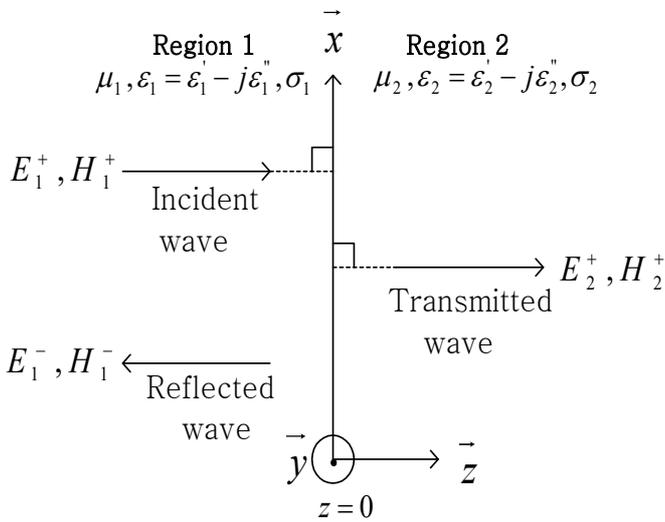


Chapter 12. Plane Wave Reflection and Dispersion

12.1 Reflection of Uniform Plane Wave at Normal Incidence

- A uniform plane wave is incident normally on the boundary between regions composed of two different materials.



- Wave traveling in the

+ \vec{z} direction in region 1:

$$E_{x1}^+ = E_{x10}^+ e^{-\alpha_1 z} \cos(\omega t - \beta_1 z)$$

positive traveling direction
region

phasor $\rightarrow E_{xs1}^+ = E_{x10}^+ e^{-jk_1 z}$ (1)

where $E_{x10}^+ : \text{real}, (jk_1 = \alpha_1 + j\beta_1)$

$$\Rightarrow H_{ys1}^+ = \frac{1}{\eta_1} E_{x10}^+ e^{-jk_1 z} \quad (2)$$

where $\eta_1 = \sqrt{\frac{j\omega\mu_1}{\sigma_1 + j\omega\epsilon_1'}} : \text{intrinsic impedance}$

$E_{x1}^+, H_{y1}^+ : \text{incident wave perpendicular to the boundary plane (경계}$

$\text{면에 수직으로 입사되는 wave)}$

- Moving (or transmitted) wave in region 2 toward $(+\vec{z})$ -direction:

$$\left\{ \begin{array}{l} E_{xs2}^+ = E_{x20}^+ e^{-jk_2 z} \\ H_{ys2}^+ = \frac{E_{x20}^+}{\eta_2} e^{-jk_2 z} \end{array} \right. \quad (3)$$

$$(4) \quad \leftarrow \eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\epsilon_2'}}$$

- (Special) Boundary condition at $z = 0$:

- i) \vec{E} -field in region 1, 2 must be equal.

Eq. (1) & (3) $\Rightarrow E_{x10}^+ = E_{x20}^+$ (tangential component) $z = 0$ 에서 고려하므로
①, ③식의 진폭이 같게 됨.

- ii) \vec{H} -field in region 1, 2 must be equal.

Eq. (2) & (4) $\Rightarrow \frac{E_{x10}^+}{\eta_1} = \frac{E_{x20}^+}{\eta_2} \Rightarrow \eta_1 = \eta_2$

\Rightarrow This is very special condition \Rightarrow Not in general.

- Let define reflected wave:

$$\left\{ \begin{array}{l} E_{xs1}^- = E_{x10}^- e^{jk_1 z} \\ H_{ys1}^- = -\frac{E_{x10}^-}{\eta_1} e^{jk_1 z} \end{array} \right. \quad (5)$$

$$(6) \quad \vec{a}_x \times (-\vec{a}_y) = -\vec{a}_z$$

- Poynting Vector $\vec{E}_1^- \times \vec{H}_1^-$ 를 만족시키려면 \vec{H} -field는 $-\vec{a}_y$ 이어야 함.

where E_{x10}^- : complex quantity (반사계수에 근거)

- Boundary condition:

i) Total electric field intensity at $z = 0$

$$\begin{aligned}
 E_{xs1} &= E_{xs2} \\
 \Rightarrow E_{xs1}^+ + E_{xs1}^- &= E_{xs2}^+ \\
 \Rightarrow E_{x10}^+ + E_{x10}^- &= E_{x20}^+ \quad (7)
 \end{aligned}$$

ii) Total magnetic field intensity at $z = 0$

$$\begin{aligned}
 H_{ys1} &= H_{ys2} \\
 \Rightarrow H_{ys1}^+ + H_{ys1}^- &= H_{ys2}^+ \\
 \Rightarrow \frac{E_{x10}^+}{\eta_1} - \frac{E_{x10}^-}{\eta_1} &= \frac{E_{x20}^+}{\eta_2} \quad (8)
 \end{aligned}$$

$$E_{x20}^+ = E_{x10}^+ + E_{x10}^- = \frac{\eta_2}{\eta_1} E_{x10}^+ - \frac{\eta_2}{\eta_1} E_{x10}^-$$

$$\left(1 + \frac{\eta_2}{\eta_1}\right) E_{x10}^- = \frac{\eta_2 + \eta_1}{\eta_1} E_{x10}^- = \left(\frac{\eta_2}{\eta_1} - 1\right) E_{x10}^+ = \frac{\eta_2 - \eta_1}{\eta_1} E_{x10}^+$$

$$\therefore E_{x10}^- = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_{x10}^+$$

- Reflection coefficient (Γ): ratio of the amplitudes of the reflected and incident electric fields

$$\Gamma = \frac{E_{x10}^-}{E_{x10}^+} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = |\Gamma| e^{j\phi} \quad (9) \quad \begin{array}{l} \text{complex} \\ \downarrow \\ \text{phase shift} \end{array} \quad (\text{complex인 경우 위상지연 발생})$$

where $\eta_i = \sqrt{\frac{j\omega\mu_i}{j\omega\epsilon_i + \sigma_i}}$

$$\begin{aligned}
E_{x20}^+ &= E_{x10}^+ + E_{x10}^- = (1 + \Gamma) E_{x10}^+ \\
&= \left(1 + \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \right) E_{x10}^+ = \frac{2\eta_2}{\eta_2 + \eta_1} E_{x10}^+ \\
&= \tau E_{x10}^+
\end{aligned}$$

- Transmit coefficient (τ): ratio of the amplitudes of the transmitted and incident electric fields

$$\tau = 1 + \Gamma = \frac{E_{x20}^+}{E_{x10}^+} = \frac{2\eta_2}{\eta_2 + \eta_1} = |\tau| e^{j\phi_t} \quad (10)$$

- [Ex. 1] $\left\{ \begin{array}{l} \text{Region 1: Perfect dielectric} \\ \text{Region 2: Perfect conductor} \end{array} \right. \Rightarrow \sigma_2 = \infty$

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\epsilon_2}} = 0 \quad \tau = \frac{2\eta_2}{\eta_1 + \eta_2} = 0$$

$$\therefore \frac{E_{x20}^+}{E_{x10}^+} = 0 \quad \Rightarrow \quad E_{x20}^+ = 0$$

region 2에서는 \vec{E} -field 가 존재하지 못함.
 \Rightarrow skin depth 의 영향으로 생각할 수도 있음

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1$$

$$\therefore E_{x10}^- = -E_{x10}^+ \quad \Rightarrow \quad \text{All wave is reflected}$$

$$E_{xs1} = E_{xs1}^+ + E_{xs1}^-$$

$$= E_{x10}^+ e^{-j\beta_1 z} - E_{x10}^+ e^{j\beta_1 z} \quad \text{perfect dielectric} \quad jk_1 = \alpha_1 + j\beta_1 = j\beta_1$$

$$= (e^{-j\beta_1 z} - e^{j\beta_1 z}) E_{x10}^+ = -(j2) \frac{e^{j\beta_1 z} - e^{-j\beta_1 z}}{j2} E_{x10}^+$$

$$= -j2 \sin \beta_1 z E_{x10}^+ \quad (11)$$

instantaneous form $\rightarrow E_{x1}(z,t) = \text{Re} \left[(-j) 2 \sin(\beta_1 z) E_{x10}^+ \{ \cos \omega t + j \sin \omega t \} \right]$

$$= \frac{2E_{x10}^+ \sin(\beta_1 z) \sin \omega t}{}$$

$$= E_{x10}^+ [\cos(\omega t - \beta_1 z) - \cos(\omega t + \beta_1 z)] \rightarrow \sin A \sin B = \frac{1}{2} \{ \cos(A - B) - \cos(A + B) \}$$

$$= E_{x10}^+ \left[\cos \left\{ \omega \left(t - \frac{z}{v_1} \right) \right\} - \cos \left\{ \omega \left(t + \frac{z}{v_1} \right) \right\} \right]$$

\rightarrow Wave traveling in the $\pm z$ direction

For $\beta_1 z = \frac{2\pi}{\lambda_1} z = m\pi$ ($m = 0, \pm 1, \pm 2, \dots$), $E_{x1} = 0$

Null locations: $z = m\lambda_1/2$

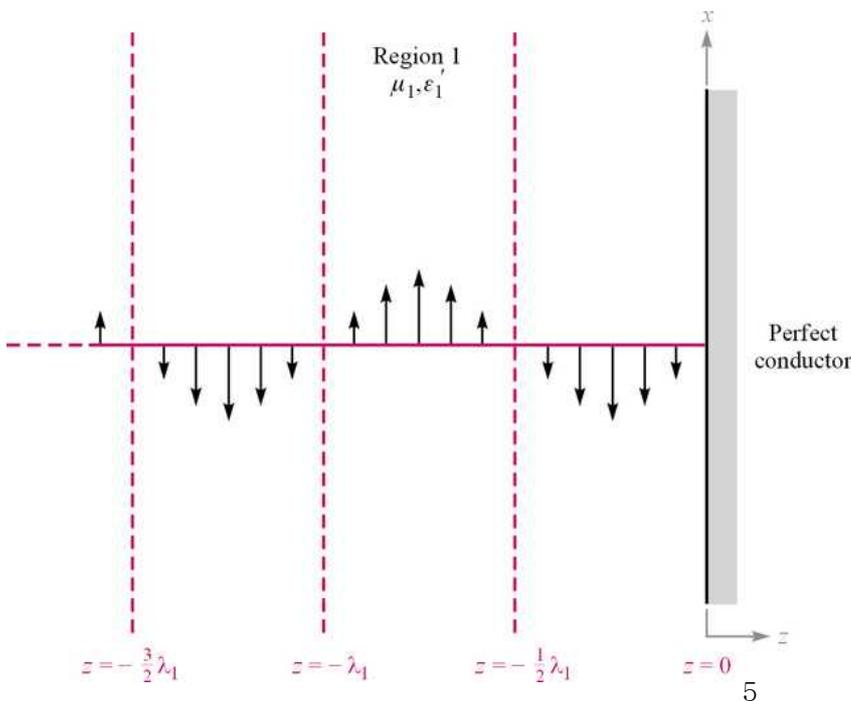
\Rightarrow Standing wave.

(Wave를 시간에 대한 term과 공간(z)에 대한 term으로 분리. $\beta_1 z = n\pi$ 이면 시간에 관계없이 $E_{x1} = 0$)

- $\bullet \omega t = m\pi, -\beta_1 z = m\pi$ 에 의해(서로 다른 두 조건) $E_{x1} = 0$ 이 될 수 있음

- For $E_{x1} = 0$,

$$\frac{2\pi}{\lambda_1} z = m\pi \quad z = m \frac{\lambda_1}{2}$$



: Instantaneous

values of the total

field E_{x1}

(at $\omega t = \frac{\pi}{2}$)

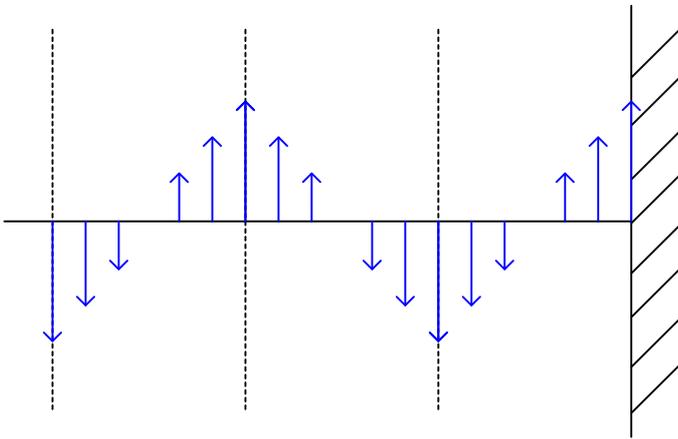
Since $E_{xs1}^+ = H_{ys1}^+ \eta_1$, $E_{xs1}^- = -H_{ys1}^- \eta_1$,

$$H_{ys1} = \frac{E_{x10}^+}{\eta_1} (e^{-j\beta_1 z} + e^{j\beta_1 z})$$

instantaneous form $\rightarrow H_{y1} = 2 \frac{E_{x10}^+}{\eta_1} \cos \beta_1 z \cos \omega t$

$$\begin{aligned} H_{ys1} &= H_{ys1}^+ e^{-j\beta_1 z} + H_{ys1}^- e^{j\beta_1 z} = \frac{E_{xs1}^+}{\eta_1} e^{-j\beta_1 z} - \frac{E_{xs1}^-}{\eta_1} e^{j\beta_1 z} \\ &= \frac{E_{xs1}^+}{\eta_1} (e^{-j\beta_1 z} - \Gamma e^{j\beta_1 z}) \quad \leftarrow \Gamma = -1 \\ &= \frac{E_{xs1}^+}{\eta_1} (e^{-j\beta_1 z} + e^{j\beta_1 z}) \\ &= 2 \frac{E_{xs1}^+}{\eta_1} \frac{e^{j\beta_1 z} + e^{-j\beta_1 z}}{2} = 2 \frac{E_{xs1}^+}{\eta_1} \cos \beta_1 z \end{aligned}$$

$\frac{2\pi}{\lambda} \cdot z$



$\Rightarrow \vec{H}$ is also standing wave
and 90° out of phase with \vec{E} .

[Ex. 12.1] Region 1, 2: perfect dielectrics.

$$\Rightarrow \alpha_1 = 0 = \alpha_2$$

$$\eta_1 = 100 [\Omega], \quad \eta_2 = 300 [\Omega]$$

$$E_{x10}^+ = 100 \text{ [V/m]}$$

$$\Rightarrow \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{300 - 100}{300 + 100} = 0.5$$

$$E_{x10}^- = 50 \text{ [V/m]}$$

$$H_{y10}^+ = \frac{E_{x10}^+}{\eta_1} = \frac{100}{100} = 1 \quad [\text{A/m}]$$

$$H_{y10}^- = -\frac{E_{x10}^-}{\eta_1} = -\frac{50}{100} = -0.5 \quad [\text{A/m}]$$

$$\therefore \langle S_{li} \rangle = \frac{1}{2} \text{Re}\{\vec{E}_s \times \vec{H}_s^*\} = \frac{1}{2} \frac{E_{x10}^2}{\eta_1} = \frac{1}{2} E_{x10}^+ H_{y10}^+ = \frac{1}{2} \times 100 \times 1 = 50 \quad [\text{W/m}^2]: \text{incident average power}$$

$$\langle S_{lr} \rangle = -\frac{1}{2} E_{x10}^- H_{y10}^- = -\frac{1}{2} \times 50 \times (-0.5) = 12.5 \quad [\text{W/m}^2]: \text{reflected average power}$$

→ $-\vec{z}$ 축으로 진행하므로

• Region 2.

$$E_{x20}^+ = \frac{2\eta_2}{\eta_1 + \eta_2} E_{x10}^+ = \frac{600}{100 + 300} \times 100 = 150 \quad [\text{V/m}]$$

$$H_{y20}^+ = \frac{E_{x20}^+}{\eta_2} = \frac{150}{300} = 0.5 \quad [\text{A/m}]$$

$$\therefore \langle S_2 \rangle = \frac{1}{2} E_{x20}^+ H_{y20}^+ = \frac{1}{2} \times 150 \times 0.5 = 37.5 \quad [\text{W/m}^2]: \text{transmit average power.}$$

$$\therefore \langle S_{li} \rangle = \langle S_{lr} \rangle + \langle S_2 \rangle \quad : \text{입사전력} = \text{반사전력} + \text{전달전력}$$

$$\Rightarrow E_{x10}^+ + E_{x10}^- = E_{x20}^+ \quad : \text{경계면에서 region 1 에서의 } \vec{E}\text{-field의 합.} \\ = \text{region 2 에서의 } \vec{E}\text{-field의 합.}$$

• General rule on the transfer of power through reflection and transmission.

- Incident power density,

$$\langle S_{li} \rangle = \frac{1}{2} \text{Re}\{E_{xs1}^+ H_{ys1}^{+*}\} = \frac{1}{2} \text{Re}\left\{E_{x10}^+ \frac{1}{\eta_1^*} E_{x10}^{+*}\right\} = \frac{1}{2} \text{Re}\left\{\frac{1}{\eta_1^*}\right\} |E_{x10}^+|^2$$

- Reflected power density:

$$\begin{aligned} \langle S_{1r} \rangle &= -\frac{1}{2} \operatorname{Re} \{ E_{x10}^- H_{y10}^- \} = \frac{1}{2} \operatorname{Re} \left\{ \Gamma E_{x10}^+ \frac{1}{\eta_1^*} \Gamma^* E_{x10}^{+*} \right\} \\ &= \frac{1}{2} \operatorname{Re} \left\{ \frac{1}{\eta_1^*} \right\} |E_{x10}^+|^2 |\Gamma|^2 \end{aligned}$$

- General relation between the reflected and incident power

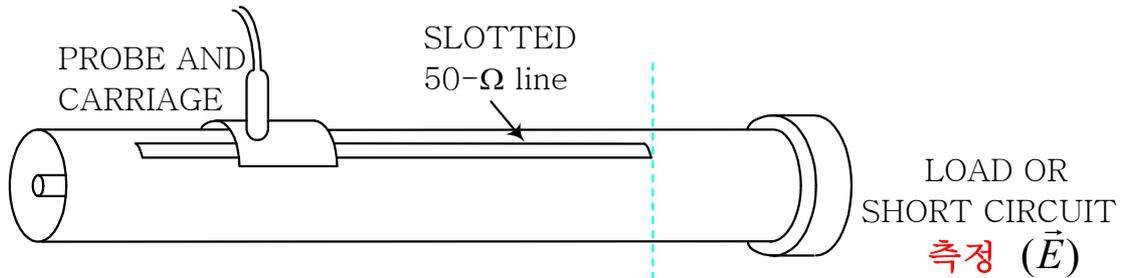
$$\langle S_{1r} \rangle = |\Gamma|^2 \langle S_{1i} \rangle \quad (15)$$

- Transmitted power:

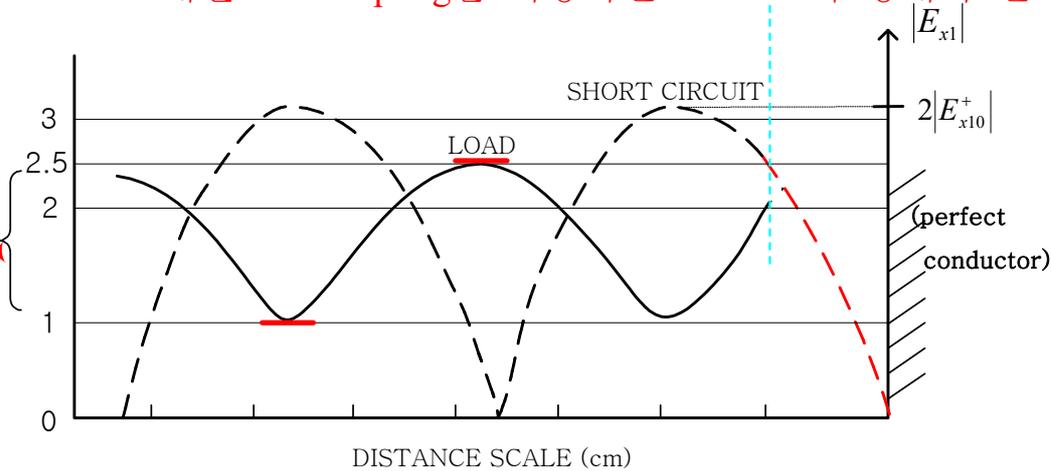
$$\begin{aligned} \langle S_2 \rangle &= \frac{1}{2} \operatorname{Re} \{ E_{x20}^+ H_{y20}^+ \} = \frac{1}{2} \operatorname{Re} \left\{ \tau E_{x10}^+ \frac{1}{\eta_2^*} \tau^* E_{x10}^{+*} \right\} = \frac{1}{2} \operatorname{Re} \left\{ \frac{1}{\eta_2^*} \right\} |E_{x10}^+|^2 |\tau|^2 \\ &= \frac{\operatorname{Re} \{ 1/\eta_2^* \}}{\operatorname{Re} \{ 1/\eta_1^* \}} |\tau|^2 \cdot \frac{1}{2} \operatorname{Re} \left\{ \frac{1}{\eta_1^*} \right\} |E_{x10}^+|^2 = \frac{\operatorname{Re} \{ 1/\eta_2^* \}}{\operatorname{Re} \{ 1/\eta_1^* \}} |\tau|^2 \langle S_{1i} \rangle \\ &= \frac{\frac{1}{2} \left(\frac{1}{\eta_2} + \frac{1}{\eta_2^*} \right)}{\frac{1}{2} \left(\frac{1}{\eta_1} + \frac{1}{\eta_1^*} \right)} |\tau|^2 \langle S_{1i} \rangle = \frac{\frac{(\eta_2 + \eta_2^*)}{(\eta_2 \eta_2^*)}}{\frac{(\eta_1 + \eta_1^*)}{(\eta_1 \eta_1^*)}} |\tau|^2 \langle S_{1i} \rangle \\ &= \left| \frac{\eta_1}{\eta_2} \right|^2 \frac{\eta_2 + \eta_2^*}{\eta_1 + \eta_1^*} |\tau|^2 \langle S_{1i} \rangle \quad (16) : \text{Too difficult to calculate} \end{aligned}$$

$$\Leftrightarrow P_{2,av}^+ = (1 - |\Gamma|^2) P_{1,av}^+ \quad (17) : \text{Alternate method.}$$

12.2 Standing Wave Ratio



- Probe를 통해 relative amplitude of electric field intensity
- Probe 대신 coil coupling을 사용하면 \vec{H} -field의 상대적 진폭 측정 가능.



- Perfect short이면 $\Gamma = -1$ 이 되어 standing wave 가 됨.

$$0 \leq |E_{x1}| \leq 2|E_{x10}^+|$$

- 그러나 임의의 load가 달린다면 임의의 신호들은 전달되고 임의의 신호는 반사가 됨.

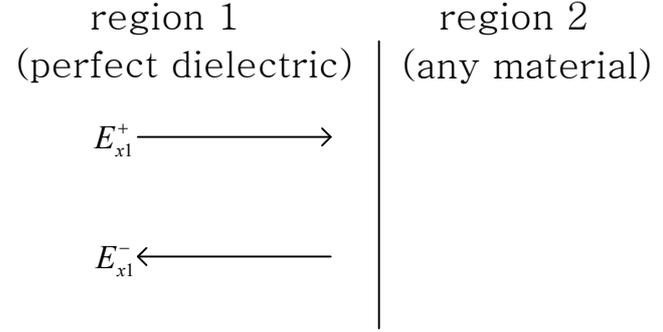
그래서 region 1에서는 traveling wave + standing wave 형태가 됨.

$$\rightarrow |E_{x1}| > 0$$

- Standing wave ratio: ratio of the maximum and minimum amplitudes of electric field intensity

- Total electric field phasor in region 1

$$\begin{aligned} E_{x1T} &= E_{x1}^+ + E_{x1}^- \\ &= E_{x10}^+ e^{-j\beta_1 z} + \Gamma E_{x10}^+ e^{j\beta_1 z} \quad (18) \end{aligned}$$



Reflection coefficient:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = |\Gamma| e^{j\phi}$$

where η_1 : positive real, η_2 : arbitrary complex

- If region 2 is a perfect conductor,

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\epsilon_2}} \cong 0 \quad \Rightarrow \quad \Gamma = -1 = 1 \angle \pi$$

- If η_2 is real and less than η_1 ,

$$\Gamma = |\Gamma| \angle \pi \quad \leftarrow \quad \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -\Delta = |\Delta| \angle \pi$$

- If η_2 is real and greater than η_1 ,

$$\Gamma = |\Gamma| \angle 0$$

- Revised total electric field phasor in region 1:

$$E_{x1T} = \left(e^{-j\beta_1 z} + |\Gamma| e^{j(\beta_1 z + \phi)} \right) E_{x10}^+ \quad (19)$$

$$|E_{x1T}|_{\max} = (1 + |\Gamma|) E_{x10}^+ \quad (20)$$

$$\Rightarrow -\beta_1 z_{\max} = \beta_1 z_{\max} + \phi + 2m\pi \quad (m = 0, \pm 1, \pm 2 \dots) \quad (21)$$

$$-\beta_1 z_{\max} = \frac{\phi}{2} + m\pi$$

$$z_{\max} = -\frac{1}{2\beta_1}(\phi + 2m\pi) \quad (22)$$

← z_{\max} : E_{x1T} 를 일으키는 z 좌표값.

- If $\phi = 0 \Rightarrow \Gamma$: real positive $\Rightarrow \eta_2 > \eta_1$ & (η_1, η_2 : real)

\Rightarrow Voltage maximum is located at the boundary plane ($z = 0$)

($\therefore @ m = 0$)

$$\Delta z_{\max} = -\frac{m\pi}{\beta_1} = -\frac{m\pi}{2\pi/\lambda_1} = -\frac{m\lambda_1}{2} \quad (z_{\max} \text{ point들의 간격})$$

- For the perfect conductor ($\phi = \pi$) ← $\Gamma = -1 = 1 \angle \pi$

$$-\beta_1 z_{\max} = \frac{\phi}{2} + m\pi \Big|_{\phi=\pi} = \frac{\pi}{2} + m\pi$$

(경계면에서 $-\pi/2$ 만큼 떨어진 곳에서 maximum point 발생.

$$-\beta_1 z_{\max} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots \quad (@ m = 0, z_{\max} = -\frac{\pi}{2\beta_1})$$

• Voltage minimum:

$$|E_{x1T}|_{\min} = (1 - |\Gamma|) E_{x10}^+ \quad (23)$$

$$\Rightarrow -\beta_1 z_{\min} = \beta_1 z_{\min} + \phi + \pi + 2m\pi \quad (m = 0, \pm 1, \pm 2, \dots) \quad (24)$$

$$-\beta_1 z_{\min} = \frac{\phi}{2} + m\pi + \frac{\pi}{2}$$

$$z_{\min} = -\frac{1}{2\beta_1} [\phi + (2m+1)\pi] \quad (25)$$

← z_{\min} : $E_{xs1, \min}$ 를 일으키는 z 좌표값.

- For the perfect conductor,

$$\Gamma = -1 = 1 \angle -\pi \quad \Rightarrow \quad \phi = -\pi$$

$$\Rightarrow \text{Voltage minimum is found at } z = 0 \quad \boxed{z_{\min} = -\frac{1}{2\beta_1} [-\pi + \pi] = 0}$$

$$\Rightarrow \Gamma: \text{real negative} \quad \Rightarrow \eta_2 < \eta_1 \text{ \& } (\eta_1, \eta_2: \text{real})$$

• Revised total electric field:

$$\begin{aligned} E_{xs1} &= (e^{-j\beta_1 z} + |\Gamma| e^{j(\beta_1 z + \phi)}) E_{x10}^+ \\ &= E_{x10}^+ (1 - |\Gamma|) e^{-j\beta_1 z} + E_{x10}^+ |\Gamma| \left(e^{-j\frac{\phi}{2}} e^{-j\beta_1 z} + e^{j\frac{\phi}{2}} e^{j\beta_1 z} \right) e^{j\frac{\phi}{2}} \\ &= (1 - |\Gamma|) E_{x10}^+ e^{-j\beta_1 z} + 2|\Gamma| E_{x10}^+ e^{j\frac{\phi}{2}} \frac{e^{j\frac{\phi}{2}} e^{j\beta_1 z} + e^{-j\frac{\phi}{2}} e^{-j\beta_1 z}}{2} \\ &= (1 - |\Gamma|) E_{x10}^+ e^{-j\beta_1 z} + 2|\Gamma| E_{x10}^+ e^{j\frac{\phi}{2}} \cos\left(\beta_1 z + \frac{\phi}{2}\right) \end{aligned}$$

instantaneous
form

$$E_{x1T}(x, t) = \underbrace{(1 - |\Gamma|) E_{x10}^+ \cos(\omega t - \beta_1 z)}_{\text{traveling wave}}$$

계속 진행되는 traveling wave

$$+ \underbrace{2|\Gamma| E_{x10}^+ \cos(\beta_1 z + \phi/2) \cos(\omega t + \phi/2)}_{\text{standing wave}} \quad (26)$$

reflected 되어 standing wave를 일으키는 성분.

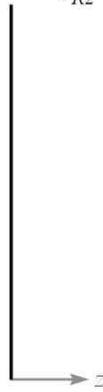
[Ex. 12.2] Propagating wave in material 1: 100 [V/m], 3 [GHz]

Dielectric 1
 $\epsilon'_{R1} = 4, \mu_{R1} = 1, \epsilon''_{R1} = 0$

$$E_{xs1}^+ = 100e^{-j40\pi z}$$

$$E_{xs1}^- = -20e^{j40\pi z}$$

Dielectric 2
 $\epsilon'_{R2} = 9, \mu_{R2} = 1, \epsilon''_{R2} = 0$



$$\omega = 2\pi f = 6\pi \times 10^9 \text{ [rad/sec]}$$

$$\beta_1 = \omega\sqrt{\mu_1\epsilon_1} = 6\pi \times 10^9 \frac{\sqrt{4 \times 1}}{3 \times 10^8} = 40\pi \text{ [rad/m]}$$

$$\beta_2 = \omega\sqrt{\mu_2\epsilon_2} = 60\pi \text{ [rad/m]}$$

$$\lambda_1 = \frac{2\pi}{\beta_1} = \frac{2\pi}{40\pi} = 0.05 \text{ [m]}$$

$$\lambda_2 = \frac{2\pi}{\beta_2} = \frac{2\pi}{60\pi} = 0.0333 \text{ [m]}$$

$$\eta_1 = \sqrt{\frac{j\omega\mu_1}{\sigma + j\omega\epsilon_1}} = \sqrt{\frac{\mu_1}{\epsilon_1}} = \frac{1}{\sqrt{\epsilon_{R1}}} \times 120\pi = 60\pi \text{ } [\Omega] \leftarrow \sigma = \omega\epsilon''$$

$$\eta_2 = \frac{1}{\sqrt{\epsilon_{R2}}} \times 120\pi = 40\pi \text{ } [\Omega]$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{40\pi - 60\pi}{40\pi + 60\pi} = -0.2 = 0.2 \angle \pi$$

$\Rightarrow \vec{E}_{\min}$ will be at the boundary and repeated at half-wavelength (0.025m) intervals in dielectric 1.

$$E_{xs1,\min} = (1 - |\Gamma|)E_{x10}^+ = 80 \text{ [V/m]}$$

$$\Rightarrow E_{xs1,\max} = (1 + |\Gamma|)E_{x10}^+ = 120 \text{ [V/m]} \text{ at } \left(\begin{array}{l} z = -0.0125, -0.0375, \\ -0.0625, \dots \text{ [m]} \end{array} \right)$$

(Region 2에서는 반사파가 없으므로 max, min점들이 없음)

- Standing wave ratio (SWR): ratio of maximum to minimum electric field amplitudes

$$s = \frac{|E_{x1T}|_{\max}}{|E_{x1T}|_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|} \quad (27)$$

Since $|\Gamma| \leq 1$, $s \geq 1$

[Ex.] ① $\Gamma = -0.2 \Rightarrow s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1.2}{0.8} = 1.5 \Rightarrow 1.5:1$

② $|\Gamma| = 1 \Rightarrow$ The reflected and incident wave amplitude are equal.

\Rightarrow All the incident wave energy is reflected.

$\Rightarrow E_{x1}$ is zero which plane is separated $\lambda_1/2$.

E_{x1} is $2E_{xs10}$ which plane is separated $\lambda_1/2$.

③ $\eta_2 = \eta_1 \Rightarrow \Gamma = 0 \Rightarrow$ No energy reflected.

$\Rightarrow s = 1$

④ If 1/2 of incident power is reflected. $\Rightarrow |\Gamma|^2 = 0.5 \Rightarrow |\Gamma| = 0.707$

$$\Rightarrow s = \frac{1+0.707}{1-0.707} = \frac{1.707}{0.293} = 5.83$$

(SWR은 반사신호의 진폭 변화 비를 의미하므로 반사신호의 진폭 측정에 의해 반사계수를 알 수 있다. $s \rightarrow |\Gamma|$)

Ex.] An uniform plane wave is reflected from an unknown material to the air. The measured electric field is 1.5 m between maxima.

$$z_{\max} = -0.75 \text{ [m]}, s = 5. \quad \eta_u = ?$$

$$\rightarrow \lambda = 1.5 \times 2 = 3 \text{ [m]} \quad \rightarrow \quad f = c / \lambda = (3 \times 10^8) / 3 = 10^8 \text{ [Hz]} \\ \text{or } 100 \text{ [MHz]}$$

$z_{\max} = -0.75 \text{ [m]} \rightarrow$ The 1st minimum occurs at the boundary.

$$|\Gamma| = \frac{s-1}{s+1} = \frac{5-1}{5+1} = \frac{2}{3}$$

$$\Gamma = -\frac{2}{3} = \frac{\eta_u - \eta_0}{\eta_u + \eta_0}$$

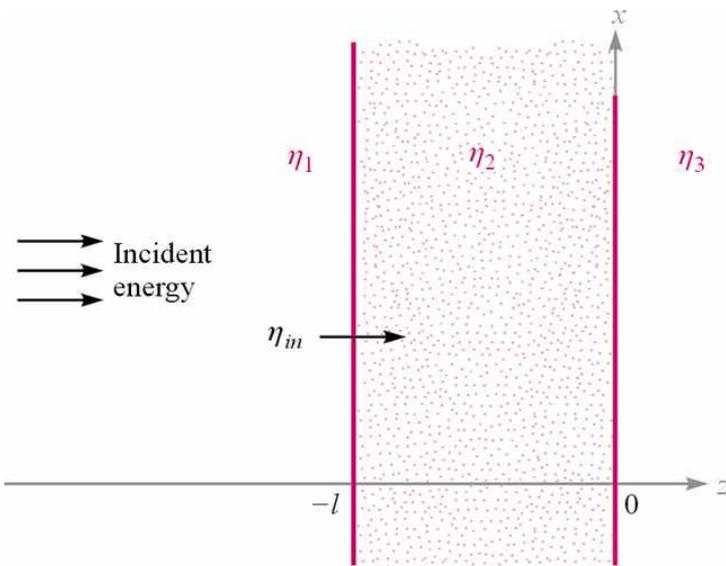
$$\eta_u = \eta_0 / 5 = 377 / 5 = 75.4 \text{ [\Omega]}$$

12.3 Wave Reflection from Multiple Interfaces

• Consider wave reflection from materials that are finite in extent.

\rightarrow Effects of the front and back surfaces.

[Ex.] Light incident on a flat piece of glass. (coated glasses, etc.)



• Let us assume all regions are composed of lossless media.

• Electric fields in region 2:

$$E_{xs2} = E_{x20}^+ e^{-j\beta_2 z} + E_{x20}^- e^{j\beta_2 z} \quad (28a)$$

$$\text{where } \beta_2 = \omega \sqrt{\mu_2 \epsilon_2} = \omega \sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon_{r2}} \\ = \frac{\omega \sqrt{\epsilon_{r2}}}{c}$$

E_{x20}^+, E_{x20}^- : complex

- Magnetic fields in region 2:

$$H_{ys2} = H_{y20}^+ e^{-j\beta_2 z} + H_{y20}^- e^{j\beta_2 z} \quad (28b)$$

Reflection coefficient at the second interface of region 2:

$$\Gamma_{23} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} \quad (29)$$

$$\rightarrow E_{x20}^- = \Gamma_{23} E_{x20}^+ \quad (30)$$

$$H_{y20}^+ = \frac{E_{x20}^+}{\eta_2} \quad (31a)$$

$$H_{y20}^- = -\frac{E_{x20}^-}{\eta_2} = -\frac{\Gamma_{23}}{\eta_2} E_{x20}^+ \quad (31b)$$

- Wave impedance (η_w): ratio of total electric field to total magnetic field at arbitrary position z in region 2

$$\begin{aligned} \eta_w(z) &= \frac{E_{xs2}}{H_{ys2}} = \frac{E_{x20}^+ e^{-j\beta_2 z} + E_{x20}^- e^{j\beta_2 z}}{H_{y20}^+ e^{-j\beta_2 z} + H_{y20}^- e^{j\beta_2 z}} \\ &= \frac{\frac{E_{x20}^+ e^{-j\beta_2 z} + \Gamma_{23} E_{x20}^+ e^{j\beta_2 z}}{\eta_2}}{\frac{E_{x20}^+ e^{-j\beta_2 z} - \Gamma_{23} E_{x20}^+ e^{j\beta_2 z}}{\eta_2}} = \eta_2 \frac{e^{-j\beta_2 z} + \Gamma_{23} e^{j\beta_2 z}}{e^{-j\beta_2 z} - \Gamma_{23} e^{j\beta_2 z}} \\ &= \eta_2 \frac{(\cos \beta_2 z - j \sin \beta_2 z) + \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} (\cos \beta_2 z + j \sin \beta_2 z)}{(\cos \beta_2 z - j \sin \beta_2 z) - \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} (\cos \beta_2 z + j \sin \beta_2 z)} \end{aligned}$$

$$\begin{aligned}
&= \eta_2 \frac{(\eta_3 + \eta_2)(\cos \beta_2 z - j \sin \beta_2 z) + (\eta_3 - \eta_2)(\cos \beta_2 z + j \sin \beta_2 z)}{(\eta_3 + \eta_2)(\cos \beta_2 z - j \sin \beta_2 z) - (\eta_3 - \eta_2)(\cos \beta_2 z + j \sin \beta_2 z)} \\
&= \eta_2 \frac{\cancel{2}\eta_3 \cos \beta_2 z - j\cancel{2}\eta_2 \sin \beta_2 z}{\cancel{2}\eta_2 \cos \beta_2 z - j\cancel{2}\eta_3 \sin \beta_2 z} \\
&= \eta_2 \frac{\eta_3 \cos \beta_2 z - j\eta_2 \sin \beta_2 z}{\eta_2 \cos \beta_2 z - j\eta_3 \sin \beta_2 z} \quad (32) \\
&= \eta_2 \frac{\eta_3 - j\eta_2 \tan \beta_2 z}{\eta_2 - j\eta_3 \tan \beta_2 z}
\end{aligned}$$

- Boundary condition at $z = -l$,

$$\begin{cases} E_{xs1}^+ + E_{xs1}^- = E_{xs2} & (33a) \\ H_{ys1}^+ + H_{ys1}^- = H_{ys2} & (33b) \end{cases}$$

$$\Rightarrow E_{x10}^+ + E_{x10}^- = \underline{E_{xs2}(z = -l)} \quad \leftarrow \text{from eq. (7), (8) in text}$$

→ region 2 에도 E_{x20}^+, E_{x20}^- 성분이 있으므로 E_{xs2} 로 표시.

$$H_{y10}^+ + H_{y10}^- = H_{ys2}(z = -l) \quad \rightarrow \quad \frac{E_{x10}^+}{\eta_1} - \frac{E_{x10}^-}{\eta_1} = \frac{E_{xs2}(z = -l)}{\eta_w(-l)}$$

where E_{x10}^+, E_{x10}^- : amplitudes of incident and reflected fields in region 1.

- Input impedance $(\eta_{in}) = \eta_w(-l)$ to the two-interface combination

$$\frac{E_{x10}^+}{\eta_1} - \frac{E_{x10}^-}{\eta_1} = \frac{E_{xs2}(z = -l)}{\eta_{in}} = \frac{E_{x10}^+ + E_{x10}^-}{\eta_{in}}$$

$$E_{x10}^+ \left(\frac{1}{\eta_1} - \frac{1}{\eta_{in}} \right) = E_{x10}^- \left(\frac{1}{\eta_{in}} + \frac{1}{\eta_1} \right)$$

$$E_{x10}^+ \frac{\eta_{in} - \eta_1}{\cancel{\eta_1 \eta_{in}}} = E_{x10}^- \frac{\eta_{in} + \eta_1}{\cancel{\eta_1 \eta_{in}}}$$

$$\frac{E_{x10}^-}{E_{x10}^+} = \Gamma = \frac{\eta_{in} - \eta_1}{\eta_{in} + \eta_1} \quad (35)$$

$$\text{where } \eta_{in} = \eta_w(z = -l) = \eta_2 \frac{\eta_3 \cos \beta_2 l + j\eta_2 \sin \beta_2 l}{\eta_2 \cos \beta_2 l + j\eta_3 \sin \beta_2 l} \quad (36)$$

- In case of $\eta_{in} = \eta_1$, $\Gamma = 0$

$$\Rightarrow \text{No reflected power } (\because P_r = (1 - |\Gamma|^2)P_{in}) \Rightarrow \text{Matched}$$

- Suppose that $\eta_3 = \eta_1$ and $\beta_2 l = m\pi$ (m : integer)

$$\beta_2 l = \frac{2\pi}{\lambda_2} l = m\pi \quad \left(\text{or } l = \frac{m\lambda_2}{2} \quad (37) \right)$$

$$\eta_{in} = \eta_2 \left. \frac{\eta_3 \cos \beta_2 l + j\eta_2 \sin \beta_2 l}{\eta_2 \cos \beta_2 l + j\eta_3 \sin \beta_2 l} \right|_{\beta_2 l = m\pi}$$

$$= \eta_2 \frac{\eta_3}{\eta_2} = \eta_3$$

$$= \eta_1 \quad \Rightarrow \text{Matched.}$$

Ex.] Antenna inside of the aircraft.

- (Refractive) Index:

$$n = \sqrt{\epsilon_r} \quad (38)$$

For lossless media ($\epsilon_r'' = 0$ and $\mu_r = 1$),

$$\beta = k = \omega \sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon_r} = \frac{n\omega}{c} \quad (39)$$

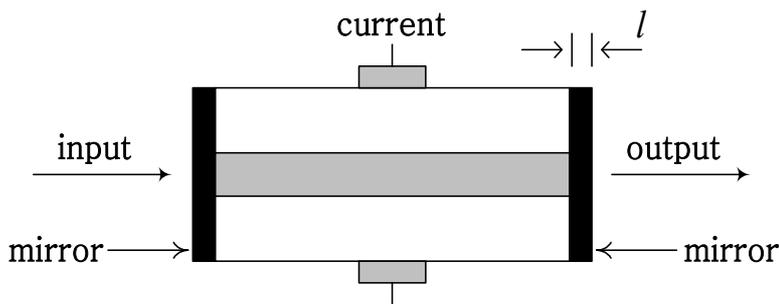
$$\eta = \frac{1}{\sqrt{\epsilon_r}} \sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{\eta_0}{n} \quad (40)$$

Phase velocity and wavelength in a material of index n :

$$v_p = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r}} = \frac{1}{\sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon_r}} = \frac{c}{n} \quad (41)$$

$$\lambda = \frac{v_p}{f} = \frac{c}{nf} = \frac{\lambda_0}{n} \quad (42)$$

Ex.] Fabry-Perot interferometer: optic filter which transmit specific



Fabry-Perot Amplifier

wavelength satisfying

the condition. $\lambda = \lambda_0 / n = 2l / m$

$$\lambda_{m-1} - \lambda_m = \Delta\lambda_f = \frac{2l}{m-1} - \frac{2l}{m} = \frac{2l}{m(m-1)} \approx \frac{2l}{m^2}$$

where m : number of half-wavelength in region 2 or

$$m = 2l / \lambda = 2nl / \lambda_0$$

λ_0 : desired free-space wavelength for transmission.

Free-space wavelength in region 2:

$$\Delta\lambda_f = \frac{2l}{m^2} = 2l \frac{\lambda^2}{4l^2} = \frac{\lambda^2}{2l} \quad (43a)$$

Free spectral range: wavelength in free space

$$\Delta\lambda_{f0} = n\Delta\lambda_f = n \frac{\lambda_0^2}{2l} = n \frac{\lambda_0^2 / n^2}{2l} = \frac{\lambda_0^2}{2nl} \quad (43b)$$

[Ex. 12.4] $\Delta\lambda_s = 50 \text{ nm}$, $\lambda_0 = 600 \text{ nm}$, ($\lambda = 600 \pm 50 \text{ nm}$)
└───────────▶ Red part of the visible spectrum.

Febry-Perot filter: $n = 1.45$

Required range of glass thickness?

⇒ Optical spectral width $\Delta\lambda_{f0} > \Delta\lambda_s$ (\because 원하는 대역폭의 신호가 모두 통과 되어야 하므로)

$$\frac{\lambda_0^2}{2l} = \Delta\lambda_f > \Delta\lambda_s$$

$$\therefore l < \frac{\lambda_0^2}{2\Delta\lambda_s} = \frac{\left(\frac{600}{\sqrt{1.45}}\right)^2 \text{ nm}^2}{2 \times 50 \text{ nm}} = \frac{600^2}{2 \times 1.45 \times 50} = 2.5 \times 10^3 \text{ nm} = 2.5 \mu\text{m}$$

///

• Another matching condition:

For $\beta_2 l = (2m - 1) \frac{\pi}{2}$ (m : integer)

$$\beta_2 l = \frac{2\pi}{\lambda_2} l = (2m-1) \frac{\pi}{2} \quad (44)$$

$$\Rightarrow l = (2m-1) \frac{\lambda_2}{4} \quad : \text{ odd multiple of a quarter wavelength}$$

$$\eta_{in} = \eta_2 \frac{\eta_3 \cos \beta_2 l + j\eta_2 \sin \beta_2 l}{\eta_2 \cos \beta_2 l + j\eta_3 \sin \beta_2 l} \Bigg|_{l=(2m-1)\frac{\lambda_2}{4}}$$

$$= \eta_2 \frac{j\eta_2}{j\eta_3} = \frac{\eta_2^2}{\eta_3} \quad (45) \quad \leftarrow \quad \eta_{in} = \eta_1 \quad (\text{ for matching })$$

$$\eta_2 = \sqrt{\eta_{in} \eta_3} = \sqrt{\eta_1 \eta_3} \quad (46) : \text{ quarter wave matching.}$$

[Ex. 12.5] Coating glass. Transmission wavelength from air to glass: 570 [nm]

$$n_3 = 1.45$$

Required index for the coating and its minimum thickness?

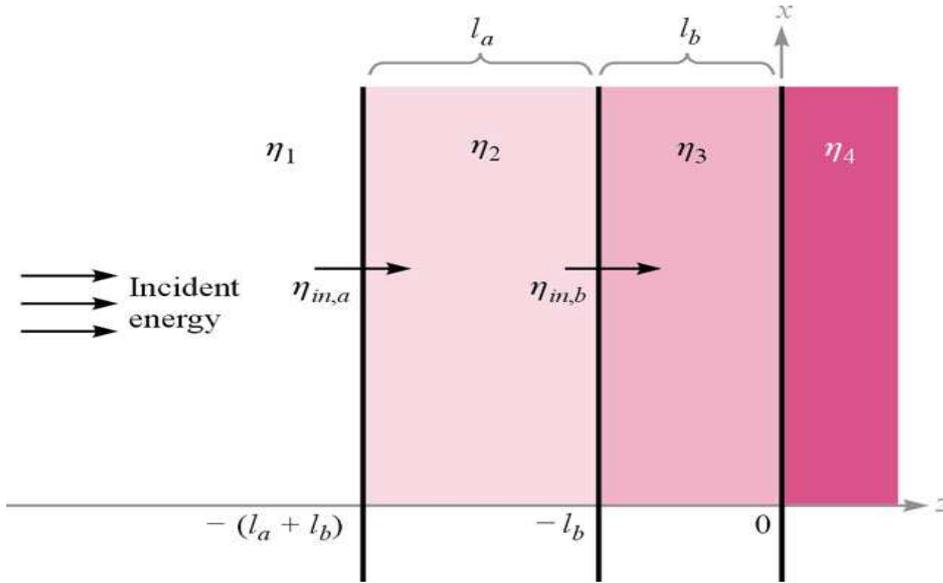
$$\Rightarrow \eta_1 = 377 \Omega \quad (\text{ free space}), \quad \eta_3 = \frac{377}{1.45} = 260 \Omega \quad (\text{ glass})$$

$$\eta_2 = \sqrt{\eta_1 \eta_3} = \sqrt{377 \times 260} = 313 \Omega$$

$$\therefore \lambda_2 = \frac{570}{1.45} = 475 \text{ nm}$$

$$l = \frac{\lambda_2}{4} = 119 \text{ nm} = 0.119 \mu\text{m} \quad : \text{ coating width}$$

- Multiple-interface situation



$$\text{Step 1) } \eta_{in,b} = \eta_3 \frac{\eta_4 \cos \beta_3 l_b + j \eta_3 \sin \beta_3 l_b}{\eta_3 \cos \beta_3 l_b + j \eta_4 \sin \beta_3 l_b} \quad (47)$$

$$\text{Step 2) } \eta_{in,a} = \eta_2 \frac{\eta_{in,b} \cos \beta_2 l_a + j \eta_2 \sin \beta_2 l_a}{\eta_2 \cos \beta_2 l_a + j \eta_{in,b} \sin \beta_2 l_a} \quad (48)$$

$$\text{Step 3) } \Gamma = \frac{\eta_{in,a} - \eta_1}{\eta_{in,a} + \eta_1}$$

$$\text{Step 4) Reflected power: } P_r = |\Gamma|^2 P_{in}$$

$$\text{Transmitted power at layer 4: } P_t = (1 - |\Gamma|^2) P_{in}$$