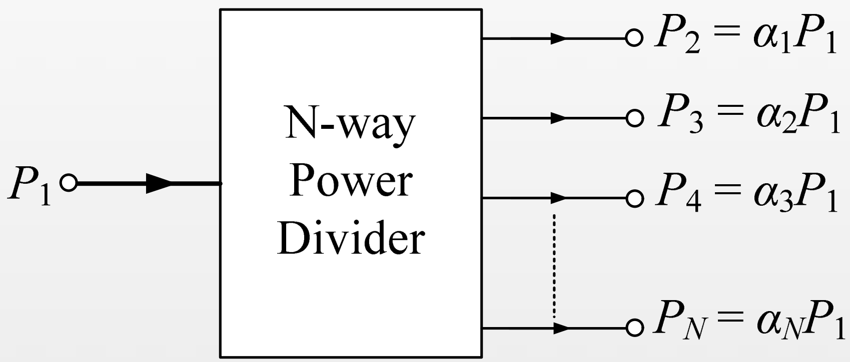
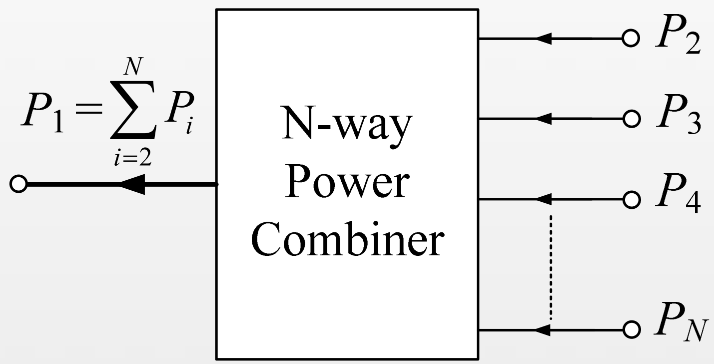
Chapter 7
Power Divider and Directional Coupler

Prof. Jeong, Yongchae





Learning Objectives

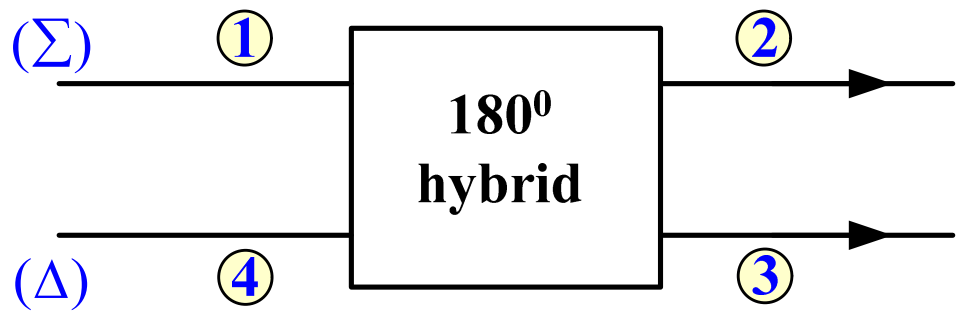
- Know about ring hybrid or 180^o hybrid
- Analyze ring hybrid
- Learn application of ring hybrid

Learning contents

- Ring hybrid
- Analysis of ring hybrid
- Application of Ring Hybrid

1 Ring Hybrid

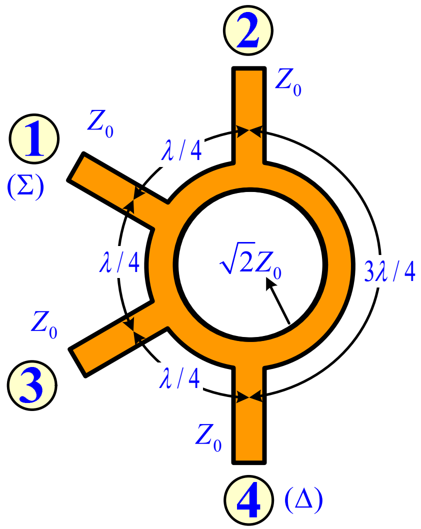
- The ring hybrid (or 180° hybrid, rat-race hybrid) is a 4-port network with 0° or 180° phase difference between the two output ports according to incident port operations.
- A signal applied to port 1 will be evenly split into two in-phased signals at ports 2 and 3, while port 4 is isolated.
- A signal applied to port 4 will be evenly split in two out-of-phased signals at ports 2 and 3, while port 1 is isolated.
- If in-phased signals are applied to ports 2 and 3, it operates as a combiner as summation at port 1 and difference at port 4.
- If out-of-phased signals are applied to ports 2 and 3, it operates as a combiner as summation at port 4 and difference at port 1.



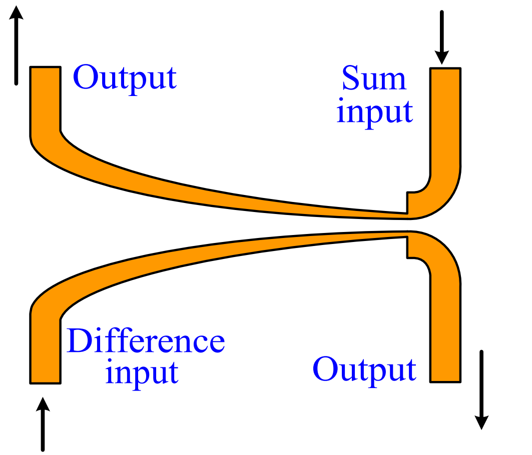
1 Ring Hybrid

- Scattering parameters of ring hybrid

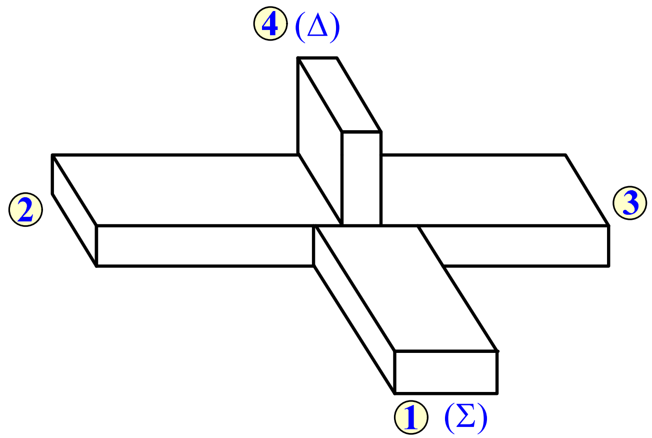
$$[S] = \frac{-j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \qquad (1)$$



(Microstrip ring hybrid)



(Tappered coupled line hybrid)

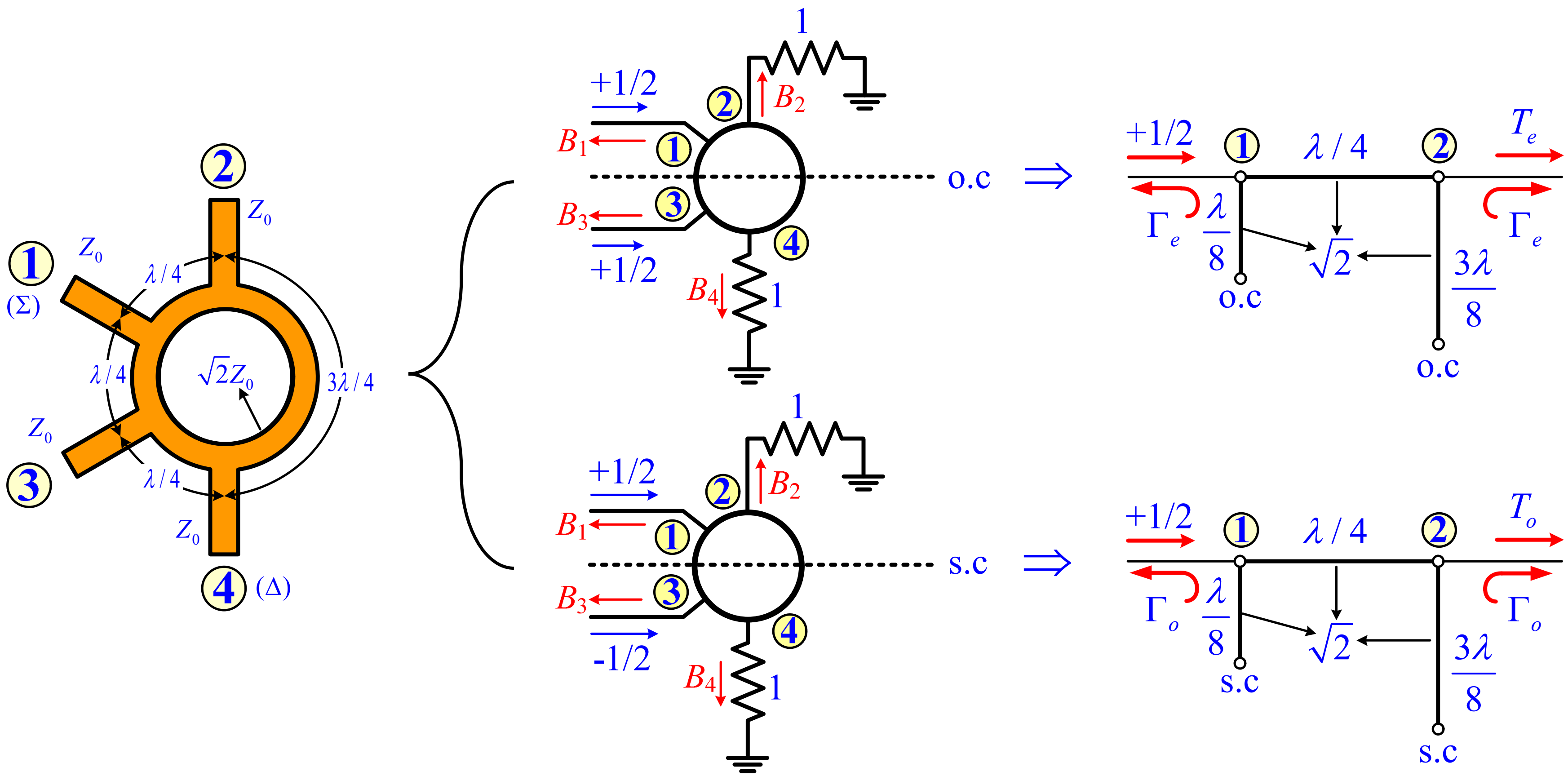


(Magic-T)

2 Analysis of Ring Hybrid

- **Even-Odd Mode Analysis:**

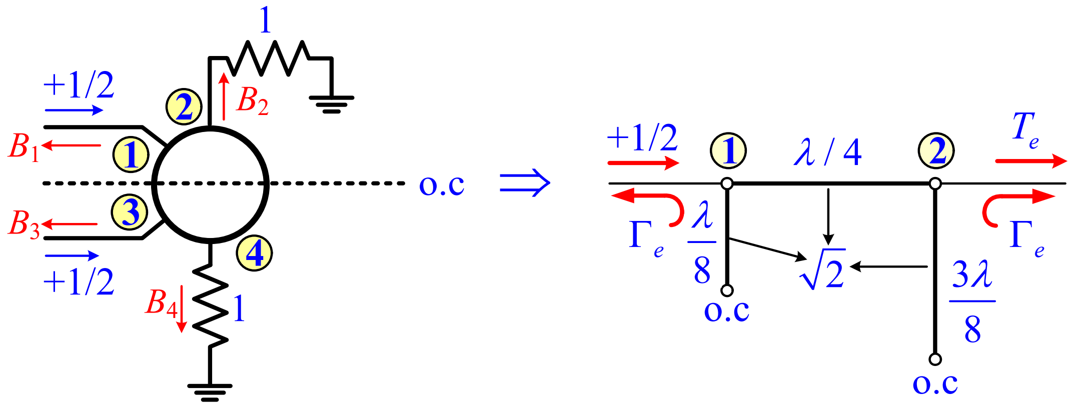
  - Ring hybrid can be analyzed with its even- and odd-mode circuits due to symmetrical structure.



2 Analysis of Ring Hybrid

- Consider **a unit amplitude wave incident at port 1 (summation port)**.

- Even- and odd-mode decompositions of ring hybrid in condition a unit amplitude incident wave is excited at port 1



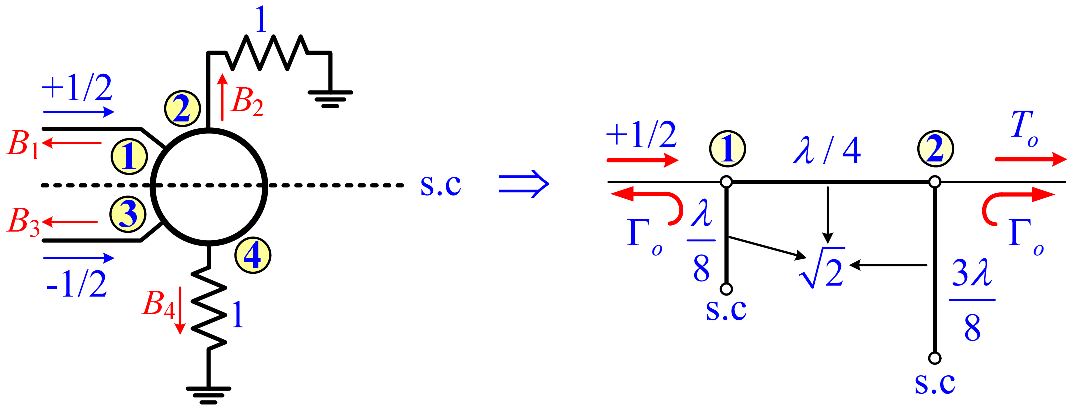
- Amplitudes of scattered waves at each port

$$B_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o \qquad (2a)$$

$$B_2 = \frac{1}{2}T_e + \frac{1}{2}T_o \qquad (2b)$$

$$B_3 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o \qquad (2c)$$

$$B_4 = \frac{1}{2}T_e - \frac{1}{2}T_o \qquad (2d)$$

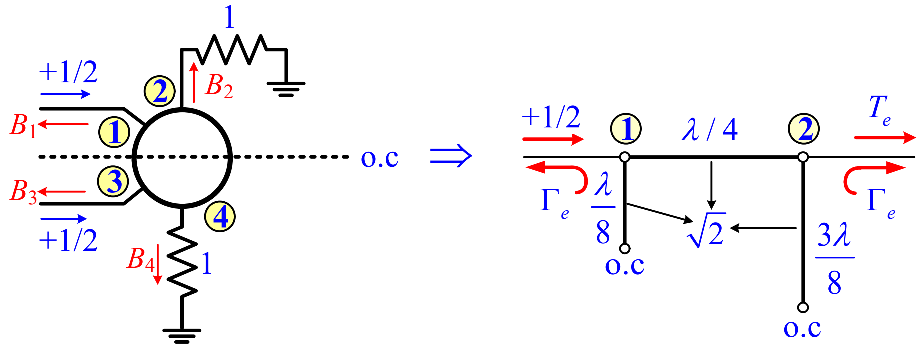


2 Analysis of Ring Hybrid

- *ABCD*-matrix for even- and odd- mode two-port circuits:

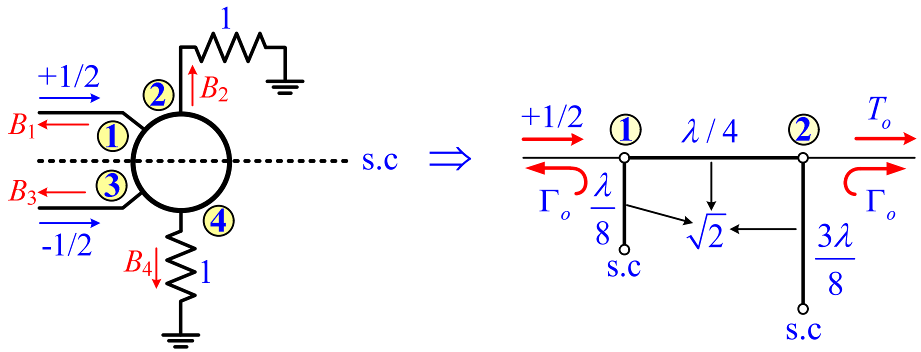
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & j\sqrt{2} \\ j\sqrt{2} & -1 \end{bmatrix} \qquad (3a)$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & j\sqrt{2} \\ j\sqrt{2} & 1 \end{bmatrix} \qquad (3b)$$

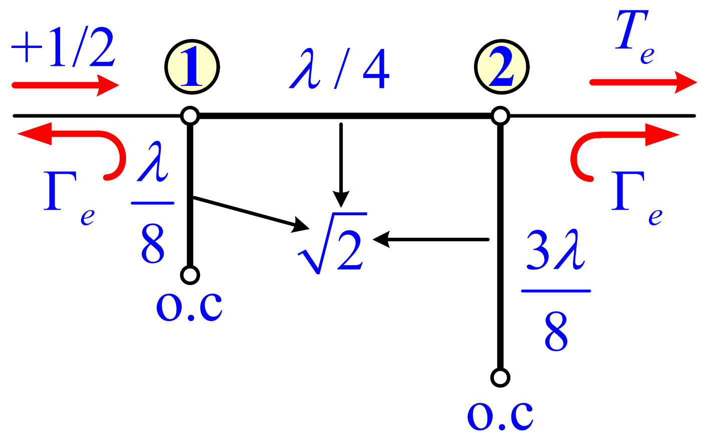


2 Analysis of Ring Hybrid

- Using conversion from *ABCD*-parameters to *S*-parameters with Z_0 = 1, the reflection and transmission coefficients of even-mode two-port circuits can be obtained as:

$$\Gamma_e = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} = \frac{1 + j\sqrt{2} - j\sqrt{2} + 1}{1 + j\sqrt{2} + j\sqrt{2} - 1} = \frac{-j}{\sqrt{2}} \qquad (4a)$$

$$T_e = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{1 + j\sqrt{2} + j\sqrt{2} - 1} = \frac{-j}{\sqrt{2}} \qquad (4b)$$



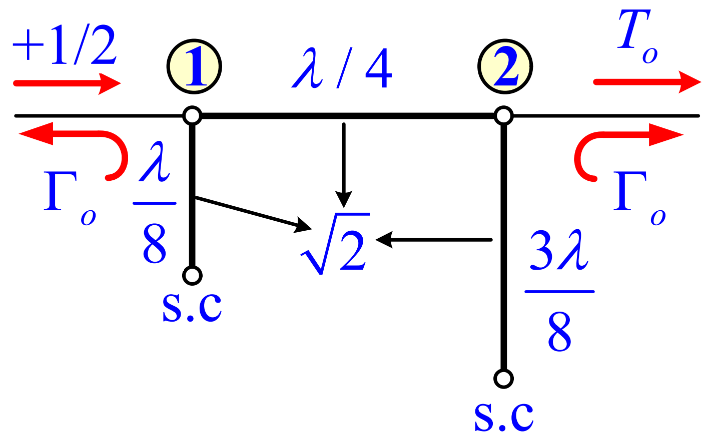
$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D}, \quad S_{12} = \frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D}$$

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D}, \quad S_{22} = \frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D}$$

- Using conversion from *ABCD*-parameters to *S*-parameters with $Z_0 = 1$, the reflection and transmission coefficients of odd-mode two-port circuits can be obtained as:

$$\Gamma_o = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} = \frac{-1 + j\sqrt{2} - j\sqrt{2} - 1}{-1 + j\sqrt{2} + j\sqrt{2} + 1} = \frac{j}{\sqrt{2}} \qquad (4c)$$

$$T_o = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{-1 + j\sqrt{2} + j\sqrt{2} + 1} = \frac{-j}{\sqrt{2}} \qquad (4d)$$



$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D}, \quad S_{12} = \frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D}$$

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D}, \quad S_{22} = \frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D}$$

2 Analysis of Ring Hybrid

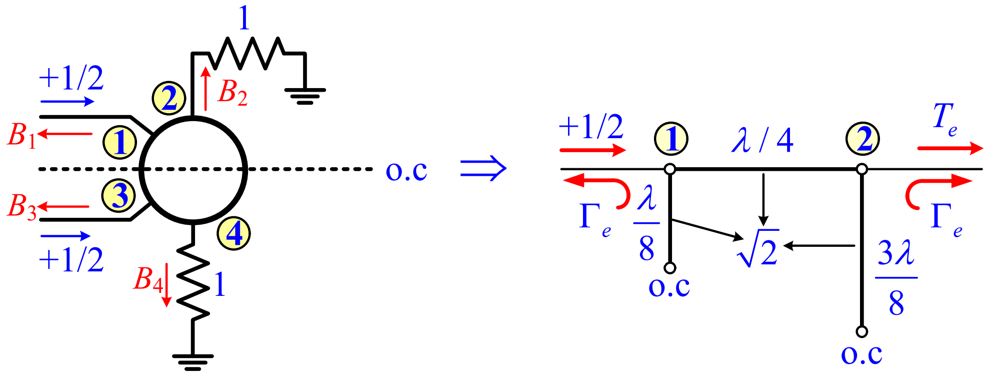
- Substitute (4) into (2), then

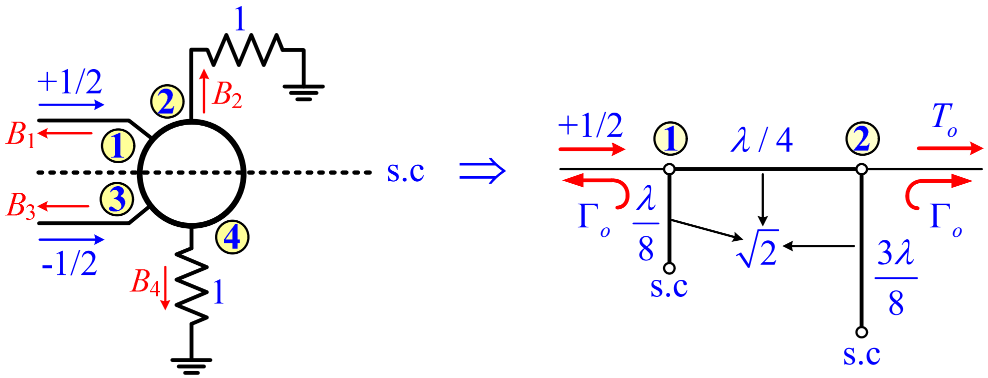
$$B_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o = \frac{-j}{2\sqrt{2}} + \frac{j}{2\sqrt{2}} = 0 \qquad (5a)$$

$$B_2 = \frac{1}{2}T_e + \frac{1}{2}T_o = \frac{-j}{2\sqrt{2}} + \frac{-j}{2\sqrt{2}} = \frac{-j}{\sqrt{2}} \qquad (5b)$$

$$B_3 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o = \frac{-j}{2\sqrt{2}} - \frac{j}{2\sqrt{2}} = \frac{-j}{\sqrt{2}} \qquad (5c)$$

$$B_4 = \frac{1}{2}T_e - \frac{1}{2}T_o = \frac{-j}{2\sqrt{2}} - \frac{-j}{2\sqrt{2}} = 0 \qquad (5d)$$





- The input port (port 1) is matched.

- Port 4 is isolated.

- Input power is evenly divided into ports 2 and 3 with in-phase (0° phase difference).

- Typical bandwidth of ring hybrid: 20 ~ 30%. → limited by frequency dependence of line lengths.

2 Analysis of Ring Hybrid

- Consider **a unit amplitude wave incident at port 4 (difference port)**.
- Even- and odd-mode decompositions of ring hybrid in condition a unit amplitude incident wave is excited at port 4

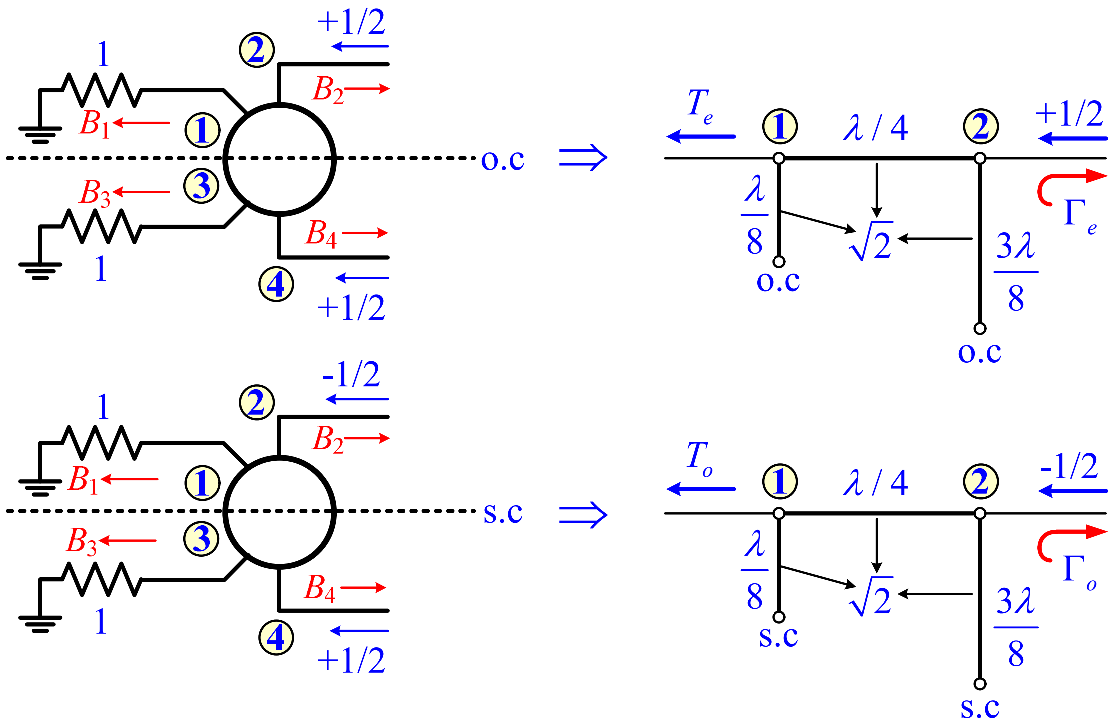
- Amplitudes of scattered waves at each port

$$B_1 = \frac{1}{2}T_e - \frac{1}{2}T_o \qquad (6a)$$

$$B_2 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o \qquad (6b)$$

$$B_3 = \frac{1}{2}T_e + \frac{1}{2}T_o \qquad (6c)$$

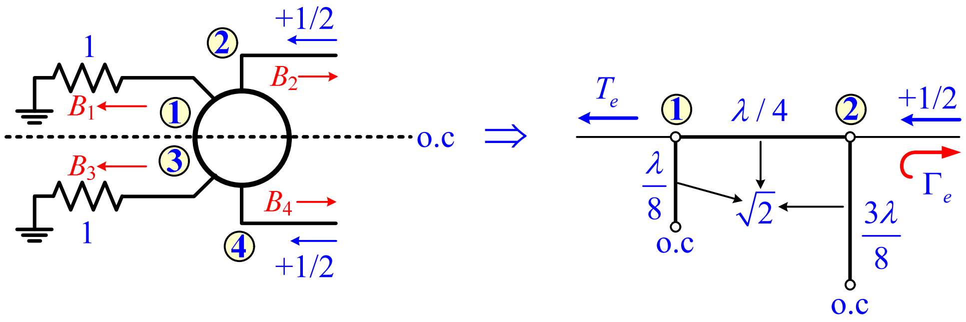
$$B_4 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o \qquad (6d)$$



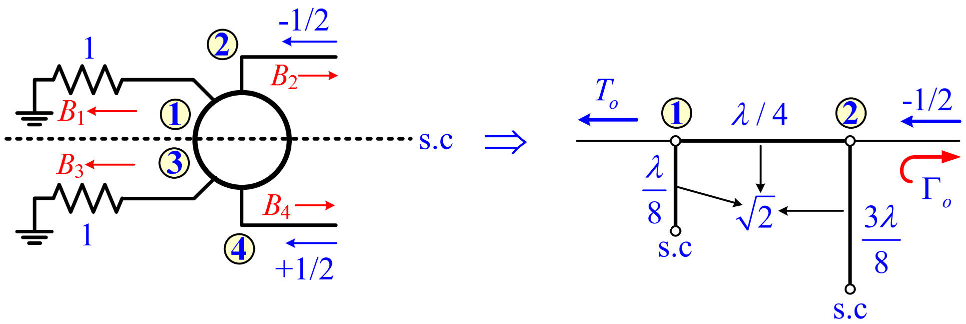
2 Analysis of Ring Hybrid

- *ABCD*-matrix for the even- and odd- mode two-port circuits:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{bmatrix} = \begin{bmatrix} -1 & j\sqrt{2} \\ j\sqrt{2} & 1 \end{bmatrix} \quad (7a)$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & j\sqrt{2} \\ j\sqrt{2} & -1 \end{bmatrix} \quad (7b)$$

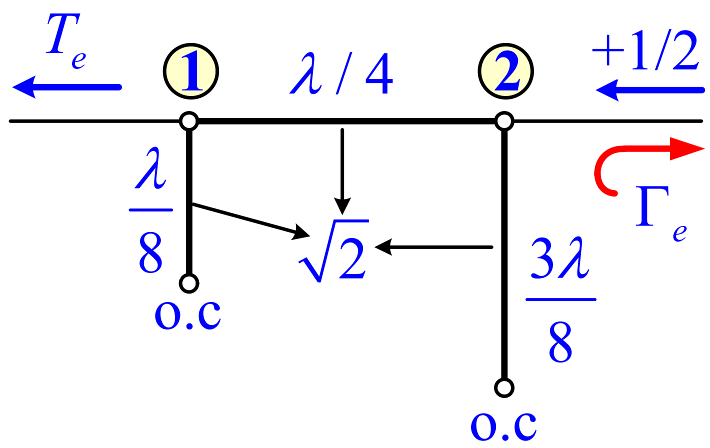


2 Analysis of Ring Hybrid

- Using conversion from *ABCD*-parameters to *S*-parameters with $Z_0 = 1$, the reflection and transmission coefficients of even-mode two-port circuits can be obtained as:

$$\Gamma_e = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} = \frac{-1 + j\sqrt{2} - j\sqrt{2} - 1}{-1 + j\sqrt{2} + j\sqrt{2} + 1} = \frac{j}{\sqrt{2}} \tag{8a}$$

$$T_e = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{-1 + j\sqrt{2} + j\sqrt{2} + 1} = \frac{-j}{\sqrt{2}} \tag{8b}$$



$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D}, \quad S_{12} = \frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D}$$

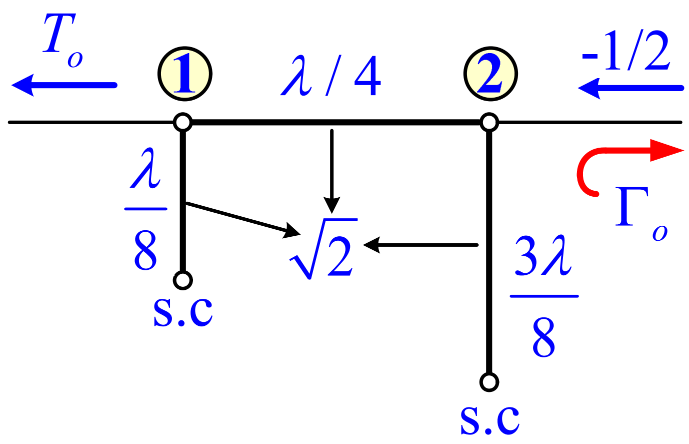
$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D}, \quad S_{22} = \frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D}$$

2 Analysis of Ring Hybrid

- Using conversion from *ABCD*-parameters to *S*-parameters with $Z_0 = 1$, the reflection and transmission coefficients of odd-mode two-port circuits can be obtained as:

$$\Gamma_o = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} = \frac{1 + j\sqrt{2} - j\sqrt{2} + 1}{1 + j\sqrt{2} + j\sqrt{2} - 1} = \frac{-j}{\sqrt{2}} \quad \text{(8c)}$$

$$T_o = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{1 + j\sqrt{2} + j\sqrt{2} - 1} = \frac{-j}{\sqrt{2}} \quad \text{(8d)}$$



$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D}, \quad S_{12} = \frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D}$$

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D}, \quad S_{22} = \frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D}$$

2 Analysis of Ring Hybrid

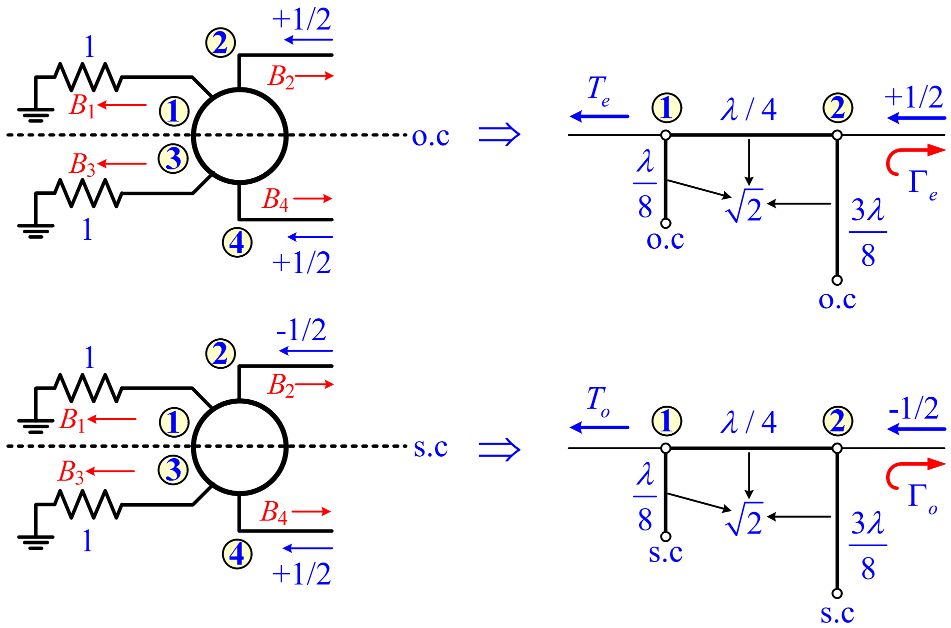
- Substitute (8) in (6), then

$$B_1 = \frac{1}{2}T_e - \frac{1}{2}T_o = \frac{-j}{2\sqrt{2}} - \frac{-j}{2\sqrt{2}} = 0 \qquad (9a)$$

$$B_2 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o = \frac{j}{2\sqrt{2}} - \frac{-j}{2\sqrt{2}} = \frac{j}{\sqrt{2}} \qquad (9b)$$

$$B_3 = \frac{1}{2}T_e + \frac{1}{2}T_o = \frac{-j}{2\sqrt{2}} + \frac{-j}{2\sqrt{2}} = \frac{-j}{\sqrt{2}} \qquad (9c)$$

$$B_4 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o = \frac{j}{2\sqrt{2}} + \frac{-j}{2\sqrt{2}} = 0 \qquad (9d)$$



- The input port (port 4) is matched.

- Port 1 is isolated.

- Input power is evenly divided into ports 2 and 3 with a 180° phase difference.

- Typical bandwidth of ring hybrid: 20 ~ 30%. → limited by frequency dependence of line lengths.

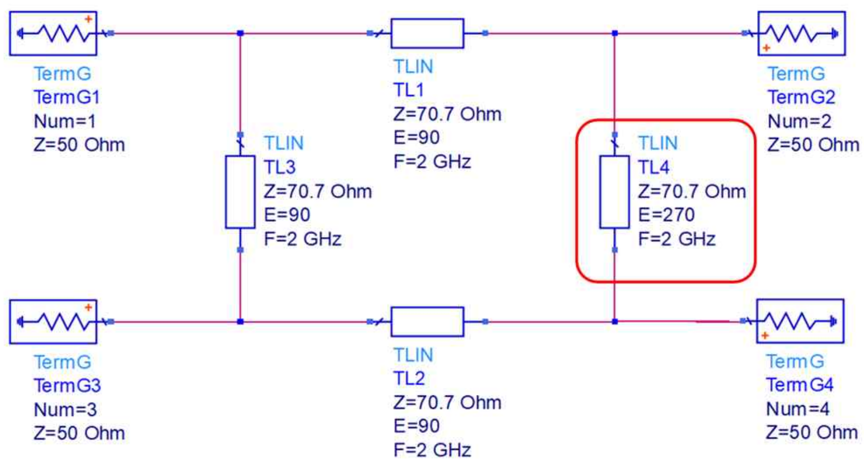
② Analysis of Ring Hybrid

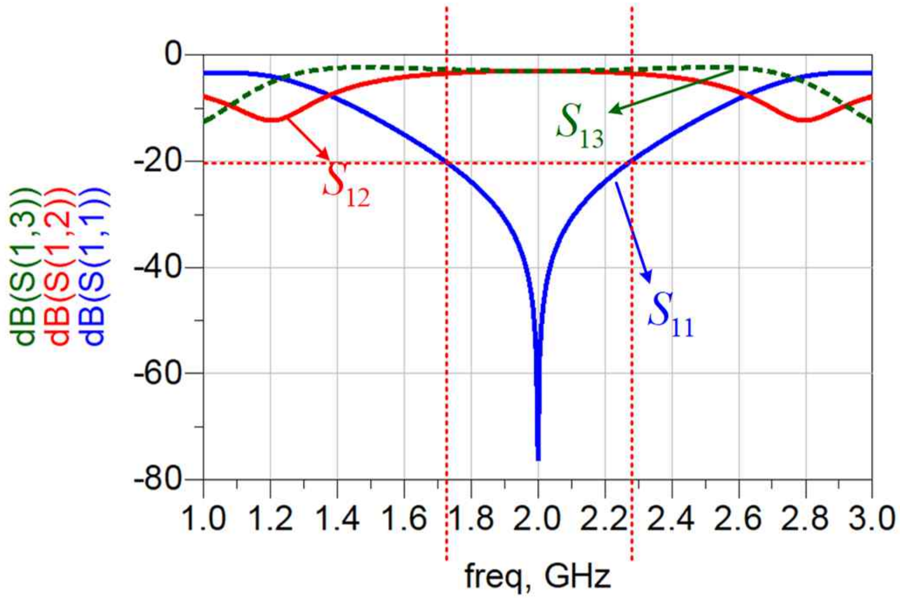
- **Example:** Design a ring hybrid for a 50 Ω system impedance and plot the magnitude of the *S*-parameters (S_{ij}) from $0.5f_0$ to $1.5f_0$, where f_0 is the design frequency.

Solution:

- Characteristic impedance of the ring transmission line: $\sqrt{2}Z_0 = 70.7\,\Omega$

- Feedline impedance: 50 Ω

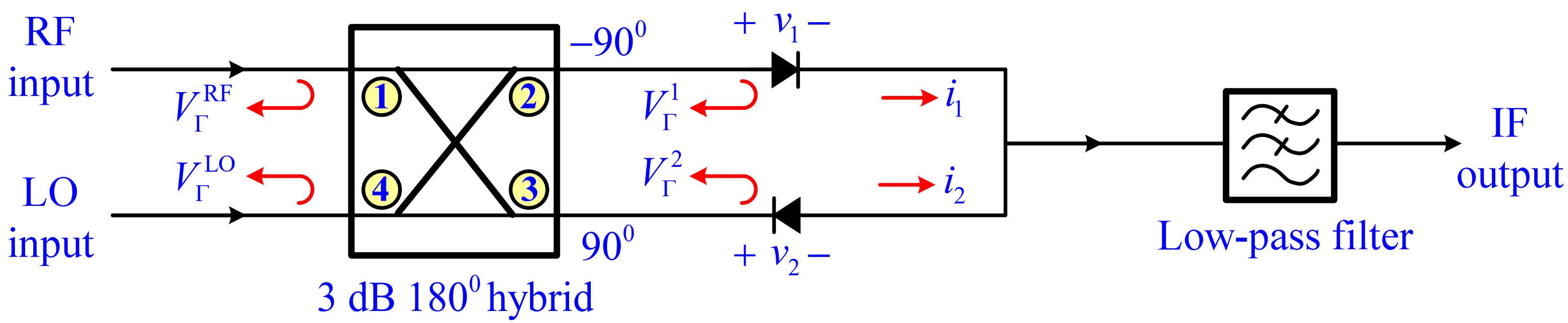




3 Application of Ring Hybrid

- **Single Balanced Mixer**

  - RF input matching and RF-LO isolation can be improved by using (single/double) balanced mixer(s), which consist of two single-ended Shottky diodes or transistors combined with ring hybrid(s).
  - The ring hybrid will ideally lead to perfect RF-LO isolation over a wide frequency range.
  - The single balanced mixer as shown in below figure will also reject all even-order intermodulation products.
  - Doubled balanced mixer is also widely used.



4 Review

- Ring hybrid
- Even- and odd-mode analysis
- Different operations according to different feeding input ports