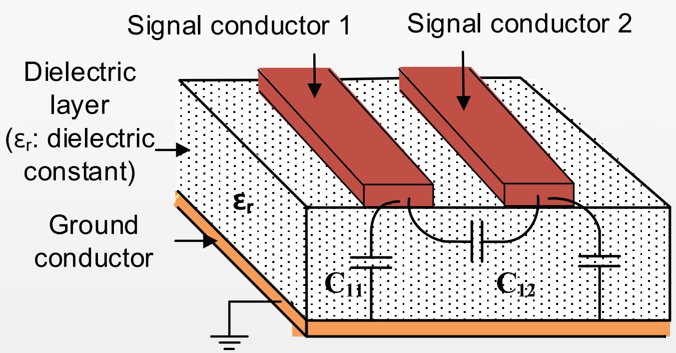
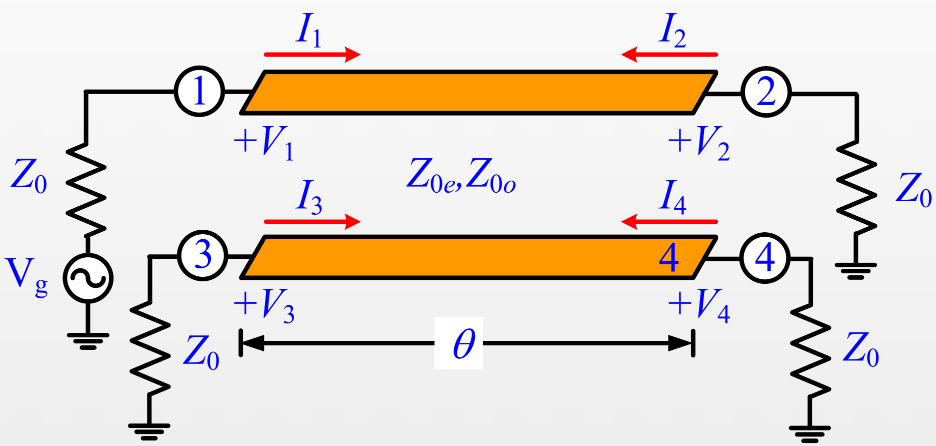


Microstrip coupled line



Learning Objectives

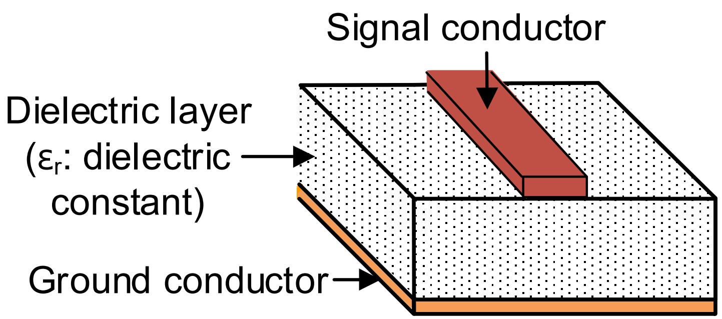
- Understanding what is coupled transmission line.
- Learn about coupled line theory
- Learn what are different configurations of coupled transmission lines.

Learning contents

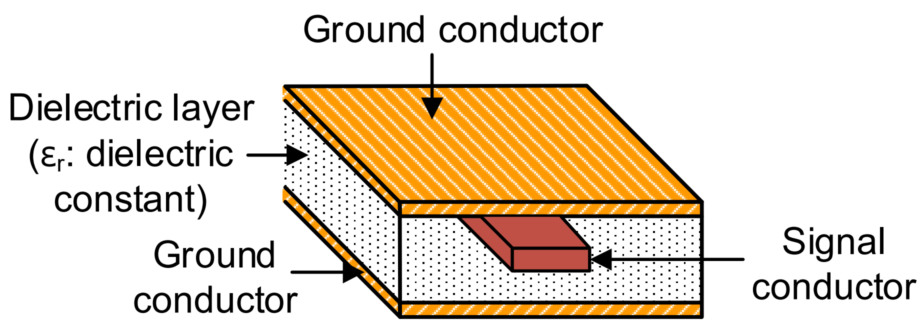
- Single and Coupled Transmission Lines
- Coupled line theory
- Different Configurations of Coupled Transmission Lines

1 Single and Coupled Transmission Lines

- Single transmission lines
  - Consists of a pair of conductor (single signal conductor and ground conductor) separated by dielectric layer



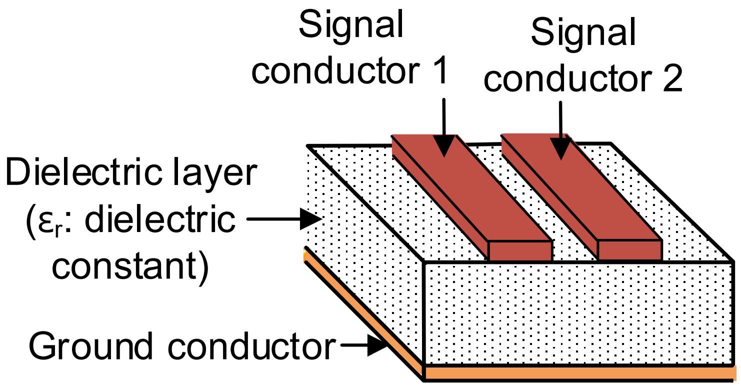
Microstrip line



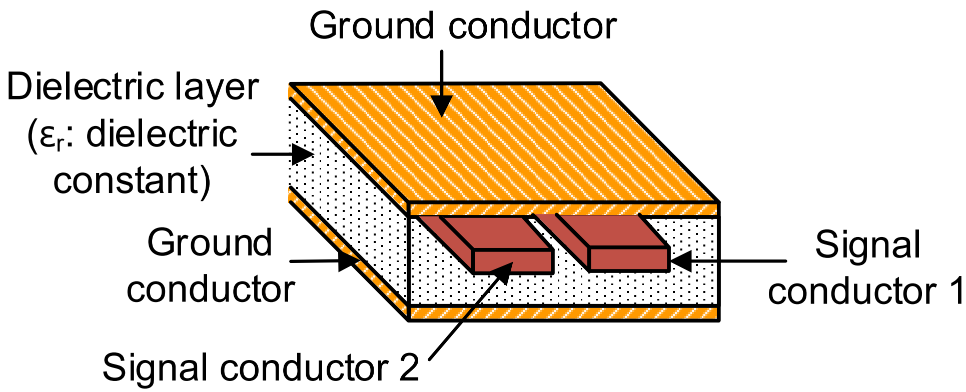
Strip line

1 Single and Coupled Transmission Lines

- Coupled transmission lines
  - Two or more signal lines are placed closely together so that they interact with each other.
  - Power can be coupled between the lines due to the interaction of the electromagnetic fields of each line.
  - TEM mode operation: strip-coupled line ← homogeneous structure
  - Quasi-TEM mode operation: microstrip coupled lines ← inhomogeneous structure



Microstrip coupled line



Strip coupled line

2 Coupled Line Theory

- Introduction of coupled line theory

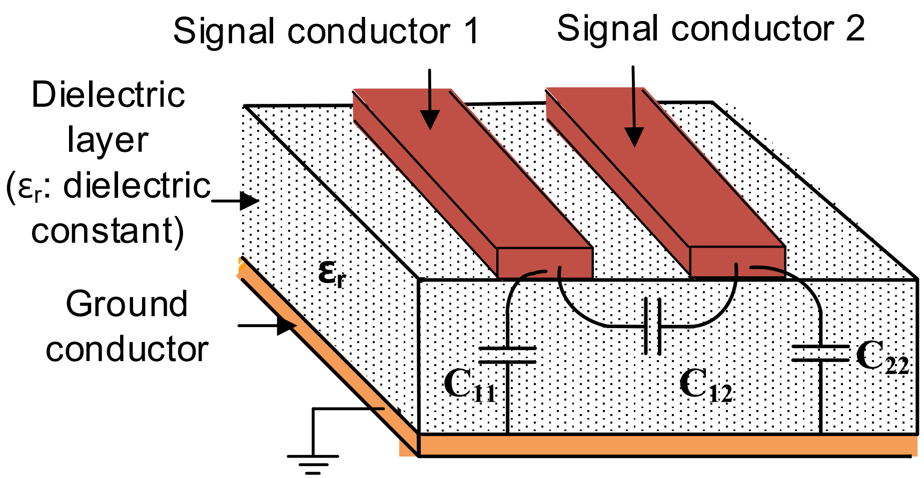
  - TEM propagation: The electrical characteristics of the coupled lines can be completely determined from the effective capacitances between the lines and the velocity of propagation on the line.

  - C_{12}: capacitance between two strip conductors in absence of ground conductor

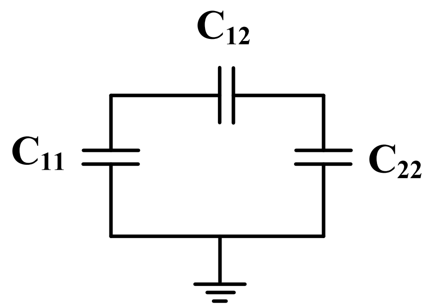
  - C_{11} and C_{22}: capacitances between one strip conductor and ground in absence of another strip conductor

  - If two transmission lines are identical,

    $C_{11} = C_{22}$

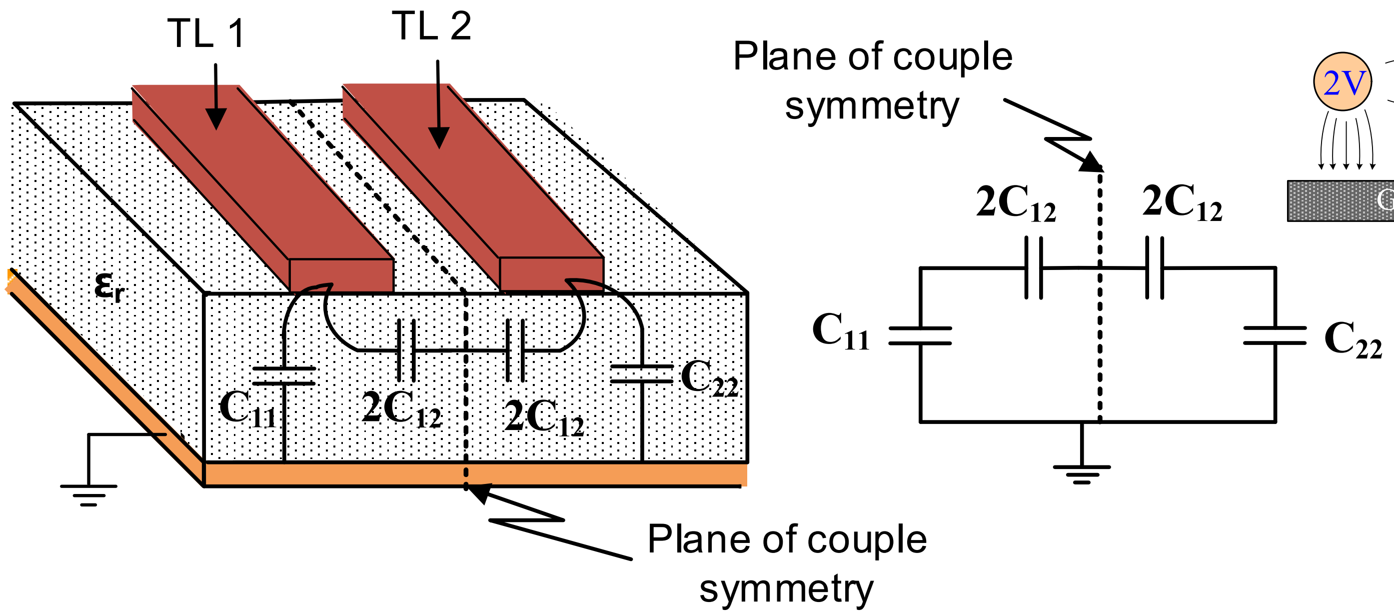


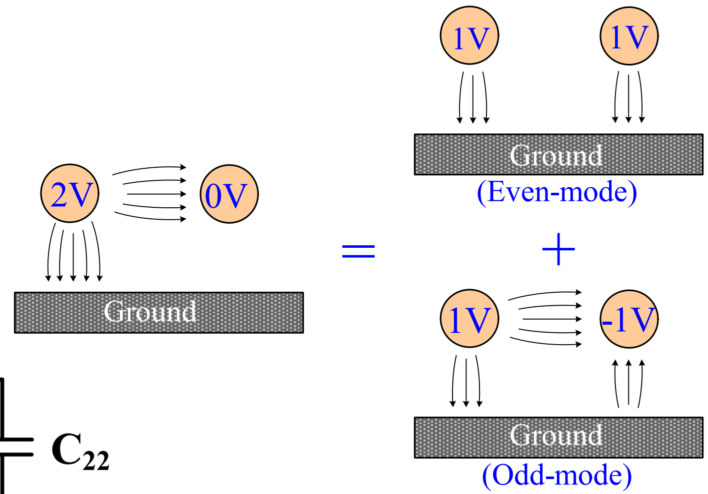
Microstrip coupled line



2 Coupled Line Theory

- Coupled line theory
  - If two transmission lines are identical, symmetric plane of circuit exists.
  - Even and odd-mode analysis can be used to find even- and odd-mode characteristic impedances of coupled line.

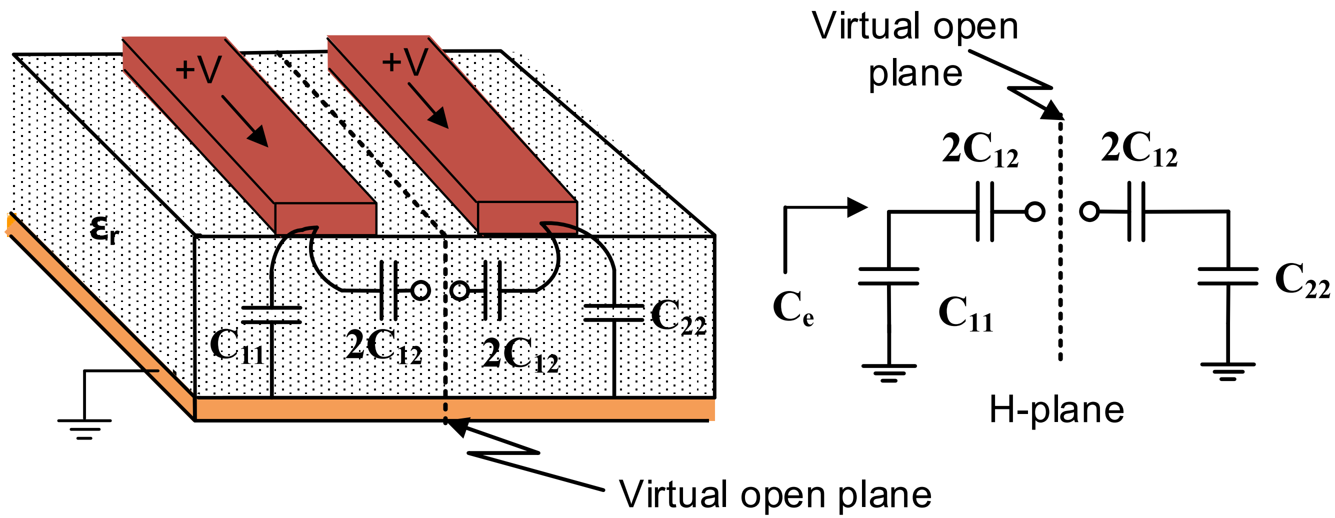




2 Coupled Line Theory

- Even-mode characteristic impedance
  - If two incident waves along two transmission lines are equal (*i.e.* equal in magnitude and phase), then virtual open plane is created at symmetric center plane of circuit.
  - Under even-mode, $2C_{12}$ capacitors are "***disconnected***".
  - Even-mode capacitance per unit length of each transmission line is given as

$$C_e = C_{11} = C_{22} \qquad (1)$$



  - Characteristic impedance for even mode

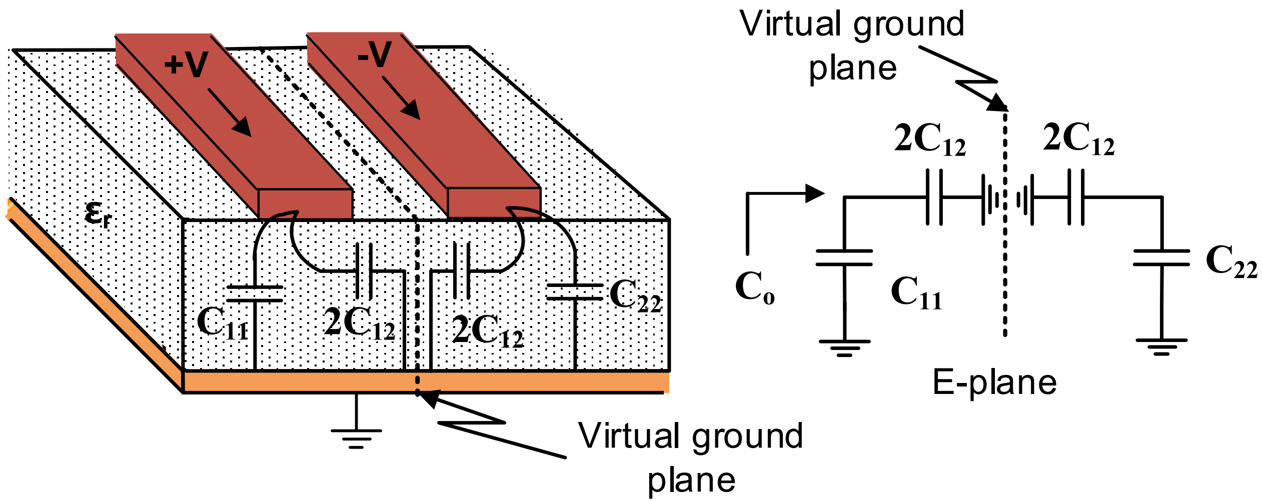
$$Z_{0e} = \sqrt{\frac{L}{C_e}} = \frac{\sqrt{LC_e}}{C_e} = \frac{1}{vC_e} \qquad (2) \leftarrow v = \frac{1}{\sqrt{LC}}$$

v: velocity of propagation on transmission line

2 Coupled Line Theory

- Odd-mode characteristic impedance
  - If two incident waves along two transmission lines are opposite (*i.e.* equal in magnitude, but out-of-phased (180°)), then virtual ground plane is created at symmetric center plane of circuit.
  - Under odd-mode, a voltage null exists between the two strip conductors
  - Odd-mode capacitance per unit length of each transmission line is given as

$$C_o = C_{11} + 2C_{12} = C_{22} + 2C_{12} \qquad (3)$$

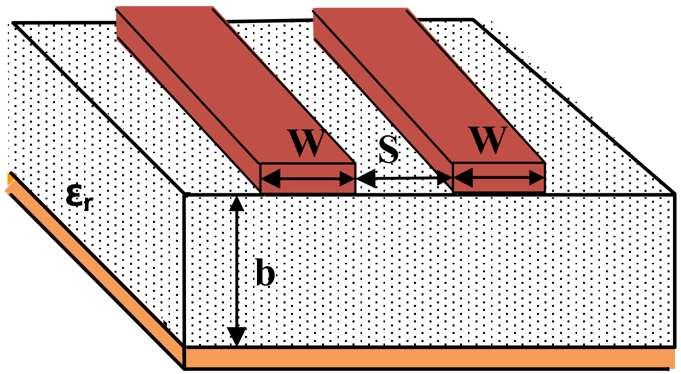


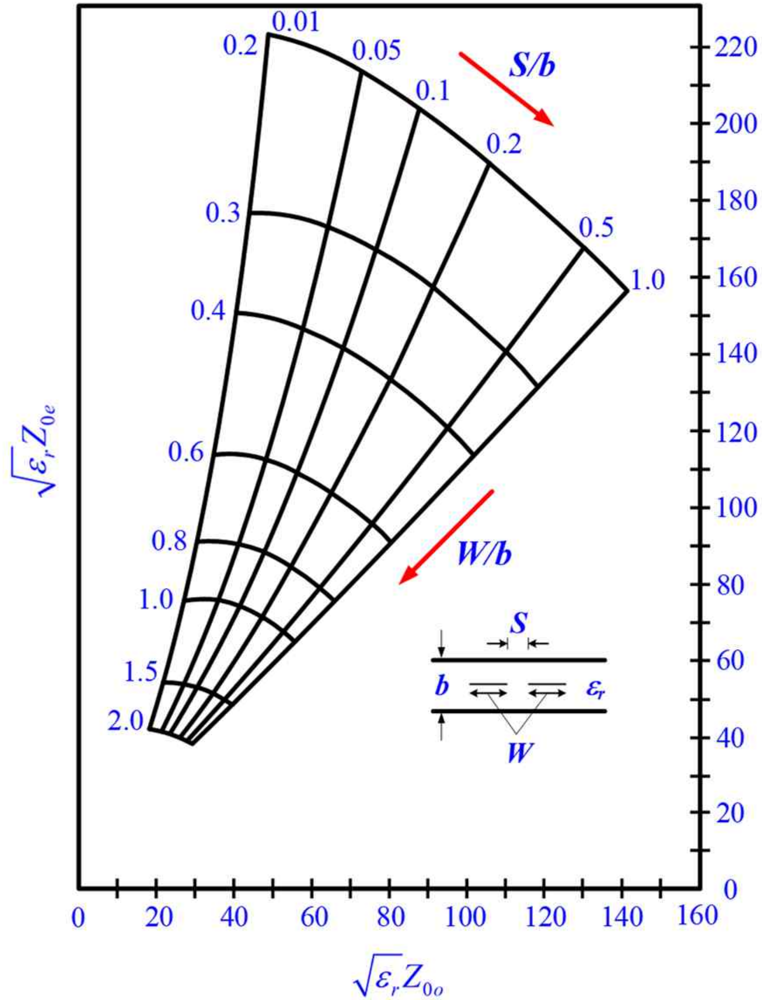
- Characteristic impedance for even mode

$$Z_{0o} = \sqrt{\frac{L}{C_o}} = \frac{\sqrt{LC_o}}{C_o} = \frac{1}{vC_o} \qquad (4) \leftarrow v = \frac{1}{\sqrt{LC}}$$

2 Coupled Line Theory

- Even- and odd-mode characteristic impedances of coupled line for given dielectric substrate
  - For quasi-TEM lines, characteristic impedances can be calculated numerically or by appropriate quasi-static technique
  - For symmetric coupled line, figure can be used to determine necessary strip line widths and scaping for given set of characteristics impedances → Educational purpose



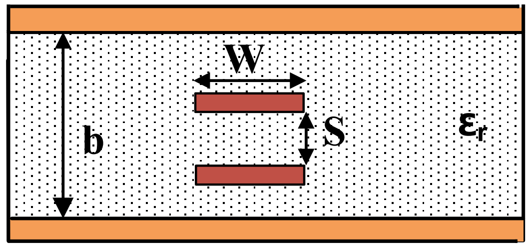


** Modern microwave circuits or EM simulators contain physical structure evaluation programs.*

2 Coupled Line Theory

- Example: Even- and odd-mode characteristic impedances of coupled line for given dielectric substrate

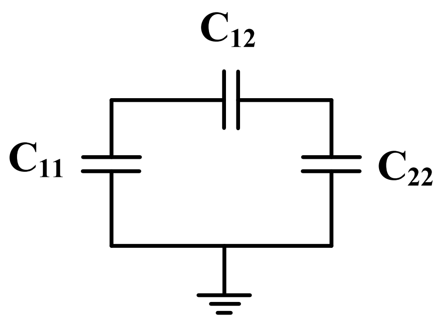
  - Assuming $W >> S$ and $W >> b$ *for coupled strip line* so that fringing fields can be ignored and determine the even- and odd-mode characteristic impedances.



Solution

- Since line is symmetric, $C_{11} = C_{22}$
- Capacitance of a parallel plate capacitor with plate area, S, and plate separation d

$$C = \frac{\varepsilon S}{d}, \quad \varepsilon : \text{permittivity of material between plates}$$



- Capacitance per unit length ($\rightarrow S = W \times L = W$)

$$C_{11} = \frac{\varepsilon_r \varepsilon_o W}{(b-S)/2} + \frac{\varepsilon_r \varepsilon_0 W}{b - \frac{(b-S)}{2}} = \frac{\varepsilon_r \varepsilon_o W}{(b-S)/2} + \frac{\varepsilon_r \varepsilon_0 W}{(b+S)/2} = \frac{4b\varepsilon_r \varepsilon_0 W}{b^2 - S^2} \text{[F/m]}$$

Assumption: t (thickness of metal line) $\rightarrow 0$

2 Coupled Line Theory

- Capacitance between coupled lines per unit length

$$C_{12} = \frac{\varepsilon_r \varepsilon_o W}{S} \text{[F/m]}$$

- Even-mode capacitance per unit length

$$C_e = C_{11} = \frac{4b\varepsilon_r \varepsilon_0 W}{b^2 - S^2} \text{[F/m]}$$

- Odd-mode capacitance per unit length

$$C_0 = C_{11} + 2C_{12}$$

$$= 2\varepsilon_r \varepsilon_0 W \left(\frac{2b}{b^2 - S^2} + \frac{1}{S} \right) \text{[F/m]}$$

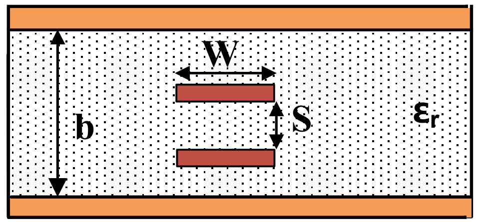
- Phase velocity on coupled lines

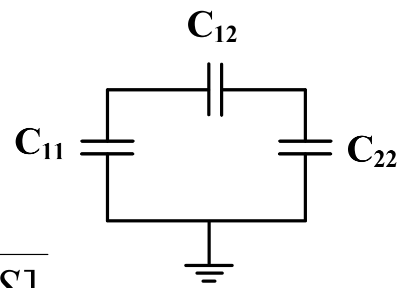
$$v = 1 / \sqrt{\varepsilon_r \varepsilon_0 \mu_0}$$

- Even-mode characteristic impedance

$$Z_{0e} = \frac{1}{vC_e} = \sqrt{\varepsilon_r \varepsilon_0 \mu_0} \frac{b^2 - S^2}{4b\varepsilon_r \varepsilon_0 W}$$

$$= \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{b^2 - S^2}{4bW\sqrt{\varepsilon_r}} = Z_0 \frac{b^2 - S^2}{4bW\sqrt{\varepsilon_r}}$$





- Odd-mode characteristic impedance

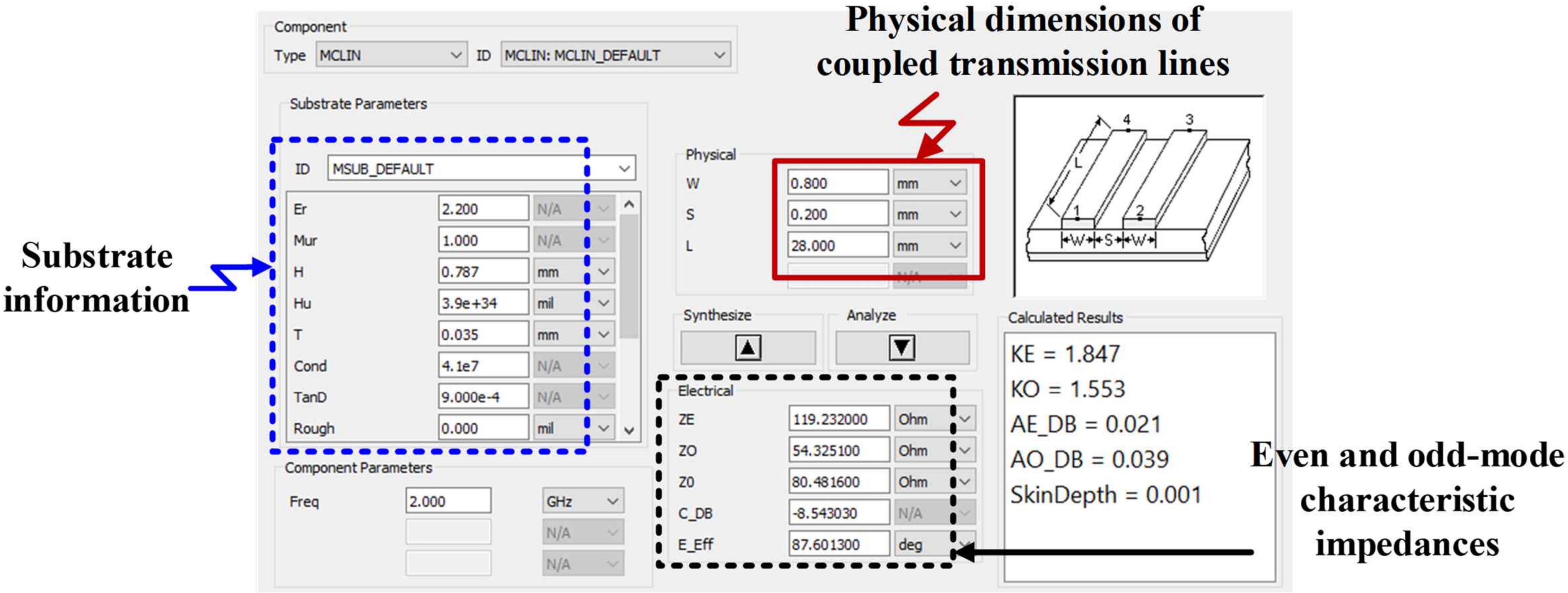
$$Z_{0o} = \frac{1}{vC_o} = \frac{\sqrt{\varepsilon_r \varepsilon_0 \mu_0}}{2\varepsilon_r \varepsilon_0 W[2b/(b^2 - S^2) + 1/S]}$$

$$= \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{2W\sqrt{\varepsilon_r}[2b/(b^2 - S^2) + 1/S]}$$

$$= \eta_0 \frac{1}{2W\sqrt{\varepsilon_r}[2b/(b^2 - S^2) + 1/S]}$$

2 Coupled Line Theory

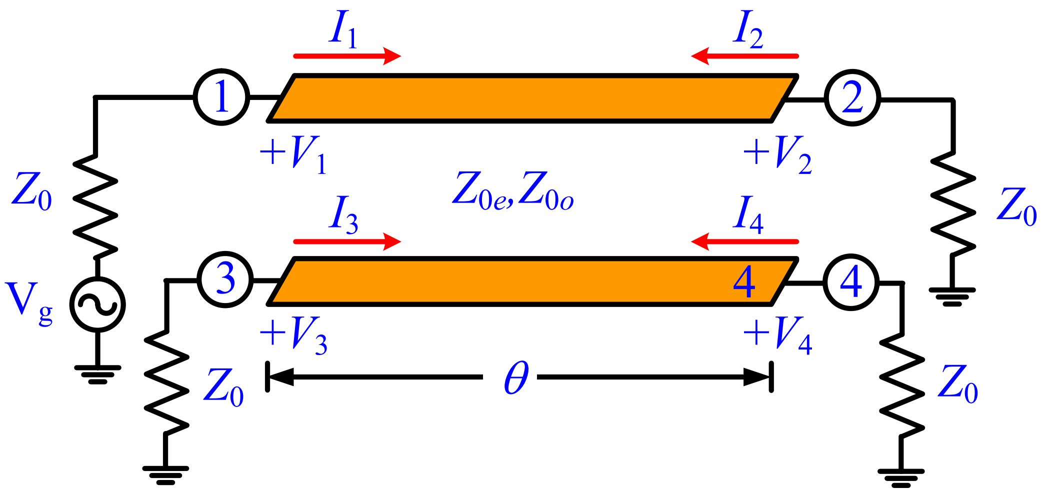
- Even- and odd-mode characteristic impedances calculation of microstrip coupled lines by using microwave circuit simulator



3 Different Coupled Transmission Lines

- 4-port coupled lines impedance matrix (Z-parameters)

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}, \quad (5)$$



where

$$Z_{11} = Z_{22} = Z_{33} = Z_{44} = -j\frac{Z_{0e} + Z_{0o}}{2}\cot\theta$$

$$Z_{12} = Z_{21} = Z_{34} = Z_{43} = -j\frac{Z_{0e} - Z_{0o}}{2}\cot\theta$$

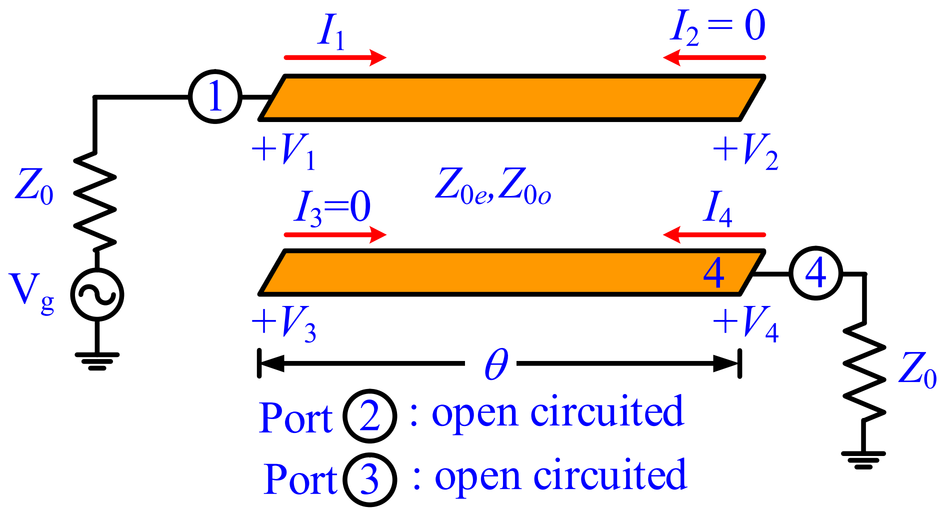
$$Z_{13} = Z_{31} = Z_{24} = Z_{42} = -j\frac{Z_{0e} - Z_{0o}}{2}\csc\theta$$

$$Z_{14} = Z_{41} = Z_{23} = Z_{32} = -j\frac{Z_{0e} + Z_{0o}}{2}\csc\theta$$

- Z_{0e}: even-mode characteristic impedance
- Z_{0o}: odd-mode characteristic impedance
- θ : electrical length of coupled line

3 Different Coupled Transmission Lines

- 2-port open-circuited coupled lines
  - Terminating ports 2 and 3 with open of 4-port coupled transmission line
  - Setting $I_2 = I_3 = 0$ in (5), Z-parameters of two-port coupled line can be found as



$$\begin{bmatrix} Z'_{11} & Z'_{12} \\ Z'_{21} & Z'_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{24} \\ Z_{42} & Z_{44} \end{bmatrix}$$

$$= \begin{bmatrix} -j\frac{Z_{0e}+Z_{0o}}{2}\cot\theta & -j\frac{Z_{0e}-Z_{0o}}{2}\csc\theta \\ -j\frac{Z_{0e}-Z_{0o}}{2}\csc\theta & -j\frac{Z_{0e}+Z_{0o}}{2}\cot\theta \end{bmatrix} = -jZ_{0o}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + j\frac{Z_{0e}-Z_{0o}}{2}\begin{bmatrix} -\cot\theta & -\csc\theta \\ -\csc\theta & -\cot\theta \end{bmatrix}$$

- Equivalent two open stubs with Z_{0o} are connected series with ports 1 and 4, and coupled transmission line $(Z_{0e} - Z_{0o})/2$ with electrical length θ is between open stubs.

3 Different Coupled Transmission Lines

- 4-port coupled lines admittance matrix (Y-parameters)

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}, \qquad (6)$$

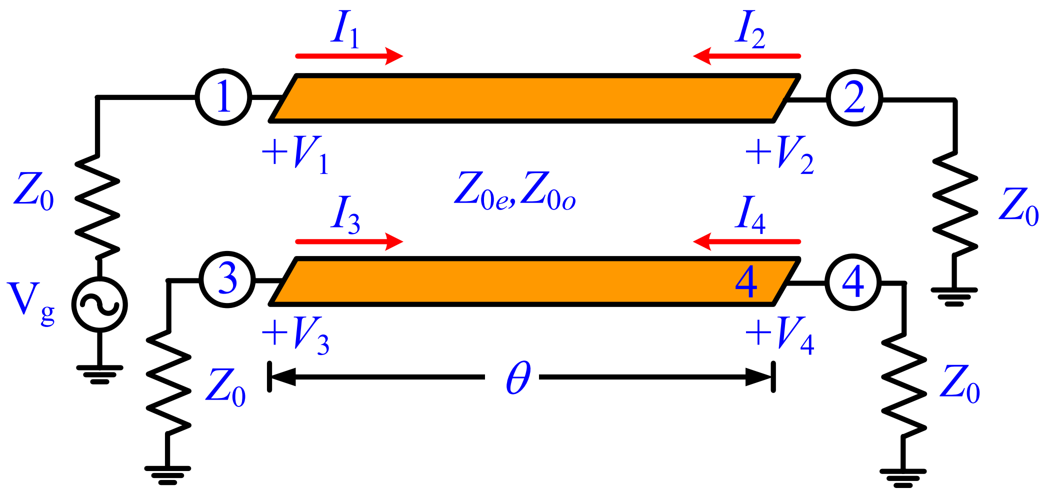
where

$$Y_{11} = Y_{22} = Y_{33} = Y_{44} = -j\frac{Y_{0e} + Y_{0o}}{2}\cot\theta$$

$$Y_{12} = Y_{21} = Y_{34} = Y_{43} = -j\frac{Y_{0o} - Y_{0e}}{2}\cot\theta$$

$$Y_{13} = Y_{31} = Y_{24} = Y_{42} = -j\frac{Y_{0o} - Y_{0e}}{2}\csc\theta$$

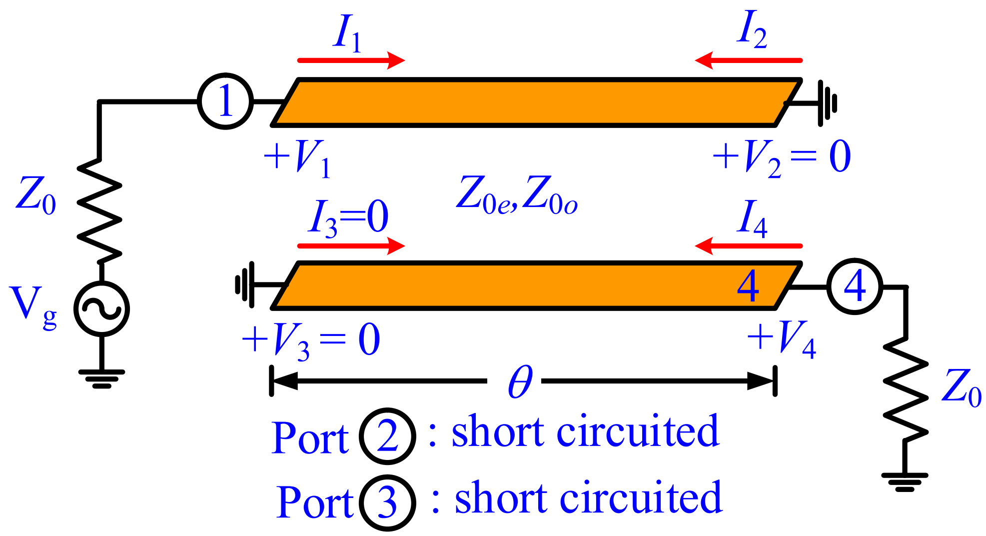
$$Y_{14} = Y_{41} = Y_{23} = Y_{32} = -j\frac{Y_{0e} + Y_{0o}}{2}\csc\theta$$



- $Y_{0e} = 1 / Z_{0e}$: even-mode characteristic admittance
- $Y_{0o} = 1 / Z_{0o}$: odd-mode characteristic admittance
- θ : electrical length of coupled line

3 Different Coupled Transmission Lines

- 2-port short-circuited coupled lines
  - Terminating ports 2 and 3 with short of 4-port coupled transmission line
  - Setting $V_2 = V_3 = 0$ in (6), Y-parameters of two-port coupled line can be found as



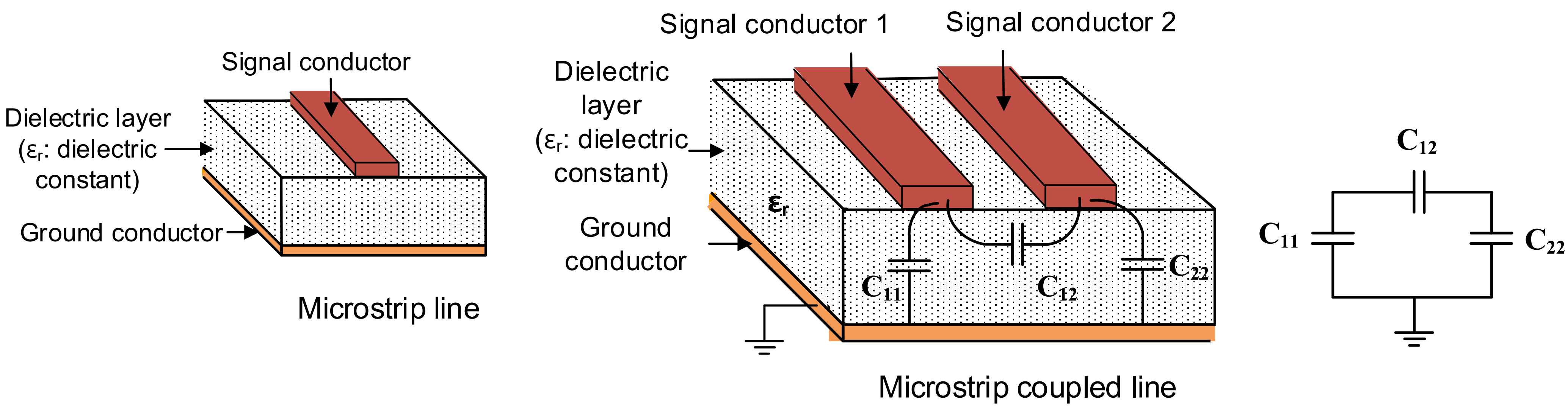
$$\begin{bmatrix} Y'_{11} & Y'_{12} \\ Z'_{21} & Z'_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{24} \\ Y_{42} & Y_{44} \end{bmatrix}$$

$$= \begin{bmatrix} -j\dfrac{Y_{0e}+Y_{0o}}{2}\cot\theta & -j\dfrac{Y_{0o}-Y_{0e}}{2}\csc\theta \\ -j\dfrac{Y_{0o}-Y_{0e}}{2}\csc\theta & -j\dfrac{Y_{0e}+Y_{0o}}{2}\cot\theta \end{bmatrix} = -jY_{0e}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + j\dfrac{\left(Y_{0o}-Y_{0e}\right)}{2}\begin{bmatrix} -\cot(\pi+\theta) & \csc(\pi+\theta) \\ \csc(\pi+\theta) & -\cot(\pi+\theta) \end{bmatrix}$$

- Equivalent two short stubs with characteristic admittance of Y_{0e} are connected in parallel with ports 1 and 4, and that transmission line $(Y_{0o} - Y_{0e})/2$ with electrical length of $(\theta + \pi)$ is between short stubs.

4 Review

- Single transmission line and coupled lines



- Coupled line theory
  - Even- and odd-mode characteristic impedances