Chapter 7
Power Dividers and Couplers

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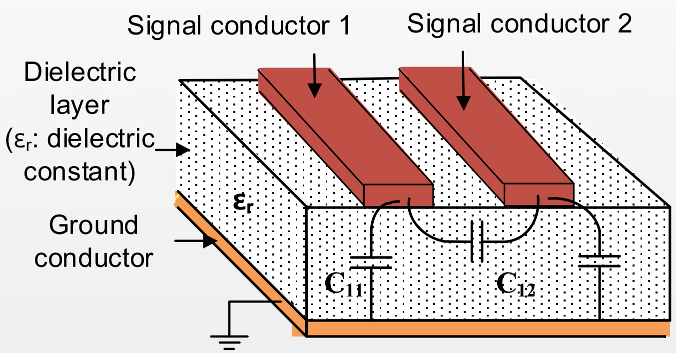
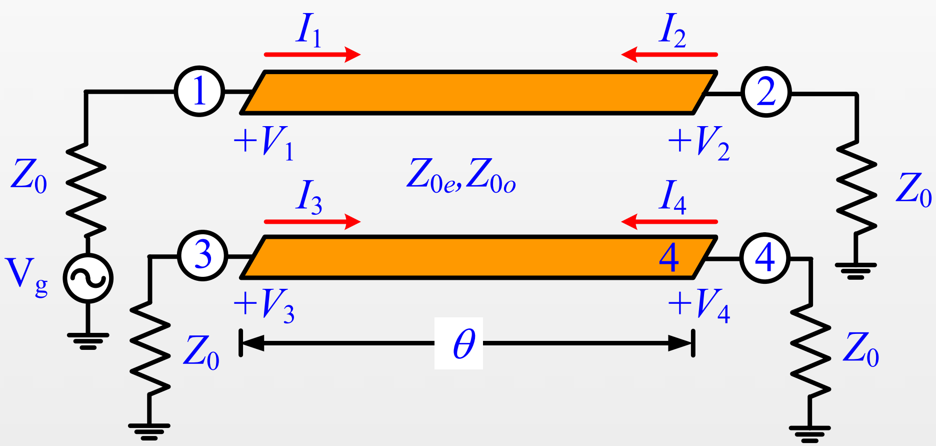


Microstrip coupled line



Learning Objectives

- Learn how to design and simulate coupled line coupler.
- Learn how to enhance bandwidth coupled line coupler with multi-section design.
- Learn how to design and obtain frequency response of multi-section coupled line coupler.

Learning contents

- Design Example of Single Section Coupled Line Coupler
- Multi-Section Coupled Line Coupler
- Design Example of Multi-Section Coupled Line Coupler

1 Design Example of Single-Section Coupled Line Coupler

- Design Example: 20 dB coupler
  - Design a 20 dB single-section coupled line coupler with characteristic impedance of 50 Ω using PCB with a dielectric constant of 2.2 and thickness of 0.787 mm and a center frequency $f_0 = 3$ GHz.
  - Plot coupling and directivity from 1 to 5 GHz

Solution

- Voltage coupling factor C (dB) = 20 dB

$$C_{\mathrm{dB}} = -20\log_{10}(C)$$

$$\log_{10}(C) = -\frac{C_{\mathrm{dB}}}{20}$$

$$C = 10^{\left(-\frac{C_{\mathrm{dB}}}{20}\right)} = 10^{\left(-\frac{20}{20}\right)} = 0.1$$

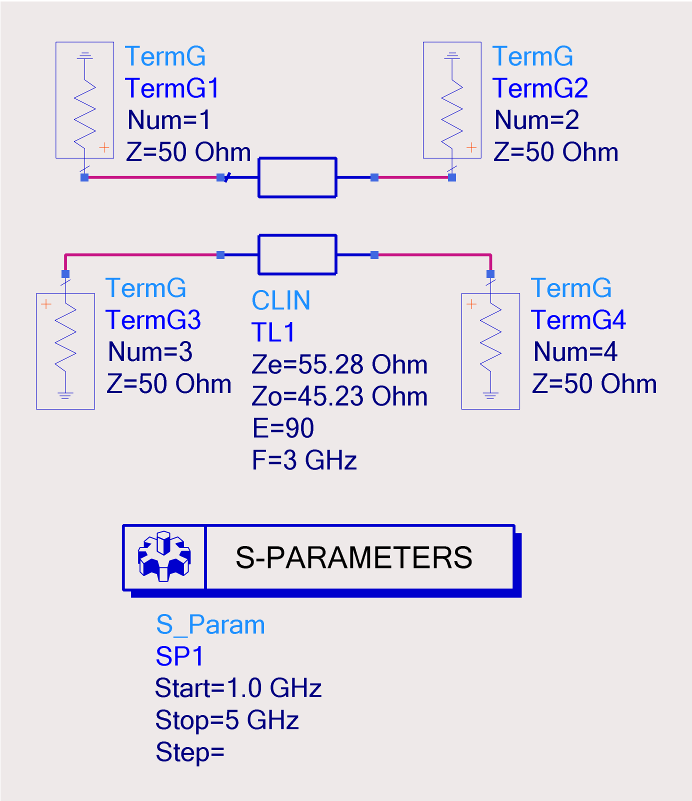
- Even and odd-mode characteristic impedances

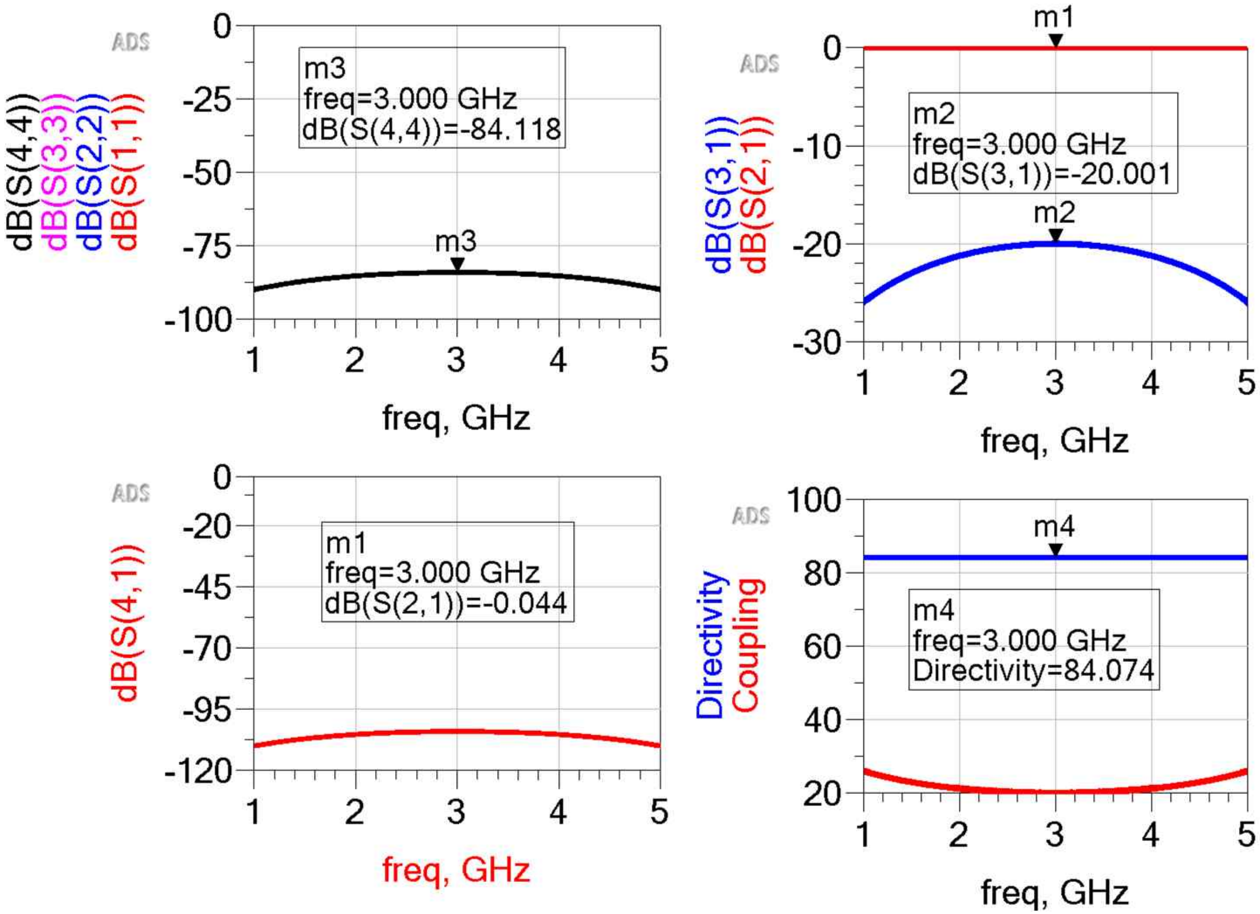
$$Z_{0e} = Z_0\sqrt{\frac{1+C}{1-C}} = 50\sqrt{\frac{1+0.1}{1-0.1}} = 50\sqrt{\frac{1.1}{0.9}} = 55.28\ \Omega$$

$$Z_{0o} = Z_0\sqrt{\frac{1-C}{1+C}} = 50\sqrt{\frac{1-0.1}{1+0.1}} = 50\sqrt{\frac{0.9}{1.1}} = 45.23\ \Omega$$

1 Design Example of Single-Section Coupled Line Coupler

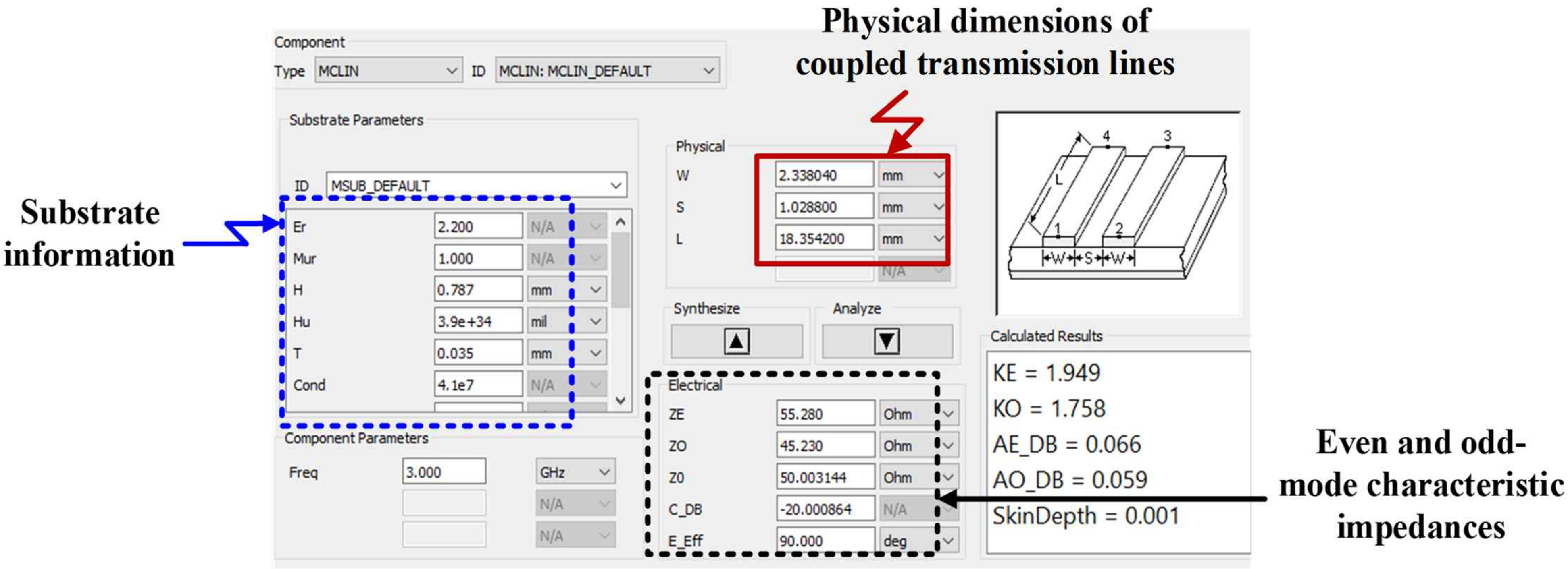
- Simulation results with microwave circuit simulator: Ideal coupled line





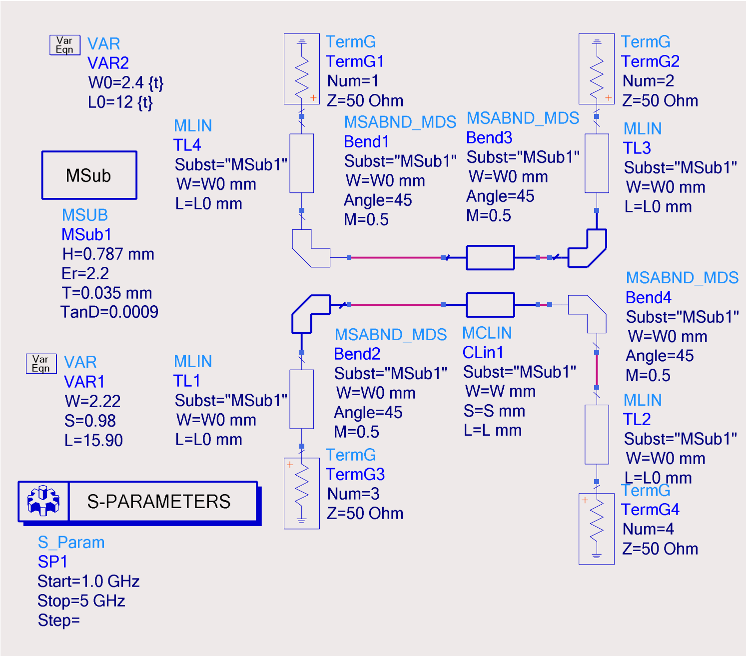
1 Design Example of Single-Section Coupled Line Coupler

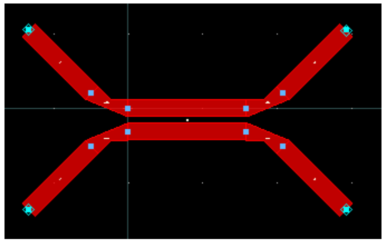
- Substrate dielectric constant $\varepsilon_r = 2.20$, thickness $h = 0.787$ mm, and $f_0 = 3$ GHz
- Using microwave circuit simulator, $W = 2.34$ mm, $S = 1.03$ mm, and $L = 18.35$ mm

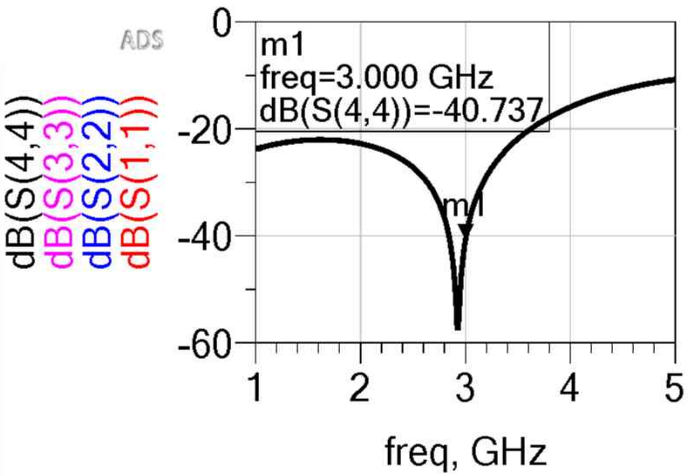


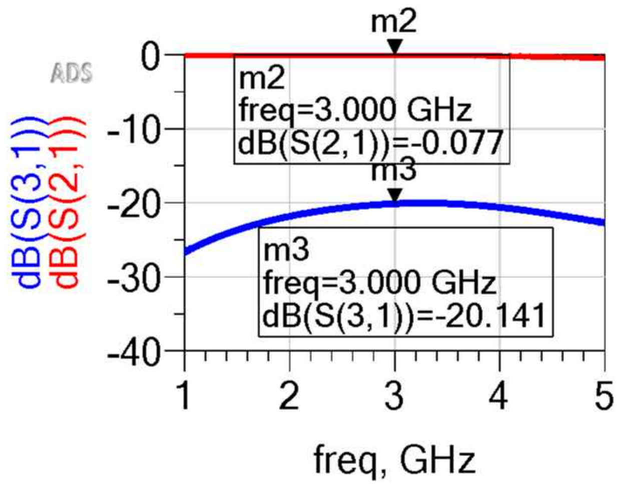
1 Design Example of Single-Section Coupled Line Coupler

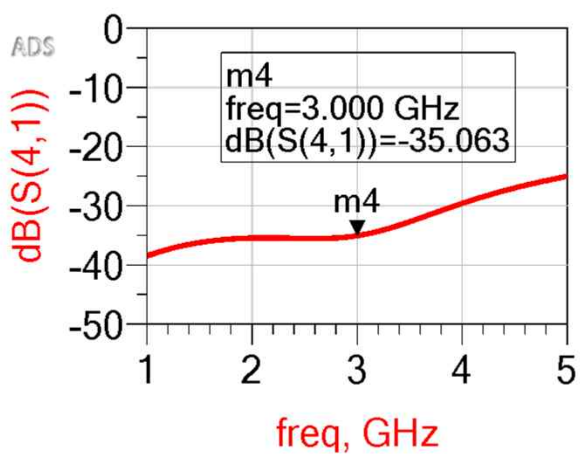
- ADS layout and Simulation results

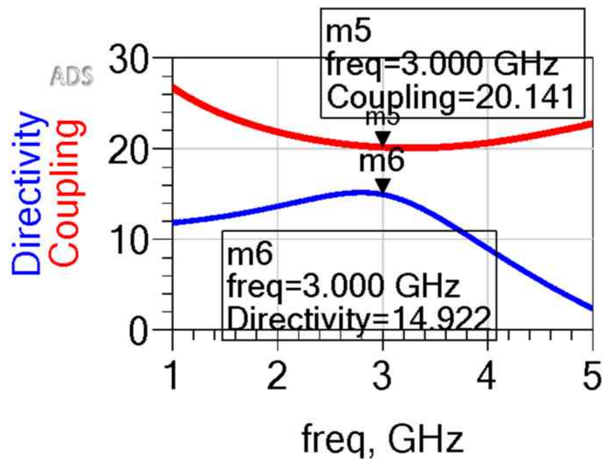












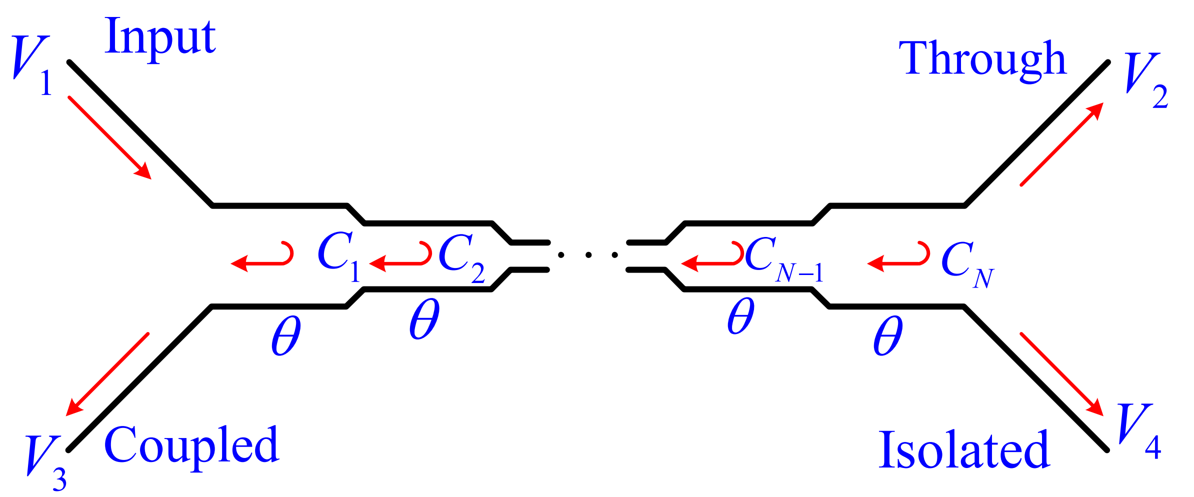
1 Design Example of Single-Section Coupled Line Coupler

- Single section coupled line coupler
  - Narrow frequency bandwidth characteristics
  - Coupling of single-section coupled line coupler is limited in bandwidth due to $\lambda/4$ length characteristics.
  - As like matching transformers and waveguide couplers, bandwidth can be increased by using multiple sections.
  - There is close relationship between multi-section coupled line couplers and multi-section quarter-wavelength transformers.

- Multi-section coupled line coupler
  - Consists of multi-section of single-coupled line coupler
  - Multi-section coupled line couplers can achieve broader bandwidth as compared to single section.
  - For multi-section coupler design, the coupling coefficient is weak ($C \geq 10$ dB)

2 Multi-Section Coupled Line Coupler

- Multi-section coupled line coupler
  - Because the phase characteristics are usually better, multi-section coupled line couplers are generally made with **odd number of sections**.
  - Coupling is week ($C \geq 10$ dB)
  - Each section is $\lambda/4$ long ($\theta = \pi/2$) at the center frequency.



- For a single-section coupled line

$$\frac{V_3}{V_1} = \frac{jC\tan\theta}{\sqrt{1-C^2} + j\tan\theta}$$

$$\frac{V_2}{V_1} = \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta}$$

- If $C << 1$, then design-equations of single-section coupled line can be simplified as

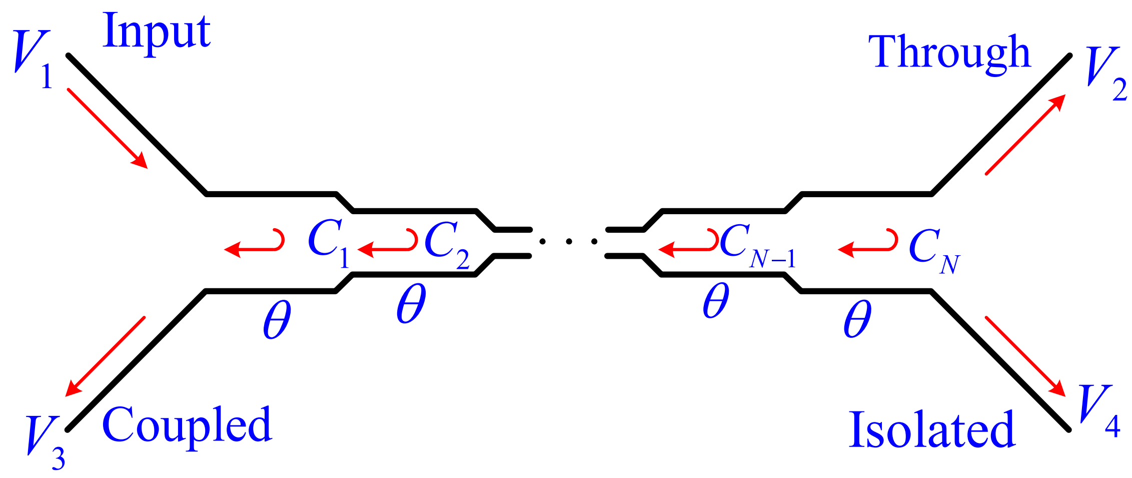
$$\frac{V_3}{V_1} = \frac{jC\tan\theta}{\sqrt{1-C^2} + j\tan\theta} \approx \frac{jC\tan\theta}{1 + j\tan\theta} = \frac{jC\sin\theta}{\cos\theta + j\sin\theta} = jC\sin\theta e^{-j\theta} \quad (1)$$

$$\frac{V_2}{V_1} = \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta} \approx \frac{1}{\cos\theta + j\sin\theta} = e^{-j\theta} \quad (2)$$

2 Multi-Section Coupled Line Coupler

- For $\theta = \pi/2$

$$\frac{V_3}{V_1} = jC\sin\left(\frac{\pi}{2}\right)e^{-j\left(\frac{\pi}{2}\right)} = jC\times(-j)$$
$$= -j^2C = C$$



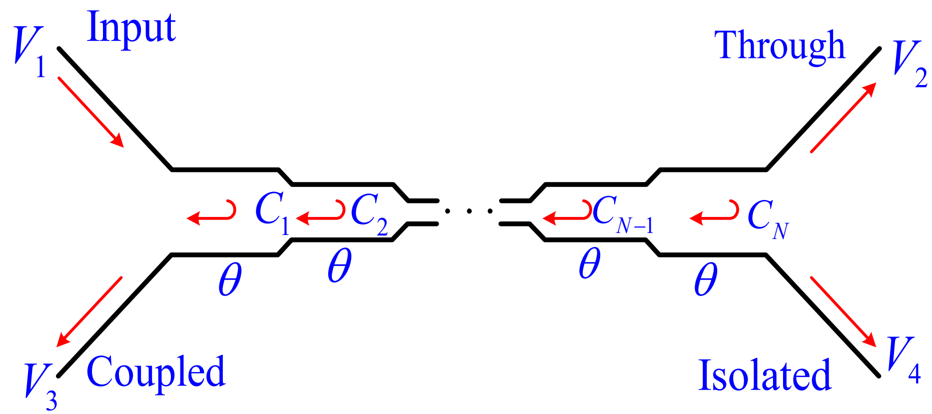
$$\frac{V_2}{V_1} = e^{-j\left(\frac{\pi}{2}\right)} = \cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right) = -j$$
$$\rightarrow \frac{V_2}{V_1} = -j$$

- No power is lost on the through path from one section to the next.
- *It is a good assumption for small C, even though power conservation law is violated*.

2 Multi-Section Coupled Line Coupler

- Total voltage at coupled port (port 3) of cascaded coupler

$$V_3 = \left\{ \begin{aligned} &\left(jC_1 \sin\theta e^{-j\theta}\right)V_1 + \left(jC_2 \sin\theta e^{-j\theta}\right)V_1 e^{-j2\theta} \\ &+ \cdots + \left(jC_N \sin\theta e^{-j\theta}\right)V_1 e^{-j2(N-1)\theta} \end{aligned} \right\} \quad (3)$$



where C_n: voltage coupling coefficient of n^{th} section

- If we assume that the coupler is symmetric,

$$C_1 = C_N,\ C_2 = C_{N-1},\ \cdots$$

- Then equation (3) can be simplified as

$$\begin{aligned} V_3 &= jV_1 \sin\theta e^{-j\theta} \left\{ C_1\left(1 + e^{-j2(N-1)\theta}\right) + C_2\left(e^{-j2\theta} + e^{-j2(N-2)\theta}\right) + \cdots + C_M e^{-j(N-1)\theta} \right\} \\ &= 2jV_1 \sin\theta e^{-jN\theta} \left\{ C_1\left(e^{j(N-1)\theta} + e^{-j(N-1)\theta}\right)/2 + C_2\left(e^{j(N-3)\theta} + e^{-j(N-3)\theta}\right)/2 + \cdots + C_M/2 \right\} \\ &= 2jV_1 \sin\theta e^{-jN\theta} \left\{ C_1 \cos(N-1)\theta + C_2 \cos(N-3)\theta + \cdots + C_M/2 \right\} \quad (4) \end{aligned}$$

where $M = (N+1)/2$

2 Multi-Section Coupled Line Coupler

- Coupling factor (C_0) at center frequency

$$C_0 = \left|\frac{V_3}{V_1}\right|_{\theta=\pi/2} \qquad (5)$$

- Wideband desired coupling characteristic can be obtained by choosing the coupling coefficients, C_n.
- *Equation (4) is in form of a Fourier series of coupling as frequency function.*
- Multi-section couplers of this form can achieve decade bandwidths, but coupling levels must be low.
- Because of the longer electrical length, it is more critical to have equal even- and odd-mode phase velocities than the single-section coupler.
- *Stripline is the preferred medium* for good coupler directivity.

3 Design Example of Multi-Section Coupled Line Coupler

- Design three-section 20 dB coupler with Butterworth response for system impedance of 50 Ω and center frequency of 3 GHz.

Solution

- For Butterworth response of a three section ($N = 3$) coupler

$$\left.\frac{d^n}{d\theta^n} C(\theta)\right|_{\theta=\frac{\pi}{2}} = 0 \quad \text{for } n = 1, 2$$

- From (4), we can write for $N = 3$ $\leftarrow V_3 = 2jV_1 \sin\theta e^{-jN\theta} \{C_1 \cos(N-1)\theta + C_2 \cos(N-3)\theta + \cdots + C_M / 2$

$$C = \left|\frac{V_3}{V_1}\right| = 2\sin\theta\left\{C_1 \cos 2\theta + \frac{C_2}{2}\right\} = 2C_1 \sin\theta \cos 2\theta + C_2 \sin\theta$$

$$= C_1(\sin 3\theta - \sin\theta) + C_2 \sin\theta = C_1 \sin 3\theta + (C_2 - C_1)\sin\theta$$

3 Design Example of Multi-Section Coupled Line Coupler

- First derivative

$$\frac{dC}{d\theta} = \frac{d}{d\theta}\left\{C_1 \sin 3\theta + \left(C_2 - C_1\right)\sin\theta\right\}\Big|_{\theta=\frac{\pi}{2}} = 3C_1\cos 3\theta + \left(C_2 - C_1\right)\cos\theta\Big|_{\theta=\frac{\pi}{2}} = 0$$

- Second derivative

$$\frac{d^2C}{d\theta^2} = \frac{d}{d\theta}\left\{3C_1 \cos 3\theta + \left(C_2 - C_1\right)\cos\theta\right\}\Big|_{\theta=\frac{\pi}{2}} = -9C_1\sin 3\theta - \left(C_2 - C_1\right)\sin\theta\Big|_{\theta=\frac{\pi}{2}} = 10C_1 - C_2 = 0$$

- At midband, $\theta = \pi/2$, $C_0 = 20$ dB

$$C = 10^{\left(\frac{-C_0}{20}\right)} = 10^{\left(\frac{-20}{20}\right)} = 0.1$$

$$C_1 \sin 3\theta + \left(C_2 - C_1\right)\sin\theta\Big|_{\theta=\frac{\pi}{2}} = 0.1$$ ← Total coupling for $N = 3$

$$C_2 - 2C_1 = 0.1 \qquad (6)$$

$$\frac{d^2C}{d\theta^2} = 0 \rightarrow 10C_1 - C_2 = 0 \qquad (7)$$

- Solving (6) and (7),

$$C_1 = C_3 = 0.0125$$

$$C_2 = 0.1 + 2C_1 = 0.1 + 2\times 0.0125 = 0.125$$

3 Design Example of Multi-Section Coupled Line Coupler

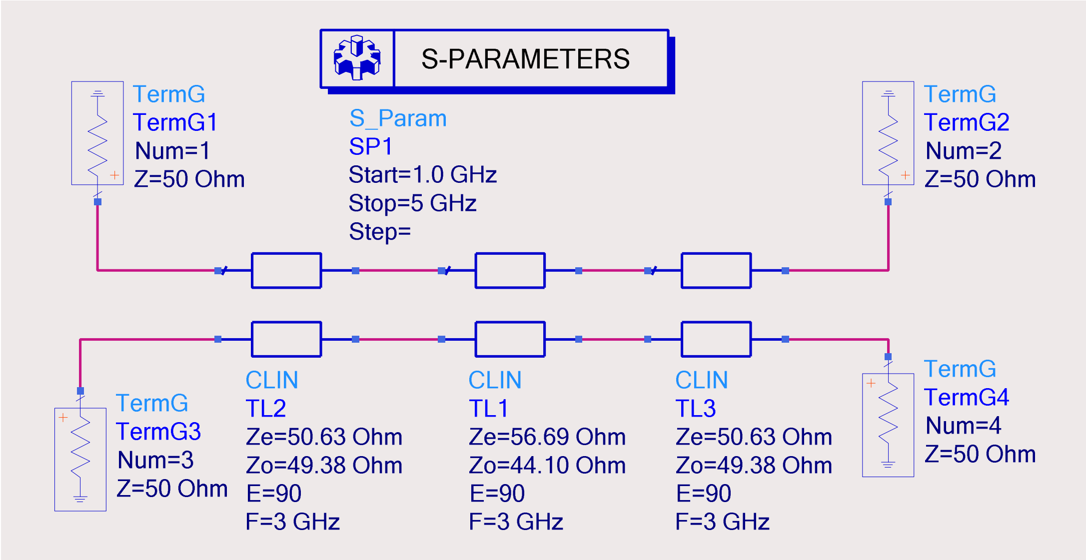
- Even and odd-mode impedances of each section

$$Z_{0e}^{1} = Z_{0e}^{3} = Z_0 \sqrt{\frac{1+C_1}{1-C_1}} = 50\sqrt{\frac{1+0.0125}{1-0.0125}} = 50.63$$

$$Z_{0o}^{1} = Z_{0o}^{3} = Z_0 \sqrt{\frac{1-C_1}{1+C_1}} = 50\sqrt{\frac{1-0.0125}{1+0.0125}} = 49.38$$

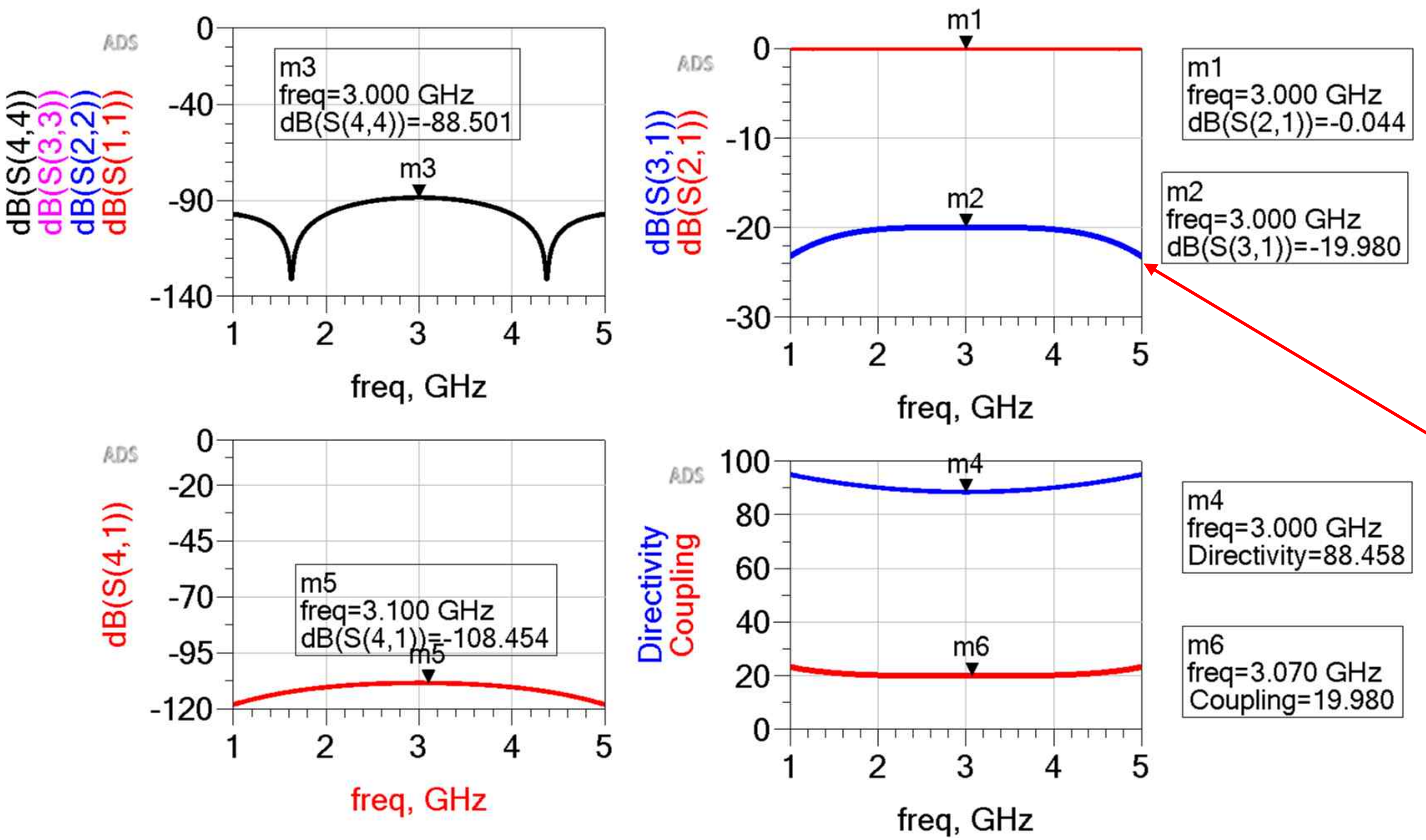
$$Z_{0e}^{2} = Z_0 \sqrt{\frac{1+C_2}{1-C_2}} = 50\sqrt{\frac{1+0.125}{1-0.125}} = 56.69$$

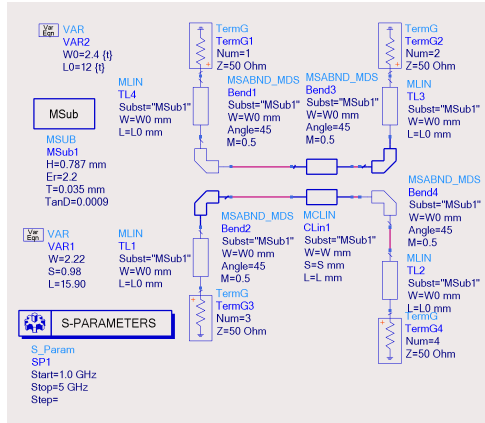
$$Z_{0o}^{2} = Z_0 \sqrt{\frac{1-C_2}{1+C_2}} = 50\sqrt{\frac{1-0.125}{1+0.125}} = 44.10$$



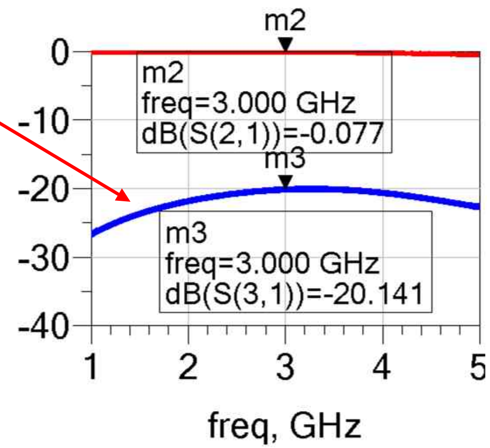
3 Design Example of Multi-Section Coupled Line Coupler

- Microwave simulator simulation results of 20-dB coupler with 3-sections
- Broader frequency characteristics than single section coupler



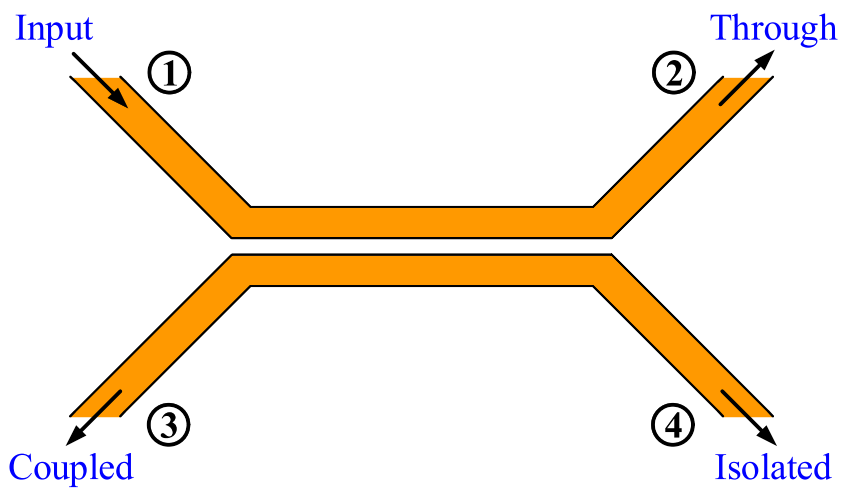


Vs.



4 Review

- Single-Section: coupled line directional coupler
  - Narrow band performances
  - Reflection, coupling, directivity, and isolation
- Design equations of coupled line coupler
  - Matched condition
  - Even- and odd-mode excitations



4 Review

- Multi-section coupled line coupler
  - Consists of several single section coupleds
  - For multi-section coupler design, loose coupling (C ≥ 10 dB) is assumed.
  - Multi-section couplers of this form can achieve decade bandwidths, but coupling levels must be low.
  - Because of longer electrical length, it is more critical to have equal even- and odd-mode phase velocities than single-section coupler.