Chapter 7
Power Divider and Directional Coupler

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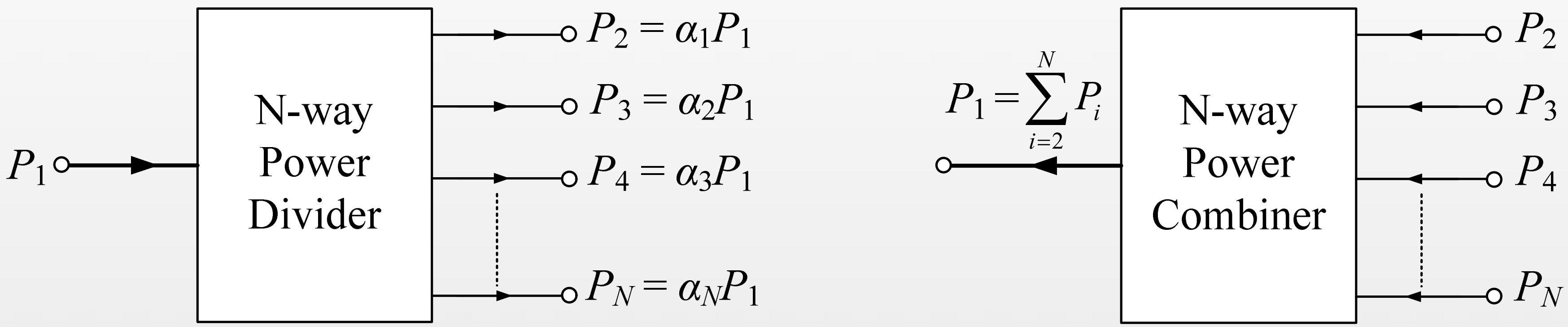


Learning Objectives

- Know about quadrature hybrid
- Analyze quadrature hybrid
- Know about some applications of quadrature hybrid

Learning contents

- Quadrature hybrid
- Analysis of quadrature hybrid
- Applications of quadrature hybrid

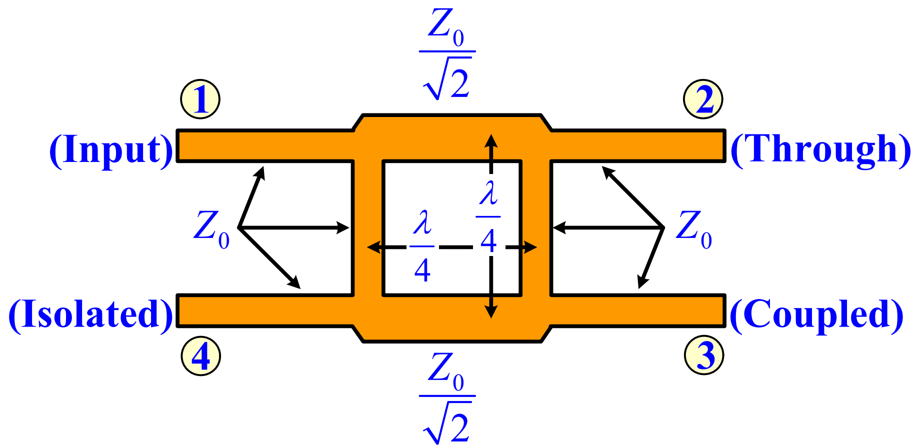
1 Quadrature (90º) Hybrid

- **Quadrature hybrid** (also known **branch-line hybrid**) is a 4-port device that split an input signal equally/unequally between two output ports (through and coupled ports) with 90º phase difference.
- Often made in microstrip line or stripline form.
- The structure of this hybrid is high degree of symmetry. → Each port can be used as input port.
- If port 1 is an input port, then

  - Port 2: through port

  - Port 3: coupled port with a 90º phase shift

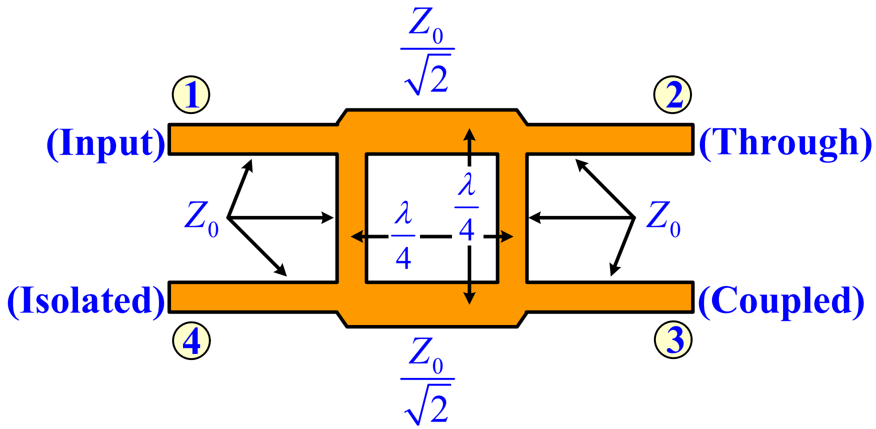
  - Port 4: isolated port
- There is no power coupled to port 4 ideally.



1 Quadrature (90^0) Hybrid

- The output ports will always be on the opposite side of the junction from the input port.
- The isolated port will be the remaining port on the same side as the input port.
- This symmetry is reflected in the scattering matrix, as each row can be obtained as a transposition of the first row.
- Scattering matrix of quadrature hybrid

$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix} \qquad (1)$$



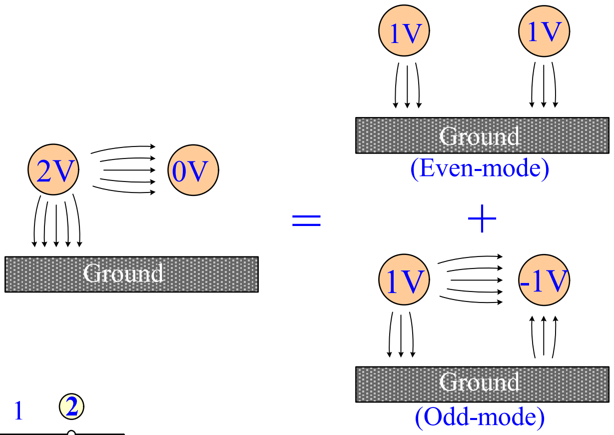
2 Analysis of Quadrature Hybrid

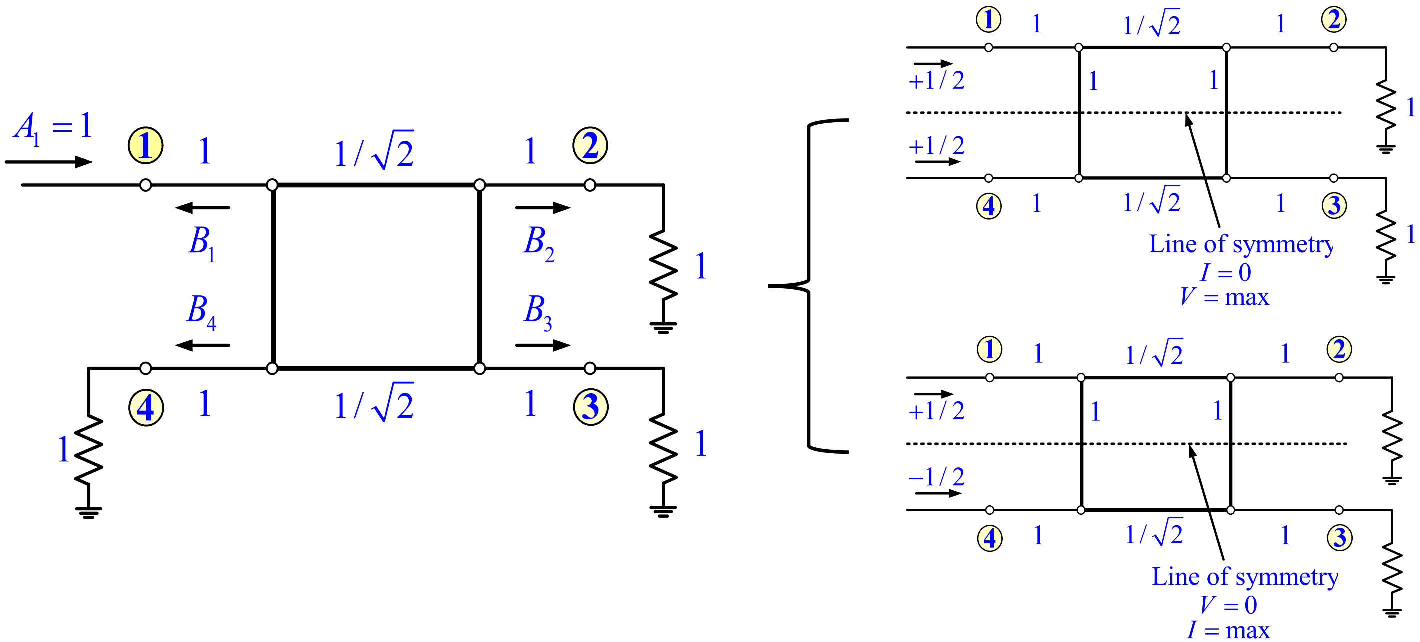
- **Even-Odd Mode Analysis**

  - The branch-line coupler is normalized to Z_0 as shown in below figure.

  - Unit amplitude of input voltage wave: $A_1 = 1$

  - Decomposed into the superpositions of even- and odd-mode excitations.

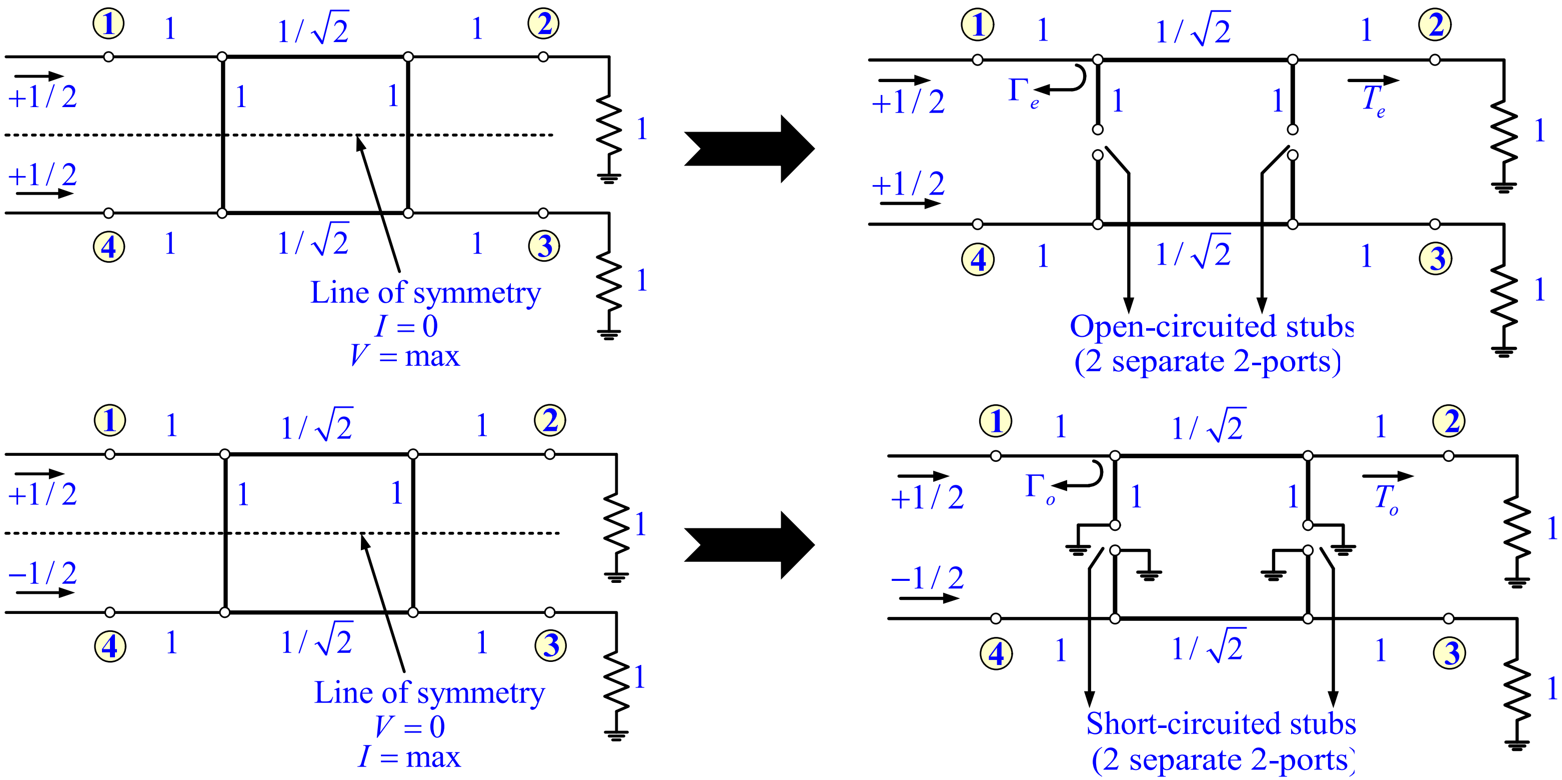




2 Analysis of Quadrature Hybrid

- **Even-Odd Mode Analysis**

  - The actual response (of the scattered waves) can be obtained from the sum of the responses to the even- and odd-mode excitations.



2 Analysis of Quadrature Hybrid

- Since the amplitudes of the incident waves for these two-ports are $\pm 1/2$, the amplitudes of the emerging wave at each port can be expressed as;

$$B_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o \qquad \text{(2a)}$$

$$B_2 = \frac{1}{2}T_e + \frac{1}{2}T_o \qquad \text{(2b)}$$

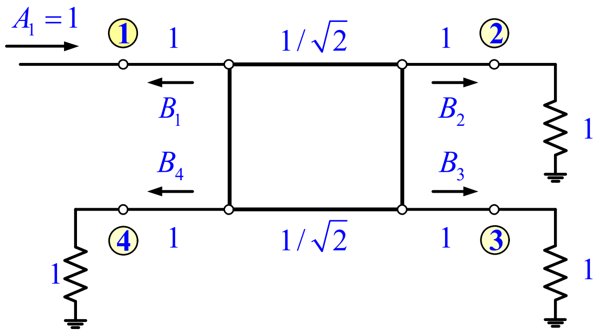
$$B_3 = \frac{1}{2}T_e - \frac{1}{2}T_o \qquad \text{(2c)}$$

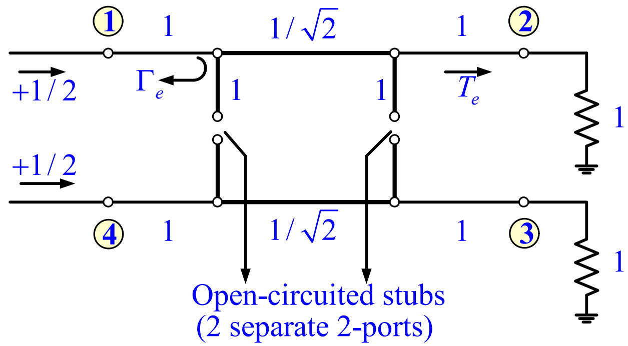
$$B_4 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o, \qquad \text{(2d)}$$

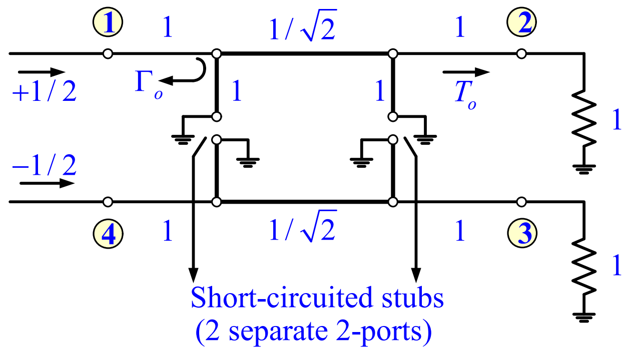
where

Γ_e and Γ_o: even- and odd-mode reflection coefficients

T_e and T_o: even- and odd-mode transmission coefficients



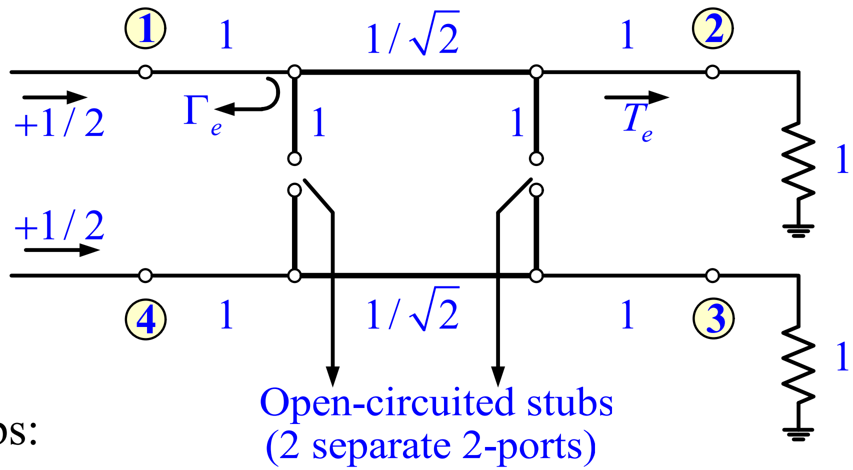




2 Analysis of Quadrature Hybrid

- Even-mode *ABCD*-matrices of each cascade component:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \underbrace{\begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix}}_{\substack{\text{shunt} \\ \text{arm} \\ Y=j}} \underbrace{\begin{bmatrix} 0 & j/\sqrt{2} \\ j\sqrt{2} & 0 \end{bmatrix}}_{\substack{\lambda/4 \\ \text{Transmission} \\ \text{line}}} \underbrace{\begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix}}_{\substack{\text{shunt} \\ \text{arm} \\ Y=j}}$$



- Admittance of shunt open-circuited $\lambda/8$ normalized stubs:

$$Y = j\tan\beta l = j\tan\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{8}\right) = j$$

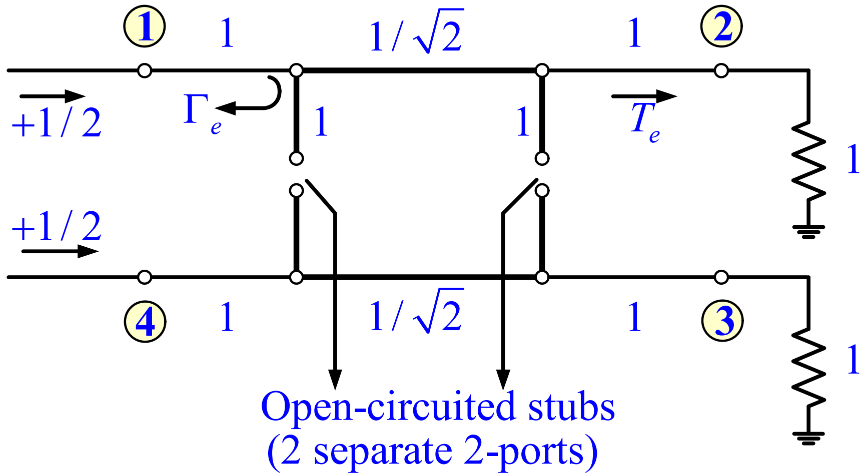
$$\Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & j \\ j & -1 \end{bmatrix} \qquad (3)$$

2 Analysis of Quadrature Hybrid

- Using conversion from *ABCD*-parameters to *S*-parameters with $Z_0 = 1$, the reflection and transmission coefficients can be defined as below;

$$\Gamma_e = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} = \frac{A + B - C - D}{A + B + C + D} = \frac{(-1 + j - j + 1)/\sqrt{2}}{(-1 + j + j - 1)/\sqrt{2}} = 0 \qquad (4a)$$

$$T_e = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{A + B + C + D} = \frac{2}{(-1 + j + j - 1)/\sqrt{2}} = \frac{-1}{\sqrt{2}}(1 + j) \qquad (4b)$$



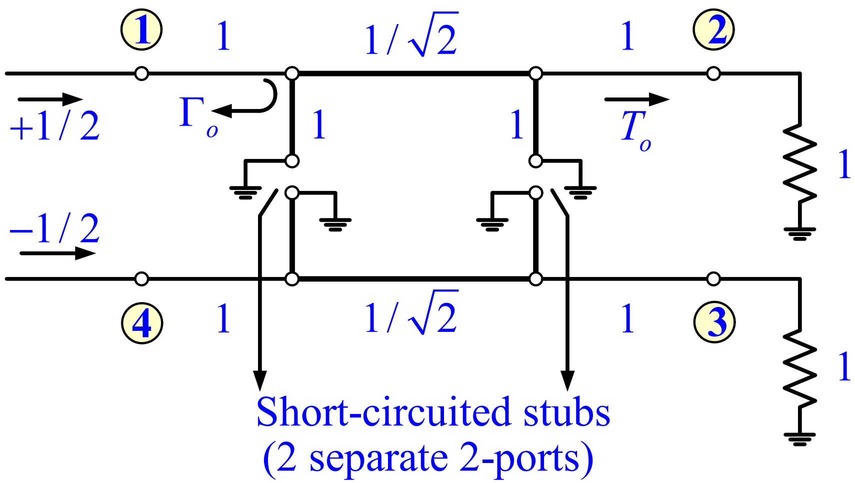
$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D}, \quad S_{12} = \frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D}$$

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D}, \quad S_{22} = \frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D}$$

2 Analysis of Quadrature Hybrid

- Odd-mode $ABCD$-matrix:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \underbrace{\begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix}}_{\substack{\text{shunt} \\ \text{arm} \\ Y=-j}} \underbrace{\begin{bmatrix} 0 & j/\sqrt{2} \\ j\sqrt{2} & 0 \end{bmatrix}}_{\substack{\lambda/4 \\ \text{Transmission} \\ \text{line}}} \underbrace{\begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix}}_{\substack{\text{shunt} \\ \text{arm} \\ Y=-j}}$$



- admittance of shunt short-circuited $\lambda/8$ normalized stubs

$$Y = -j\tan\beta l = -j\tan\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{8}\right) = -j$$

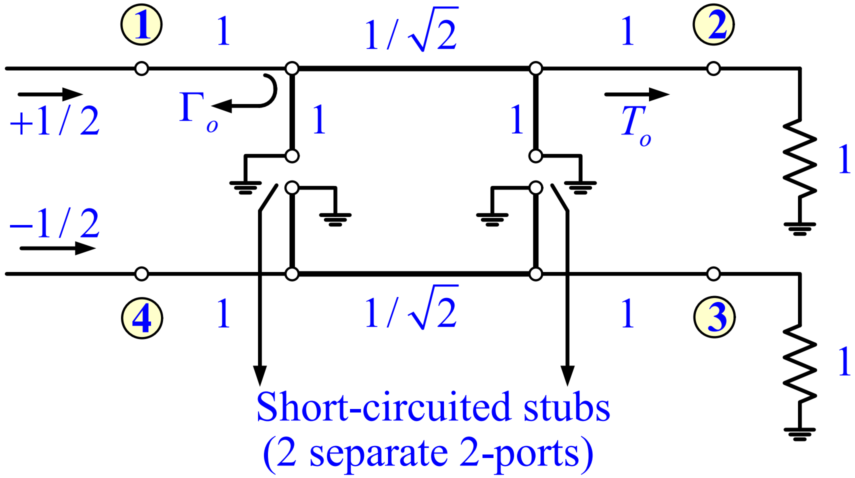
$$\Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix} \qquad (5)$$

2 Analysis of Quadrature Hybrid

- Using conversion from *ABCD*-parameters to *S*-parameters with $Z_0 = 1$, the reflection and transmission coefficients can be defined as below;

$$\Gamma_o = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} = \frac{A+B-C-D}{A+B+C+D} = \frac{(1+j-j-1)/\sqrt{2}}{(1+j+j+1)/\sqrt{2}} = 0 \qquad (6a)$$

$$T_o = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{A+B+C+D} = \frac{2}{(1+j+j+1)/\sqrt{2}} = \frac{1}{\sqrt{2}}(1-j) \qquad (6b)$$



$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D}, \quad S_{12} = \frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D}$$

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D}, \quad S_{22} = \frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D}$$

2 Analysis of Quadrature Hybrid

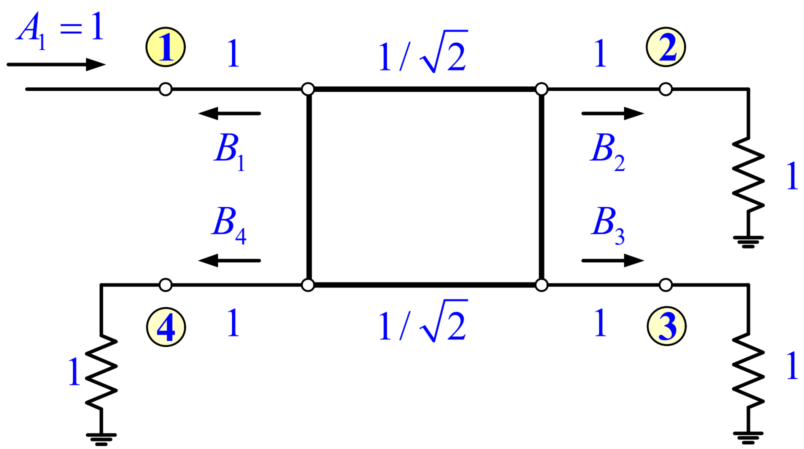
- Using (2) with (4) and (6),

$$B_1 = 0 \quad \text{: port 1 is matched} \tag{7a}$$

$$B_2 = -\frac{j}{\sqrt{2}} \quad \text{: half-power, } -90^{\circ} \text{ phase shift from ports 1 to 2} \tag{7b}$$

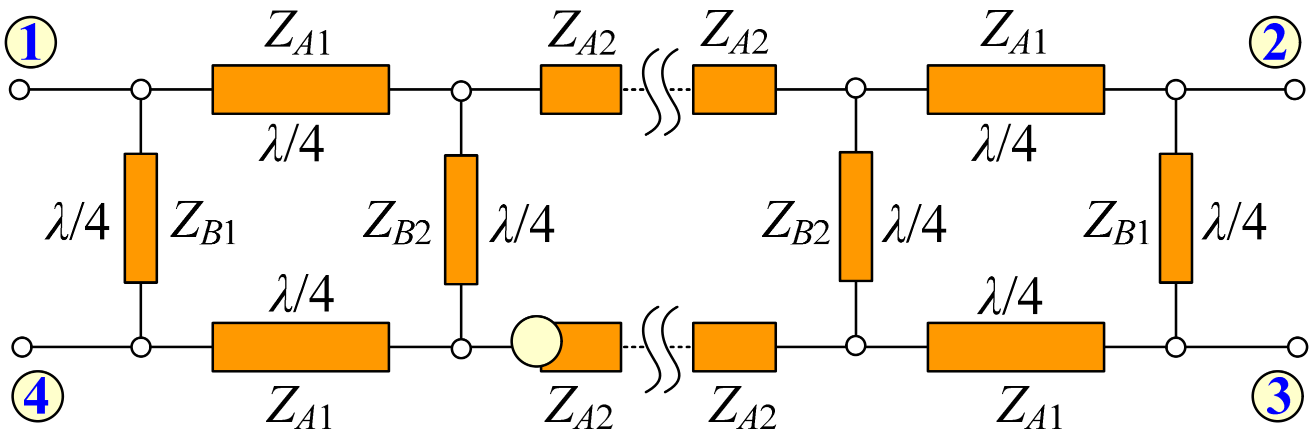
$$B_3 = -\frac{1}{\sqrt{2}} \quad \text{: half-power, } -180^{\circ} \text{ phase shift from ports 1 to 3} \tag{7c}$$

$$B_4 = 0 \quad \text{: no power dilivered to port 4} \tag{7d}$$



2 Analysis of Quadrature Hybrid

- Due to the quarter-wavelength characteristics, the bandwidth of a branch-line hybrid is limited about 10 ~ 20%.
- Multi-section quadrature hybrid is required for broader bandwidth.
- The structure of multi-section quadrature hybrid should be symmetric and the electrical lengths of all transmission lines must be $\lambda/4$.

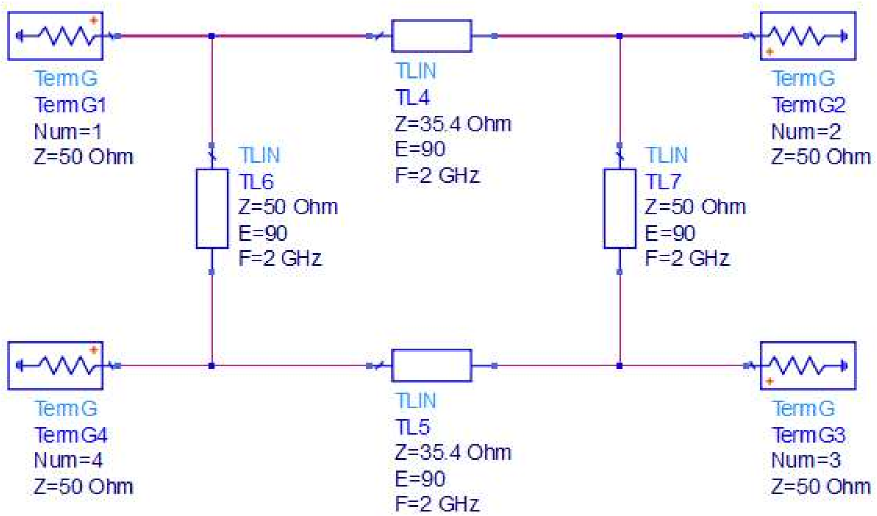


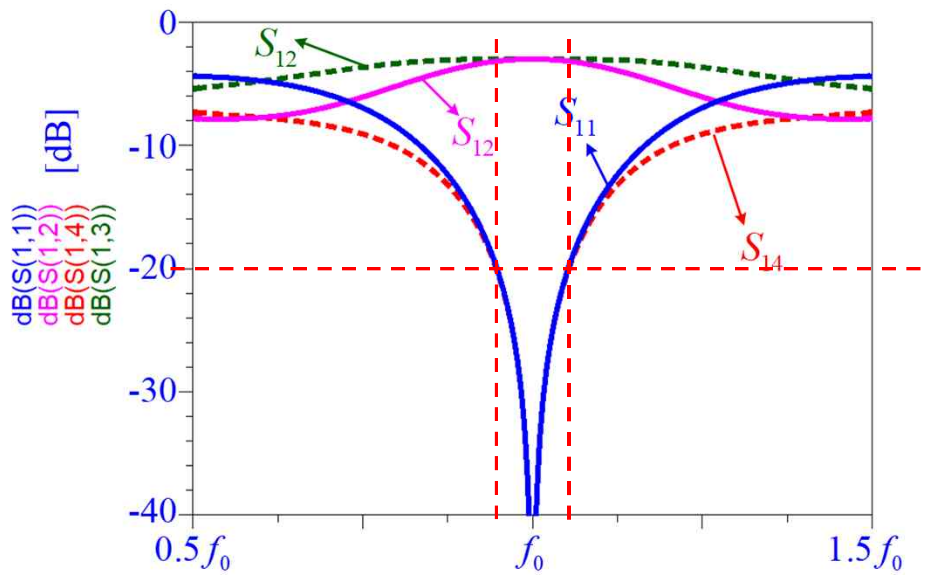
2 Analysis of Quadrature Hybrid

- **Example:** Design a 50 Ω branch-line quadrature hybrid at f_0 = 2 GHz and plot the scattering parameter magnitudes from $0.5f_0$ to$1.5f_0$.

Solution

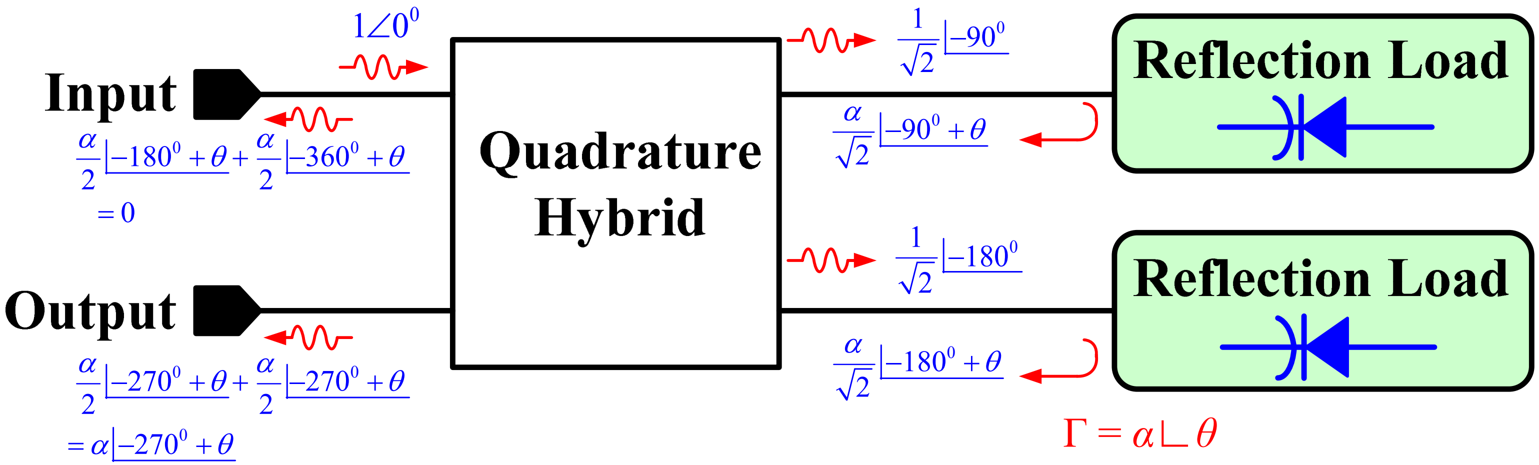
- Line length of each branch is $\lambda/4$, and branch-line impedances are: $\dfrac{Z_0}{\sqrt{2}} = \dfrac{50}{\sqrt{2}} = 35.4[\Omega]$





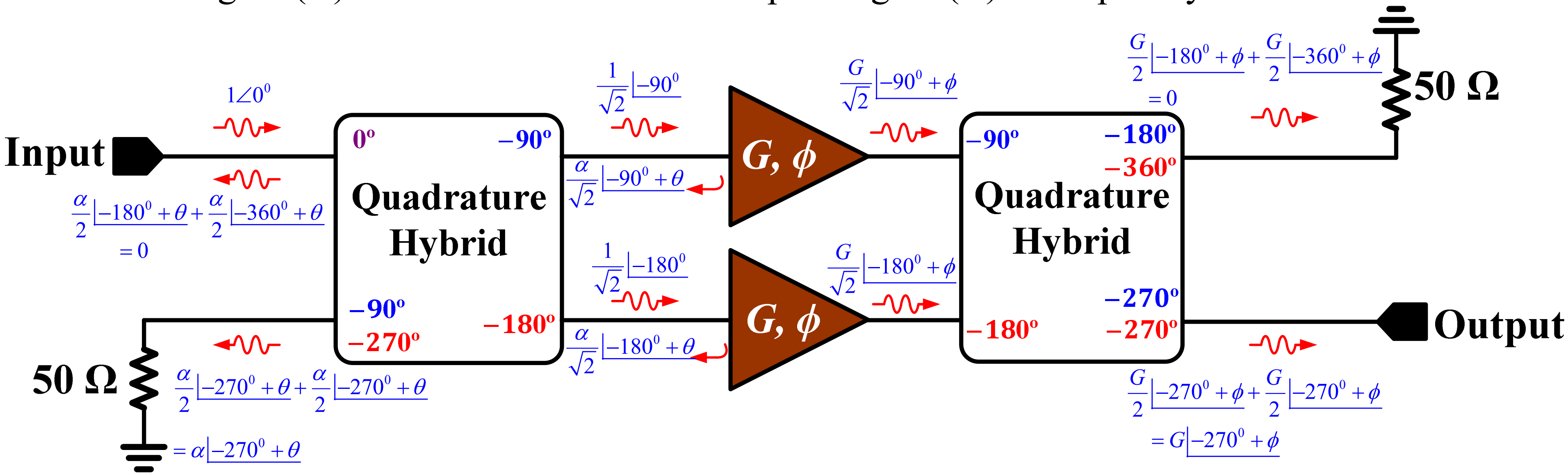
3 Some Applications of Quadrature Hybrid

- **Reflective-type microwave circuit structure**
  - Input signal is delivered to coupled and through port. Reflected signals from identical reflection load are combined at the isolation port of hybrid (port 4).
  - Perfect reflection characteristics
  - Relatively small insertion loss due to quadrature hybrid
  - Overall circuit gain (α) is same with reflection coefficient of individual reflection load (α) conceptually.
  - Variable attenuator using PIN diodes, variable phase shifter using varactor diode, etc.



3 Some Applications of Quadrature Hybrid

- **Balanced microwave circuit structure**
  - Typical example: balanced amplifier
  - Input signal is delivered to coupled and through port. Most divided signals will be amplified by amplifier and some portion of input signal will be reflected and delivered to isolation load (50 Ω).
  - Two amplified signals are combined at output port. → Twice output power
  - Perfect isolation with neighbor circuits.
  - Overall circuit gain (*G*) is same with individual amplifier gain (*G*) conceptually.



4 Review

- Quadrature hybrid
- Even- and odd-mode analysis
- Applications of quadrature hybrid