Chapter 7
Power Dividers and Couplers

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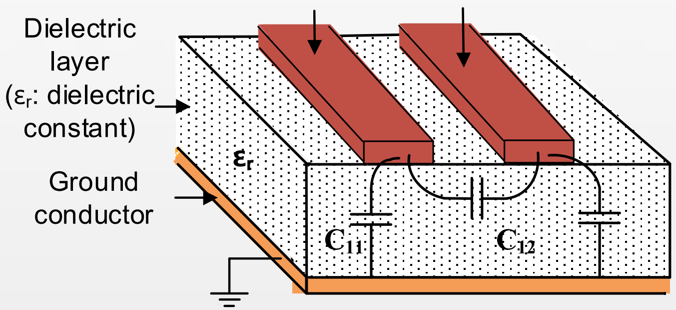
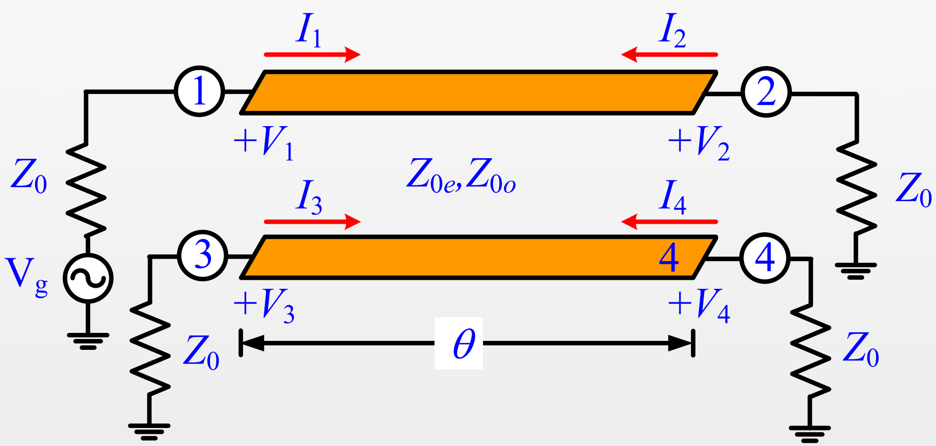


Microstrip coupled line



Learning Objectives

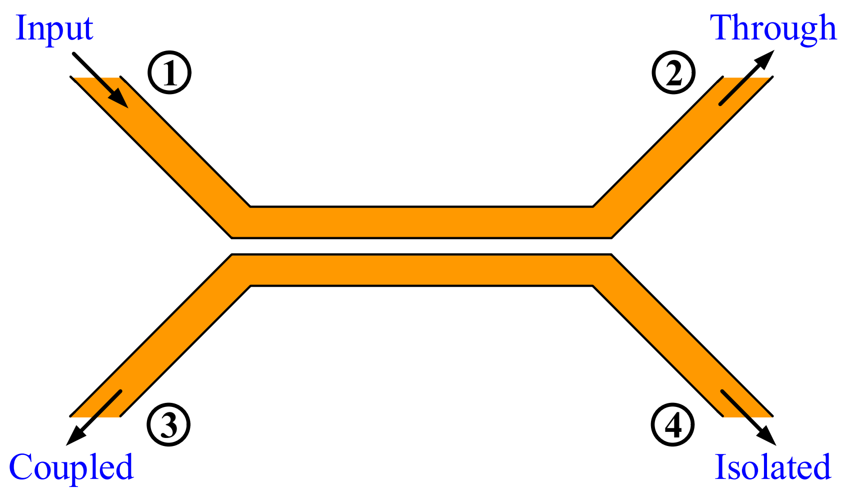
- Learn what is coupled line directional coupler.
- Learn how to find design equations of directional coupler.

Learning contents

- Brief Introduction of Directional Coupler
- Design Equations of Coupled Line Coupler

1 Brief Introduction of Directional Coupler

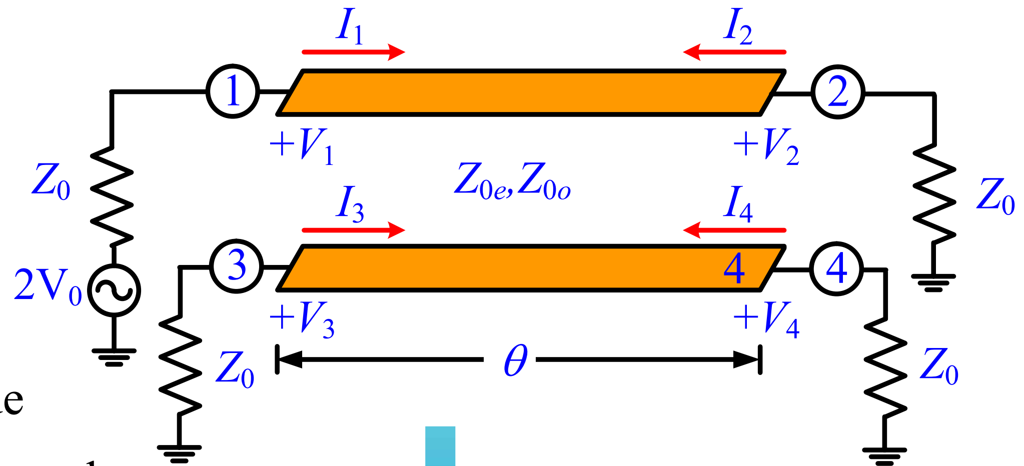
- What is directional coupler?
  - One of fundamental components in microwave and RF engineering
  - Four-port passive circuit
  - **Input port (port 1)**: signal entering port
  - **Through port (port 2)**: signal exiting port with minimum insertion loss
  - **Coupled port (port 3)**: A small fraction coupling port of input signal
  - **Isolated port (port 4)**: Ideally, no signal appearing port



2 Design Equations of Coupled Line Coupler

- 4-port coupled line coupler

  - 4-port network is terminated with impedance Z_0.
  - Driven with a voltage generator of 2V at port 1
  - By superposition, the excitation at port 1 can be treated as the sum of the even- and odd-mode excitations.



  - From symmetry

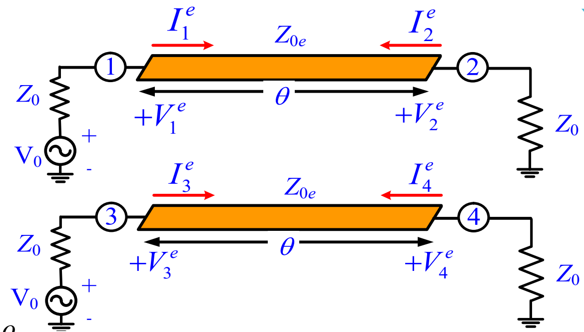
$$I_1^e = I_3^e, I_4^e = I_2^e, V_1^e = V_3^e, \text{ and } V_4^e = V_2^e \quad \text{for even-mode}$$

$$I_1^o = -I_3^o, I_4^o = -I_2^o, V_1^o = -V_3^o, \text{ and } V_4^o = -V_2^o \quad \text{for odd-mode}$$

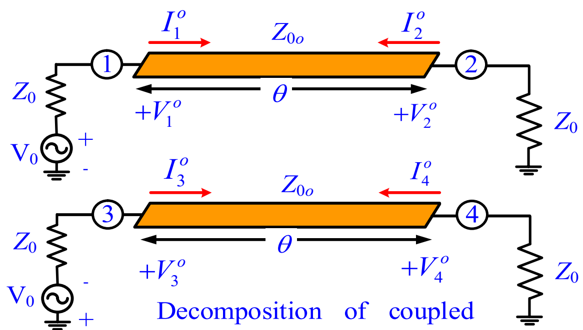
  - $P_{i,i=1,2,3,4} = f(V_1, I_1, V_2, I_2, V_3, I_3, V_4, I_4)$

$$= g(V_1^e, V_1^o, I_1^e, I_1^o, V_2^e, V_2^o, I_2^e, I_2^o, V_3^e, V_3^o, I_3^e, I_3^o, V_4^e, V_4^o, I_4^e, I_4^o)$$

$$= h(V_1^e, V_1^o, I_1^e, I_1^o, V_2^e, V_2^o, I_2^e, I_2^o)$$



Decomposition of coupled line



Decomposition of coupled line coupler under odd-mode

  - Input impedance at port 1: $Z_{in} = \dfrac{V_1}{I_1} = \dfrac{V_1^e + V_1^o}{I_1^e + I_1^o}$ (1)

2 Design Equations of Coupled Line Coupler

- Even- and odd-mode input impedances

  Z_{in}^{e} : input impedance at port 1 for even-mode

  Z_{in}^{o} : input impedance at port 1 for odd-mode

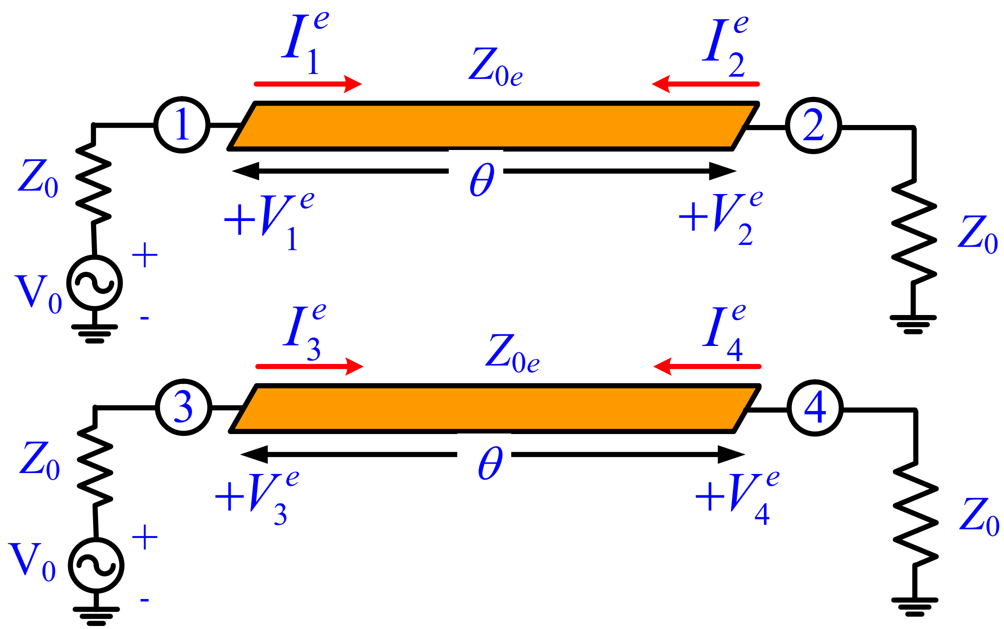
- Input impedances for even and odd-modes

$$Z_{\text{in}}^{e} = Z_{0e}\frac{Z_0 + jZ_{0e}\tan\theta}{Z_{0e} + jZ_0\tan\theta} \quad (2)$$

$$Z_{\text{in}}^{o} = Z_{0o}\frac{Z_0 + jZ_{0o}\tan\theta}{Z_{0o} + jZ_0\tan\theta} \quad (3)$$

- By voltage division for even-mode

$$V_1^e = \mathrm{V}_0\frac{Z_{\text{in}}^{e}}{Z_{\text{in}}^{e} + Z_0} \quad (4) \qquad I_1^e = \frac{\mathrm{V}_0}{Z_{\text{in}}^{e} + Z_0} \quad (5)$$



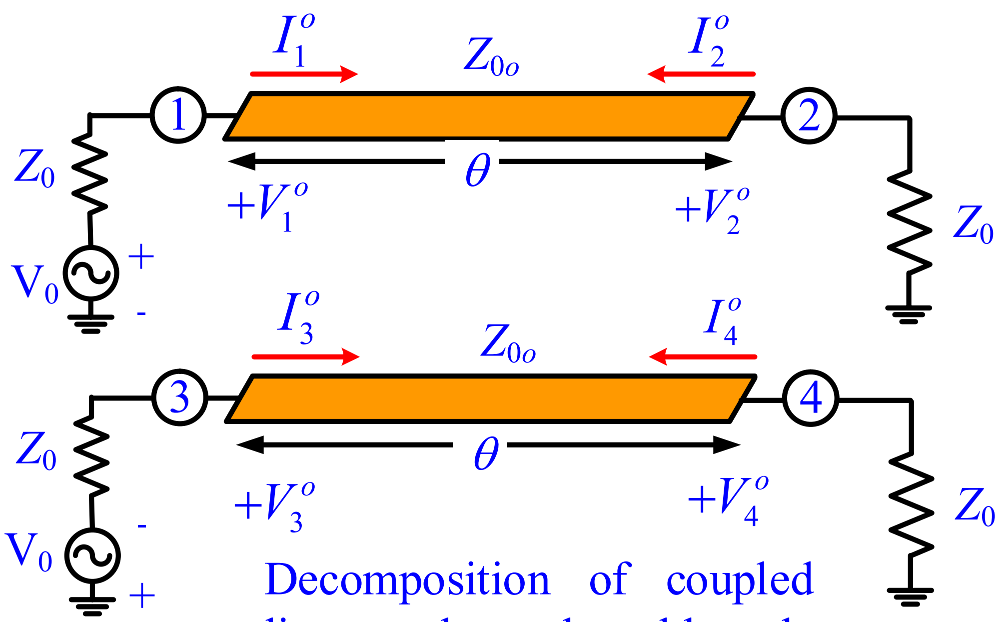
Decomposition of coupled line coupler under even-mode excitation

2 Design Equations of Coupled Line Coupler

- By voltage division for odd-mode

$$V_1^o = V_0 \frac{Z_{in}^o}{Z_{in}^o + Z_0} \quad (6)$$

$$I_1^o = \frac{V_0}{Z_{in}^o + Z_0} \quad (7)$$



Decomposition of coupled line coupler under odd-mode excitation

- Input impedance using (1)

$$Z_{in} = \frac{V_1^e + V_1^o}{I_1^e + I_1^o} = \frac{V_0 \frac{Z_{in}^e}{Z_{in}^e + Z_0} + V_0 \frac{Z_{in}^o}{Z_{in}^o + Z_0}}{\frac{V_0}{Z_{in}^e + Z_0} + \frac{V_0}{Z_{in}^o + Z_0}}$$

$$= \frac{Z_{in}^e (Z_{in}^o + Z_0) + Z_{in}^o (Z_{in}^e + Z_0)}{Z_{in}^e + Z_{in}^o + 2Z_0} = \frac{2Z_{in}^e Z_{in}^o + Z_0 (Z_{in}^e + Z_{in}^o) + 2Z_0^2 - 2Z_0^2}{Z_{in}^e + Z_{in}^o + 2Z_0} = Z_0 + \frac{2(Z_{in}^o Z_{in}^e - Z_0^2)}{Z_{in}^e + Z_{in}^o + 2Z_0} \quad (8)$$

2 Design Equations of Coupled Line Coupler

- If let $Z_0 = \sqrt{Z_{0e} Z_{0o}}$ (9)

- Equations (2) and (3) are reduced as

$$Z_{in}^e = Z_{0e} \frac{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}} \tan\theta}{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}} \tan\theta}, \quad Z_{in}^o = Z_{0o} \frac{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}} \tan\theta}{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}} \tan\theta} \quad \leftarrow$$

$$Z_{in}^e = Z_{0e} \frac{Z_0 + jZ_{0e} \tan\theta}{Z_{0e} + jZ_0 \tan\theta} \quad (2)$$

$$Z_{in}^o = Z_{0o} \frac{Z_0 + jZ_{0o} \tan\theta}{Z_{0o} + jZ_0 \tan\theta} \quad (3)$$

- Then

$$Z_{in}^e Z_{in}^o = Z_{0e} \frac{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}} \tan\theta}{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}} \tan\theta} \times Z_{0o} \frac{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}} \tan\theta}{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}} \tan\theta} = Z_{0e} Z_{0o} = Z_0^2$$

$$\rightarrow Z_{in} = \underbrace{Z_0 + \frac{2(Z_{in}^o Z_{in}^e - Z_0^2)}{Z_{in}^e + Z_{in}^o + 2Z_0}}_{\text{Equation (8)}} = Z_0 + \frac{2(Z_0^2 - Z_0^2)}{Z_{in}^e + Z_{in}^o + 2Z_0}$$

$$\rightarrow Z_{in} = Z_0 \quad (10)$$

❖ ***As long as equation (10) is satisfied, port 1 (by symmetry, all other ports) will be matched.***

2 Design Equations of Coupled Line Coupler

- If $Z_0 = \sqrt{Z_{0e} Z_{0o}}$ is satisfied, so that we have $V_1 = V_0$, voltage at port 3 can be written as

$$V_3 = V_3^e + V_3^o = V_1^e - V_1^o = V_0 \left[\frac{Z_{\text{in}}^e}{Z_{\text{in}}^e + Z_0} - \frac{Z_{\text{in}}^o}{Z_{\text{in}}^o + Z_0} \right] \qquad (11)$$

- From (2) and (3),

$$\frac{Z_{\text{in}}^e}{Z_{\text{in}}^e + Z_0} = \frac{\overbrace{Z_{0e} \frac{Z_0 + jZ_{0e} \tan\theta}{Z_{0e} + jZ_0 \tan\theta}}^{\text{Equation (2)}}}{Z_{0e} \frac{Z_0 + jZ_{0e} \tan\theta}{Z_{0e} + jZ_0 \tan\theta} + Z_0} = \frac{Z_{0e}(Z_0 + jZ_{0e} \tan\theta)}{2Z_{0e} Z_0 + j\left(Z_{0e}^2 + Z_0^2\right) \tan\theta} = \frac{Z_0 + jZ_{0e} \tan\theta}{2Z_0 + j(Z_{0e} + Z_{0o}) \tan\theta}$$

$$\frac{Z_{\text{in}}^o}{Z_{\text{in}}^o + Z_0} = \frac{Z_{0o} \frac{Z_0 + jZ_{0o} \tan\theta}{Z_{0o} + jZ_0 \tan\theta}}{\underbrace{Z_{0o} \frac{Z_0 + jZ_{0o} \tan\theta}{Z_{0o} + jZ_0 \tan\theta}}_{\text{Equation (3)}} + Z_0} = \frac{Z_{0o}\left(Z_0 + jZ_{0o} \tan\theta\right)}{2Z_{0o} Z_0 + j\left(Z_{0o}^2 + Z_0^2\right) \tan\theta} = \frac{Z_0 + jZ_{0o} \tan\theta}{2Z_0 + j(Z_{0e} + Z_{0o}) \tan\theta}$$

2 Design Equations of Coupled Line Coupler

- Equation (11) reduces

$$V_3 = V_0\left\{\frac{Z_{in}^e}{Z_{in}^e + Z_0} - \frac{Z_{in}^o}{Z_{in}^o + Z_0}\right\} = V_0\left\{\frac{Z_0 + jZ_{0e}\tan\theta}{2Z_0 + j(Z_{0e} + Z_{0o})\tan\theta} - \frac{Z_0 + jZ_{0o}\tan\theta}{2Z_0 + j(Z_{0e} + Z_{0o})\tan\theta}\right\}$$

$$= V_0\frac{j(Z_{0e} - Z_{0o})\tan\theta}{2Z_0 + j(Z_{0e} + Z_{0o})\tan\theta} \quad (12)$$

- Coupling coefficient: $C = \dfrac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}$ (13)

- From (13), even and odd-mode characteristic impedances can be derived as

$$Z_{0e} = \frac{Z_{0o}(1+C)}{1-C} = \frac{Z_0^2(1+C)}{Z_{0e}(1-C)} \rightarrow Z_{0e} = Z_0\sqrt{\frac{1+C}{1-C}}, \quad Z_{0o} = \frac{Z_{0e}(1-C)}{1+C} = \frac{Z_0^2(1-C)}{Z_{0o}(1+C)} \rightarrow Z_{0o} = Z_0\sqrt{\frac{1-C}{1+C}}$$

- From (13) with $Z_0^2 = Z_{0e}Z_{0o}$,

$$\sqrt{1-C^2} = \sqrt{1-\left(\frac{Z_{0e}-Z_{0o}}{Z_{0e}+Z_{0o}}\right)^2} = \sqrt{\frac{(Z_{0e}+Z_{0o})^2-(Z_{0e}-Z_{0o})^2}{(Z_{0e}+Z_{0o})^2}} = \sqrt{\frac{4Z_{0e}Z_{0o}}{(Z_{0e}+Z_{0o})^2}} = \sqrt{\frac{4Z_0^2}{(Z_{0e}+Z_{0o})^2}} = \frac{2Z_0}{Z_{0e}+Z_{0o}}$$

2 Design Equations of Coupled Line Coupler

- From (12)

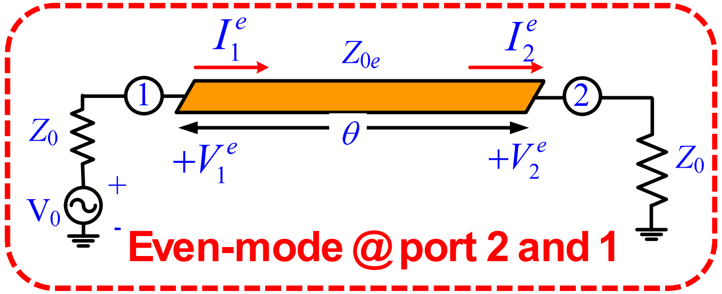
$$V_3 = \mathrm{V}_0 \frac{j(Z_{0e}-Z_{0o})\tan\theta}{2Z_0 + j(Z_{0e}+Z_{0o})\tan\theta} = \mathrm{V}_0 \frac{j(Z_{0e}-Z_{0o})/(Z_{0e}+Z_{0o})\tan\theta}{2Z_0/(Z_{0e}+Z_{0o}) + j\tan\theta} = \mathrm{V}_0 \frac{jC\tan\theta}{\sqrt{-C^2} + j\tan\theta} \quad (14)$$

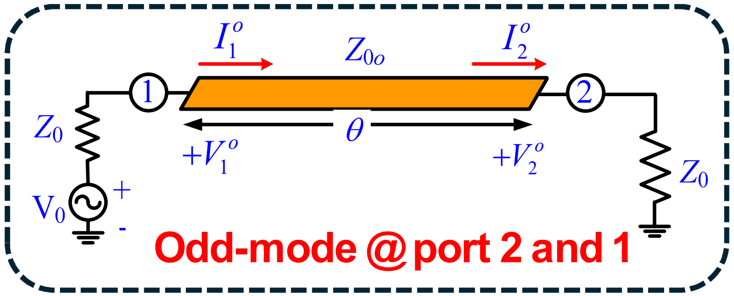
- Design equations of coupled line coupler (*another trial*)

$$\begin{bmatrix} V_1^e \\ I_1^e \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_{0e}\sin\theta \\ jY_{0e}\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_2^e \\ I_2^e \end{bmatrix} \quad (15a)$$

$$\begin{bmatrix} V_1^o \\ I_1^o \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_{0o}\sin\theta \\ jY_{0o}\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_2^o \\ I_2^o \end{bmatrix} \quad (15b)$$

$$\begin{cases} V_2^e = Z_0 I_2^e \\ V_2^o = Z_0 I_2^o \\ V_1^e + Z_0 I_1^e = \mathrm{V}_0 \\ V_1^o + Z_0 I_1^o = \mathrm{V}_0 \end{cases}$$





- From (15a)

$$I_1^e = jY_{0e}\sin\theta \times V_2^e + \cos\theta \times I_2^e = jY_{0e}\sin\theta \times Z_0 I_2^e + \cos\theta \times I_2^e$$

$$= (\cos\theta + jZ_0 Y_{0e}\sin\theta) I_2^e \rightarrow I_2^e = \frac{I_1^e}{(\cos\theta + jZ_0 Y_{0e}\sin\theta)} \quad (16)$$

2 Design Equations of Coupled Line Coupler

$$V_1^e = \cos\theta \times V_2^e + jZ_{0e}\sin\theta \times I_2^e = \cos\theta \times Z_0 I_2^e + jZ_{0e}\sin\theta \times I_2^e \leftarrow V_2^e = Z_0 I_2^e$$

$$= \left(Z_0\cos\theta + jZ_{0e}\sin\theta\right) I_2^e = \frac{Z_0\cos\theta + jZ_{0e}\sin\theta}{\cos\theta + jZ_0 Y_{0e}\sin\theta} I_1^e = \frac{Z_0\cos\theta + jZ_{0e}\sin\theta}{\cos\theta + j(Z_0\sin\theta \;\; Z_{0e})} I$$

$$\begin{bmatrix} V_1^e \\ I_1^e \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_{0e}\sin\theta \\ jY_{0e}\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_2^e \\ I_2^e \end{bmatrix}$$

$$\begin{bmatrix} V_1^o \\ I_1^o \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_{0o}\sin\theta \\ jY_{0o}\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_2^o \\ I_2^o \end{bmatrix}$$

$$= \frac{Z_0 Z_{0e}\cos\theta + jZ_{0e}^2\sin\theta}{Z_{0e}\cos\theta + jZ_0\sin\theta} I_1^e \rightarrow I_1^e = V_1^e \frac{Z_{0e}\cos\theta + jZ_0\sin\theta}{Z_0 Z_{0e}\cos\theta + jZ_{0e}^2\sin\theta}$$

- Even-mode voltage at port 1

$$V_1^e + Z_0 I_1^e = \mathrm{V}_0$$

$$V_1^e\left(1 + Z_0\frac{I_1^e}{V_1^e}\right) = \mathrm{V}_0$$

$$V_1^e\left(1 + Z_0\frac{Z_{0e}\cos\theta + jZ_0\sin\theta}{Z_0 Z_{0e}\cos\theta + jZ_{0e}^2\sin\theta}\right) = \mathrm{V}_0$$

$$V_1^e\left(1 + \frac{Z_0 Z_{0e}\cos\theta + jZ_0^2\sin\theta}{Z_0 Z_{0e}\cos\theta + jZ_{0e}^2\sin\theta}\right) = \mathrm{V}_0$$

$$V_1^e\left\{\frac{2Z_0 Z_{0e}\cos\theta + j\left(Z_0^2 + Z_{0e}^2\right)\sin\theta}{Z_0 Z_{0e}\cos\theta + jZ_{0e}^2\sin\theta}\right\} = \mathrm{V}_0$$

$$V_1^e = \mathrm{V}_0\frac{Z_0 Z_{0e}\cos\theta + jZ_{0e}^2\sin\theta}{2Z_0 Z_{0e}\cos\theta + j\left(Z_0^2 + Z_{0e}^2\right)\sin\theta} \quad (17)$$

2 Design Equations of Coupled Line Coupler

- Even-mode current at port 1

$$I_1^e = \frac{Z_{0e}\cos\theta + jZ_0\sin\theta}{Z_0 Z_{0e}\cos\theta + jZ_{0e}^2\sin\theta} V_1^e \leftarrow V_1^e \text{ from (17)}$$

$$= \frac{Z_{0e}\cos\theta + jZ_0\sin\theta}{Z_0 Z_{0e}\cos\theta + jZ_{0e}^2\sin\theta} \times \mathrm{V}_0 \frac{Z_0 Z_{0e}\cos\theta + jZ_{0e}^2\sin\theta}{2Z_0 Z_{0e}\cos\theta + j\left(Z_0^2 + Z_{0e}^2\right)\sin\theta} = \mathrm{V}_0 \frac{Z_{0e}\cos\theta + jZ_0\sin\theta}{2Z_0 Z_{0e}\cos\theta + j\left(Z_0^2 + Z_{0e}^2\right)\sin\theta}$$

- Even-mode current at port 2

$$I_2^e = \frac{I_1^e}{\cos\theta + jZ_0 Y_{0e}\sin\theta} = \frac{1}{Y_{0e}(Z_{0e}\cos\theta + jZ_0\sin\theta)} \mathrm{V}_0 \frac{Z_{0e}\cos\theta + jZ_0\sin\theta}{2Z_0 Z_{0e}\cos\theta + j\left(Z_0^2 + Z_{0e}^2\right)\sin\theta}$$

$$= \frac{Z_{0e}}{2Z_0 Z_{0e}\cos\theta + j\left(Z_0^2 + Z_{0e}^2\right)\sin\theta} \mathrm{V}_0$$

- Even-mode voltage at port 2

$$V_2^e = Z_0 I_2^e = \frac{Z_{0e} Z_0 I_1^e}{Z_{0e}\cos\theta + jZ_0\sin\theta}$$

$$= \frac{Z_{0e} Z_0}{Z_{0e}\cos\theta + jZ_0\sin\theta} \times \mathrm{V}_0 \frac{Z_{0e}\cos\theta + jZ_0\sin\theta}{2Z_0 Z_{0e}\cos\theta + j\left(Z_{0e}^2 + Z_0^2\right)\sin\theta} = \mathrm{V}_0 \frac{Z_{0e} Z_0}{2Z_0 Z_{0e}\cos\theta + j\left(Z_{0e}^2 + Z_0^2\right)\sin}$$

2 Design Equations of Coupled Line Coupler

- Odd-mode voltage at ports 1 and 2 using (15b) as like even-mode

$$V_1^o = V_0 \frac{Z_0 Z_{0o}\cos\theta + jZ_{0o}^2 \sin\theta}{2Z_0 Z_{0o}\cos\theta + j(Z_0^2 + Z_{0o}^2)\sin\theta}, \quad I_1^o = V_0 \frac{Z_{0o}\cos\theta + jZ_0 \sin\theta}{2Z_0 Z_{0o}\cos\theta + j(Z_0^2 + Z_{0o}^2)\sin\theta}$$

$$V_2^o = Z_0 I_2^o = \frac{Z_0 Z_{0o}}{Z_{0o}\cos\theta + jZ_0\sin\theta} I_1^o = V_0 \frac{Z_0 Z_{0o}}{Z_0 Z_{0o}\cos\theta + j(Z_0^2 + Z_{0o}^2)\sin\theta}$$

- Total voltage at port 2: combination of even and odd-mode excitations voltages

$$V_2 = V_2^e + V_2^o = V_0\left\{\frac{Z_{0e}Z_0}{2Z_0 Z_{0e}\cos\theta + j(Z_0^2 + Z_{0e}^2)\sin\theta} + \frac{Z_0 Z_{0o}}{Z_0 Z_{0o}\cos\theta + j(Z_0^2 + Z_{0o}^2)\sin\theta}\right\}$$

$$= V_0\left\{\frac{Z_{0e}Z_0}{2Z_0 Z_{0e}\cos\theta + j(Z_{0e}Z_{0o} + Z_{0e}^2)\sin\theta} + \frac{Z_0 Z_{0o}}{Z_0 Z_{0o}\cos\theta + j(Z_{0e}Z_{0o} + Z_{0o}^2)\sin\theta}\right\}$$

$$= V_0\left\{\frac{Z_0}{2Z_0\cos\theta + j(Z_{0e} + Z_{0o})\sin\theta} + \frac{Z_0}{Z_0\cos\theta + j(Z_{0o} + Z_{0e})\sin\theta}\right\}$$

$$= V_0 \frac{2Z_0}{2Z_0\cos\theta + j(Z_{0e} + Z_{0o})\sin\theta} = V_0 \frac{2Z_0/(Z_{0e} + Z_{0o})}{2Z_0/(Z_{0e} + Z_{0o})\cos\theta + j\sin\theta} = V_0 \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta} \quad (18)$$

2 Design Equations of Coupled Line Coupler

- Similarly, total voltage at port 4 (isolation port)

$$V_4 = V_4^e + V_4^o = V_2^e - V_2^o = \mathrm{V}_0 \left\{ \frac{Z_0}{Z_0\cos\theta + j\left(Z_{0e} + Z_{0o}\right)\sin\theta} - \frac{Z_0}{Z_0\cos\theta + j\left(Z_{0e} + Z_{0o}\right)\sin\theta} \right\} = 0 \qquad (19)$$

- Voltages at ports 2, 3, and 4

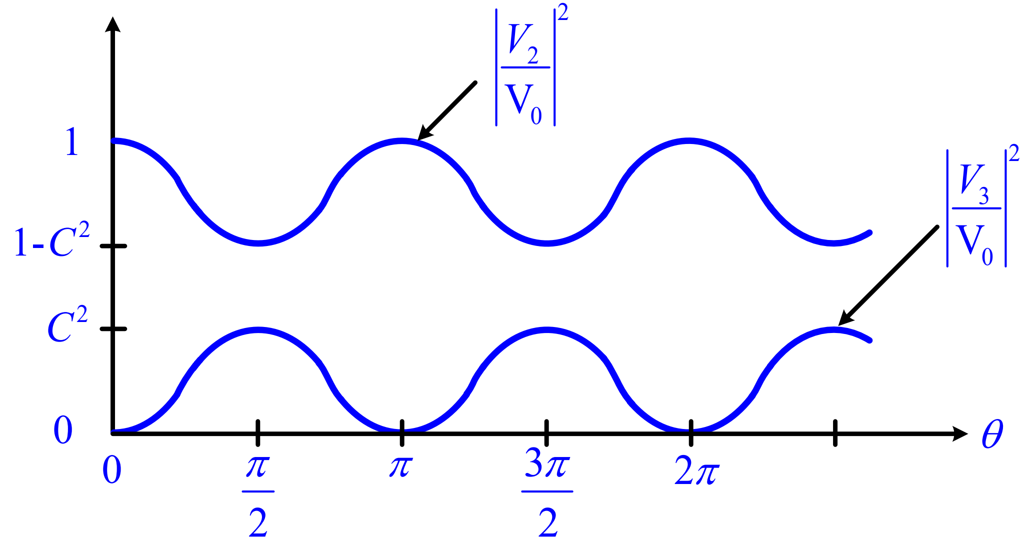
$$V_3 = \mathrm{V}_0 \frac{jC\tan\theta}{\sqrt{1-C^2} + j\tan\theta} \qquad (14)$$

$$V_2 = \mathrm{V}_0 \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta} \qquad (18)$$

$$V_4 = 0 \qquad (19)$$

- For $\theta = \pi/2$, equations (14) and (18) are

$$\frac{V_3}{V_0} = C, \quad \frac{V_2}{V_0} = -j\sqrt{1-C^2}$$



3 Review

- Coupled line directional coupler
  - Input, through, coupled, and isolation ports
  - Coupling coefficient, directivity
- Design equations of coupled line coupler
  - Matched condition
  - Even and odd-mode excitations