Smith Chart

Smith Chart

Reflection coefficient plane

- $\Gamma = |\Gamma| e^{j\theta}$

  where $0 \le |\Gamma| \le 1,\ 0^{\circ} \le \theta \le 360^{\circ}$ (or $0 \le \theta \le 2\pi$)

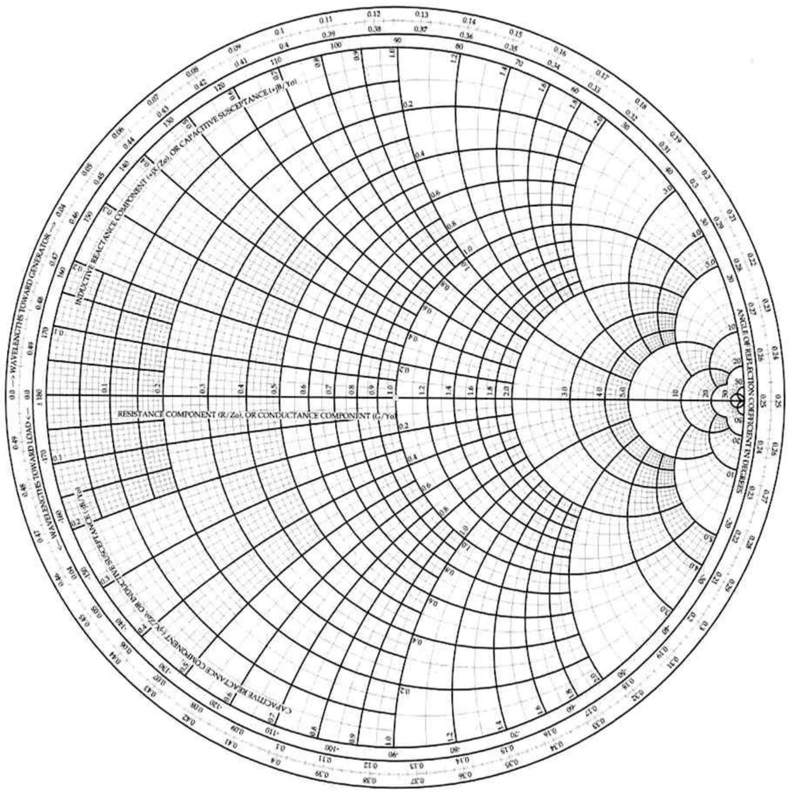
- Developed by P. Smith
  at Bell Telephone Laboratories in 1939

- Very useful when solving transmission line problems
  - ➡ Visualizing transmission line phenomenon
  - ➡ Intuition about transmission line and impedance-matching problems

- Normalized impedance (or admittance)
  : $z = Z / Z_0$ (or $y = Y / Y_0$)

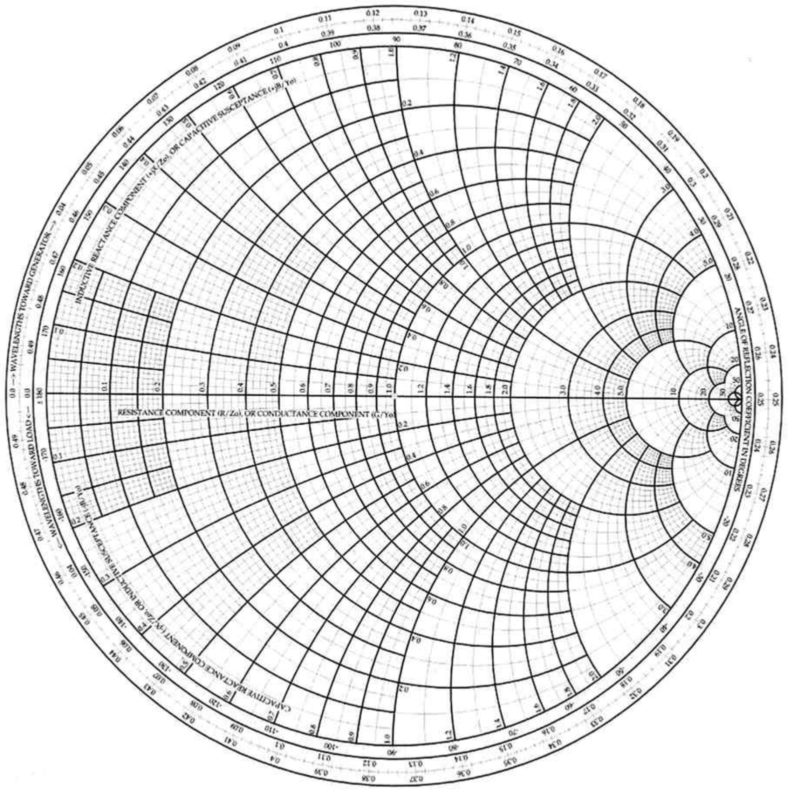
- Z_0 (or Y_0): arbitrary value

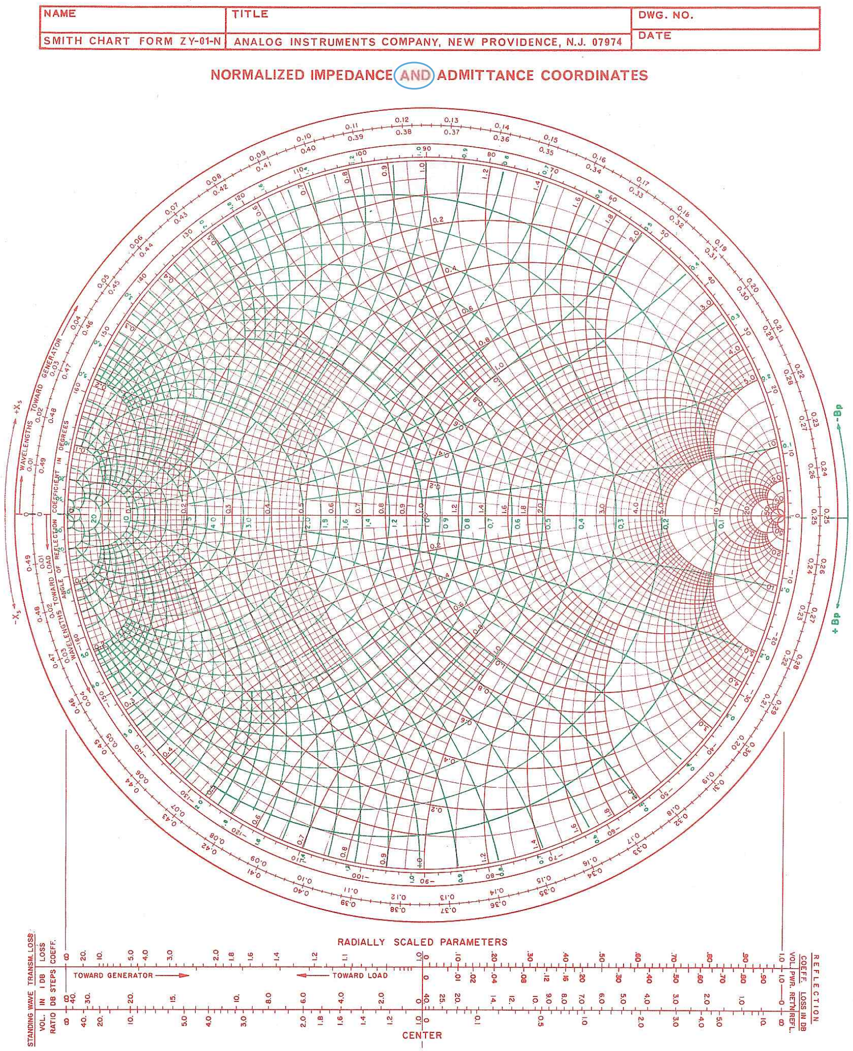
Normalized Impedance or Admittance Coordinates



Smith Chart

Normalized Impedance or Admittance Coordinates





Smith Chart

If a lossless transmission line of characteristic impedance Z_0 is terminated with a load impedance Z_L,

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(Z_L / Z_0) - 1}{(Z_L / Z_0) + 1} = \frac{z_L - 1}{z_L + 1} = |\Gamma| e^{j\theta} \quad \leftarrow \Gamma = f(Z_L)$$

where $z_L = Z_L / Z_0$: normalized load impedance

- Since $Z_{\text{in}} = \dfrac{V(-l)}{I(-l)} = \dfrac{V_0^+[e^{j\beta l} + \Gamma e^{-j\beta l}]}{V_0^+[e^{j\beta l} - \Gamma e^{-j\beta l}] / Z_0} = \dfrac{e^{j\beta l} + \Gamma e^{-j\beta l}}{e^{j\beta l} - \Gamma e^{-j\beta l}} Z_0 = \dfrac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}} Z_0$

$$Z_{\text{in}}\Big|_{l=0} = Z_L = \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}} Z_0 \Bigg|_{l=0} = \frac{1 + \Gamma}{1 - \Gamma} Z_0 \quad \leftarrow \Gamma = |\Gamma| e^{j\theta}$$

$$z_L = \frac{Z_L}{Z_0} = \frac{1 + \Gamma}{1 - \Gamma} = \frac{1 + |\Gamma| e^{j\theta}}{1 - |\Gamma| e^{j\theta}} \quad \leftarrow z_L = g(Z_L)$$

- Let $\Gamma = \Gamma_r + j\Gamma_i$ and $z_L = r_L + jx_L$.

$$z_L = r_L + jx_L = \frac{1 + \Gamma}{1 - \Gamma} = \frac{(1 + \Gamma_r) + j\Gamma_i}{(1 - \Gamma_r) - j\Gamma_i} = \frac{\{(1 + \Gamma_r) + j\Gamma_i\}\{(1 - \Gamma_r) + j\Gamma_i\}}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

$$= \frac{(1 - \Gamma_r^2) - \Gamma_i^2 + j\Gamma_i(1 + \Gamma_r) + j\Gamma_i(1 - \Gamma_r)}{(1 - \Gamma_r)^2 + \Gamma_i^2} = \frac{(1 - \Gamma_r^2 - \Gamma_i^2) + j2\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

Smith Chart

- Real and imaginary parts:

$$r_L = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2}, \qquad x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2+\Gamma_i^2}$$

- Rearrangement of real (or resistance) part

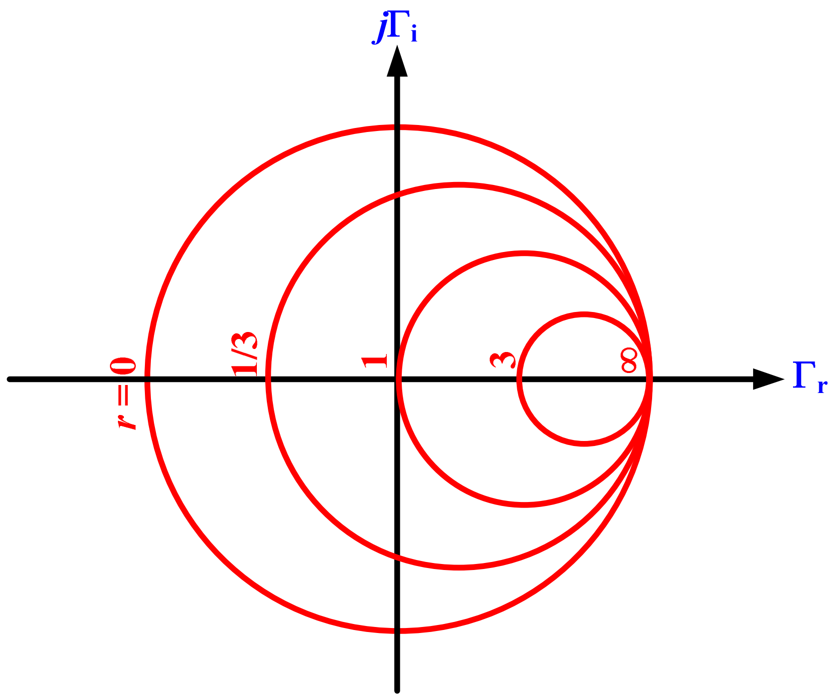
$$r_L = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2}$$

$$r_L(1-\Gamma_r)^2 + r_L\Gamma_i^2 = (1-\Gamma_r^2) - \Gamma_i^2$$

$$\Gamma_r^2(r_L+1) - 2r_L\Gamma_r + (r_L+1)\Gamma_i^2 = 1 - r_L$$

$$\Gamma_r^2 - \frac{2r_L}{r_L+1}\Gamma_r + \Gamma_i^2 = \frac{1-r_L}{1+r_L} \leftarrow x^2 - 2ax + y^2 = b$$

$$\left(\Gamma_r - \frac{r_L}{r_L+1}\right)^2 + \Gamma_i^2 = \frac{1-r_L}{1+r_L} + \left(\frac{r_L}{r_L+1}\right)^2 = \frac{1}{(1+r_L)^2}$$



- Resistance circles- Center: $\left(\frac{r_L}{r_L+1}, 0\right)$ Radius: $\frac{1}{1+r_L}$

Smith Chart

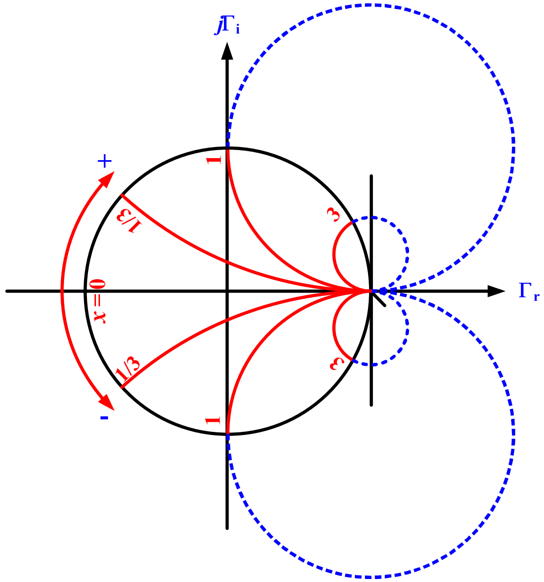
- Rearrangement of imaginary (or reactance) part

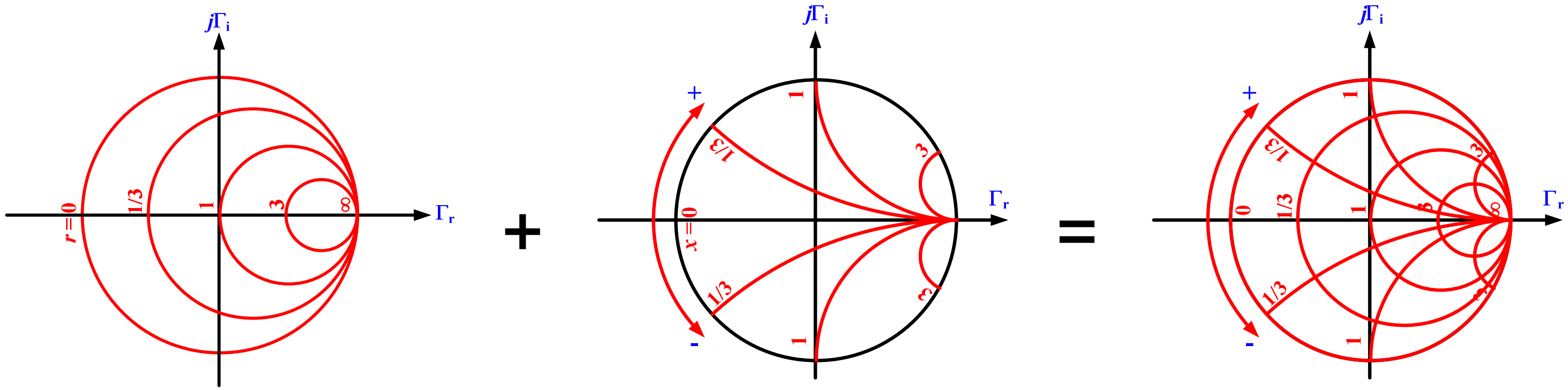
$$x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$(1-\Gamma_r)^2 + \Gamma_i^2 = \frac{2\Gamma_i}{x_L}, \qquad (1-\Gamma_r)^2 + \Gamma_i^2 - \frac{2\Gamma_i}{x_L} = 0$$

$$(\Gamma_r - 1)^2 + \Gamma_i^2 - \frac{2\Gamma_i}{x_L} + \left(\frac{1}{x_L}\right)^2 = (\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

- Reactance circles- Center: $\left(1, \frac{1}{x_L}\right)$ Radius: $\left|\frac{1}{x_L}\right|$

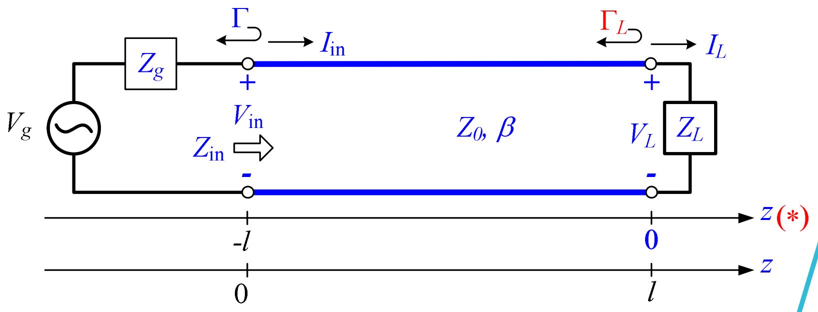




Smith Chart

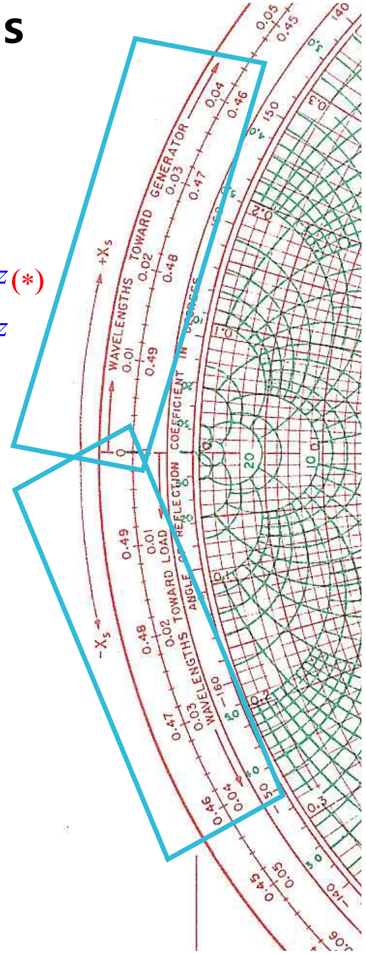
The Smith chart can also be graphical solution of the transmission line impedance equation in terms of the generalized reflection coefficient as

$$Z_{in} = Z_0 \frac{1+\Gamma e^{-2j\beta l}}{1-\Gamma e^{-2j\beta l}} \qquad \leftarrow \Gamma = |\Gamma| e^{j\theta}$$



where Γ : reflection coefficient at load

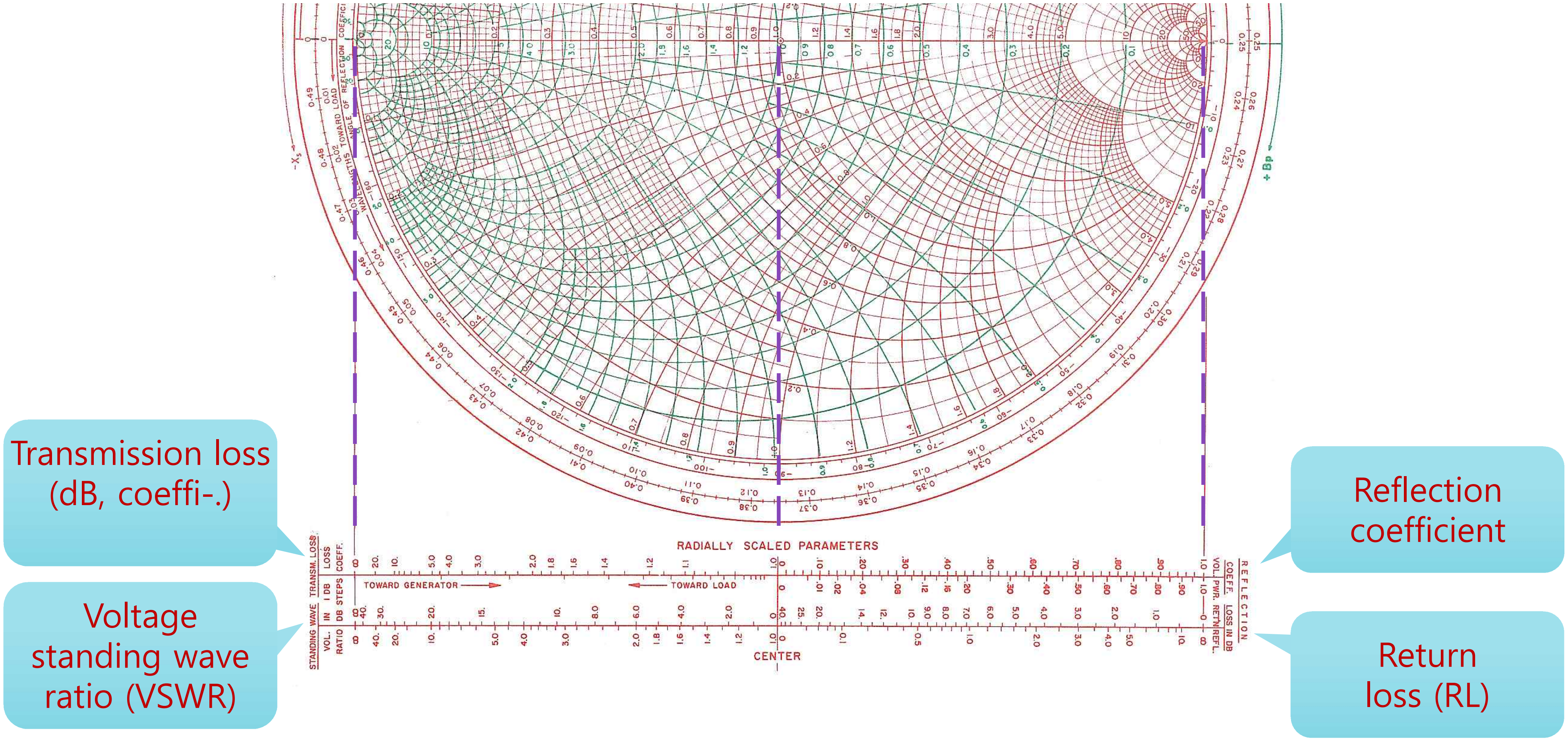
l : (positive) length of transmission line from load



- The normalized input impedance seen looking into a length l of transmission line terminated with z_L can be found by rotating the point clockwisely an amount $2\beta l$ (subtracting $2\beta l$ from θ) around the center of the chart
  - The same radius is maintained, since the magnitude of Γ does not change with position along the transmission line (∵lossless transmission line)

Smith Chart

Accessory grids



NAME		DWG. NO.
		DATE

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES