

Chapter 10. Transmission Lines

- **Transmission line:** carrier to transmit electric energy and signals from one point to another

Ex.] Source $\leftarrow^* \rightarrow$ Load, Transmitter $\leftarrow^* \rightarrow$ Receiver,

Computer $\leftarrow^* \rightarrow$ Computer,

Hydroelectric generating plant $\leftarrow^* \rightarrow$ Home

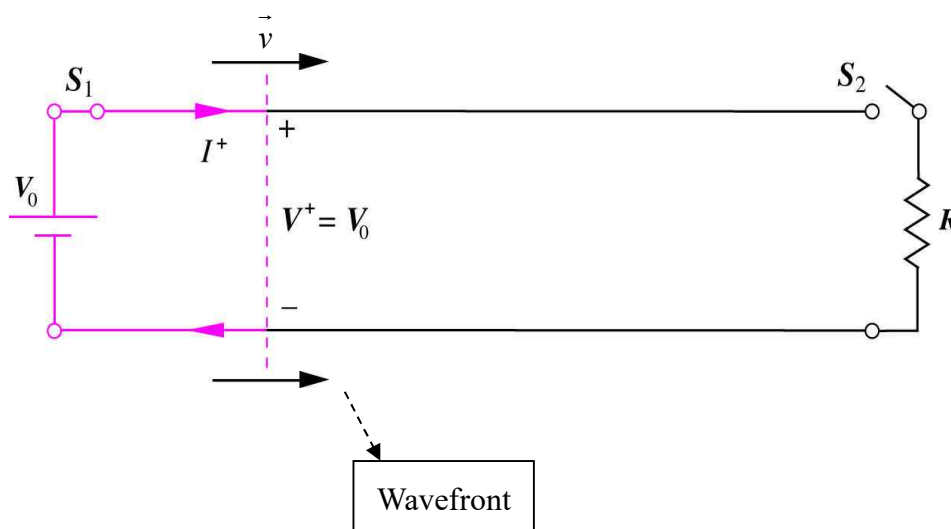
Audio System $\leftarrow^* \rightarrow$ Speaker,

Cable service provider $\leftarrow^* \rightarrow$ TV, Device $\leftarrow^* \rightarrow$ Device

- 1) **Lumped elements** (R, L, C, etc.) ignores a time delay to traverse the elements $\rightarrow$ Per-unit-distance basis
- 2) **Distributed elements** (transmission line) can't be ignored the time delay to traverse the elements $\rightarrow$ Over-unit-distance basis

10.1 Physical Description of Transmission Line Propagation

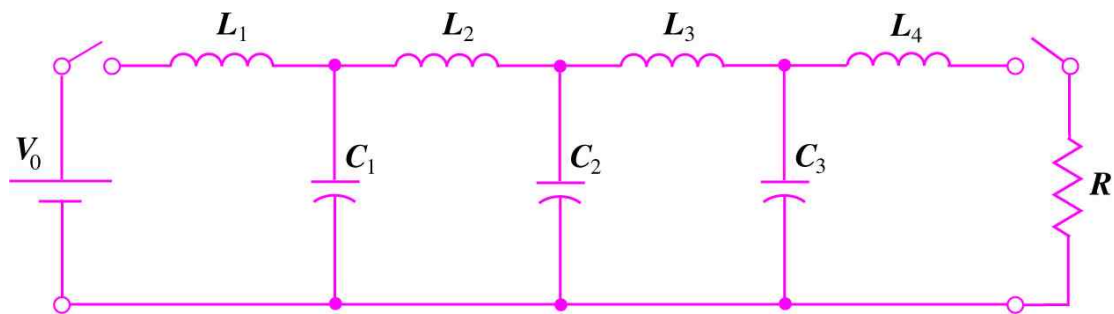
- Lossless basic transmission line circuit



- Assume the switch S_1 is closed at $t = 0$
- At $t = 0^+$, the source voltage (V_0) doesn't instantaneously appear everywhere on the line, but begins to travel from the battery toward the load (R) at a certain velocity (v).
- Wavefront: instantaneous boundary between the sections of the line to be charged or not

• Lumped-element model of transmission line

Assumptions: $L_1 = L_2 = L_3 = L_4$ [H/m] and $C_1 = C_2 = C_3$ [F/m]



S_1 On $\rightarrow$ Current through $L_1 \rightarrow$ Charging on C_1
 $\rightarrow$ Current through $L_2 \rightarrow$ Charging on $C_2 \rightarrow \dots$
 $\Leftrightarrow$ Wavefront moving from the left to right

S_2 On $\rightarrow$ Discharging on C_3 (Dissipating by R)
 $\rightarrow$ Discharging on C_2 (Dissipating by R) $\rightarrow \dots$
: Pulse-forming network (dissipating on R)

Wave velocity: $v = 1 / (LC)^{0.5}$ (to be proved in later.)

- Analyzing approaches

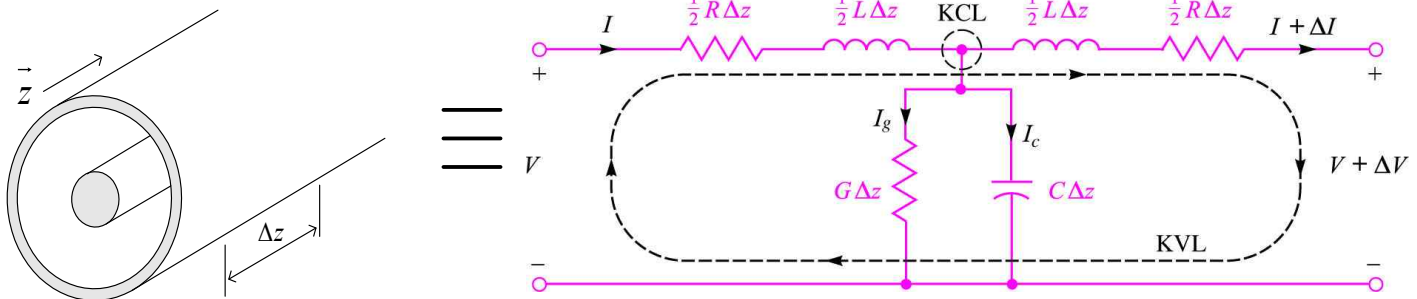
1) Maxwell's equations: electromagnetic field method
(complete & accurate)

Ex.] Wave power, velocity, ...

2) Circuit equations: equivalent voltage and current analysis method
(intuitive & inaccurate)

10.2 The Transmission Line Equations

- **Lumped-element model** of a short transmission line section **with losses**
(length : Δz)



R : series resistance per unit length [Ω/m]

← due to finite conductor conductivity σ_c

L : series inductance per unit length [H/m]

G : shunt conductance per unit length [Ω^{-1}/m]

← due to finite dielectric conductivity σ_d

C : shunt capacitance per unit length [F/m] ← due to ϵ_r

R, G : power loss components ($=f(\text{freq.})$)

- 전송선로의 미소길이에 대하여 등가 회로 model을 구하고, 이것으로부터 전압, 전류 방정식을 유도한 후 그 결과식들이 field 이론에서 구한 기본 방정식들(예: wave equation)과 성질이 같다면 uniform plane wave의 전달경로가 전송선로와 같음을 보일 수 있게 됨.

- Propagation direction: $\vec{a}_z$

- KVL

$$V = \frac{1}{2}R\Delta z I + \frac{1}{2}L\Delta z \frac{\partial I}{\partial t} + \frac{1}{2}L\Delta z \left(\frac{\partial I}{\partial t} + \frac{\partial \Delta I}{\partial t} \right) \quad \leftarrow L = \frac{\Phi}{I} \ \& \ V = \frac{d\Phi}{dt}$$

$$+ \frac{1}{2}R\Delta z (I + \Delta I) + (V + \Delta V) \quad (1)$$

$$(V + \Delta V) - V = -\frac{1}{2}R\Delta z I - \frac{1}{2}L\Delta z \frac{\partial I}{\partial t} - \frac{1}{2}L\Delta z \left(\frac{\partial I}{\partial t} + \frac{\partial \Delta I}{\partial t} \right) - \frac{1}{2}R\Delta z (I + \Delta I)$$

$$\frac{\Delta V}{\Delta z} = -\frac{1}{2}RI - \frac{1}{2}L \frac{\partial I}{\partial t} - \frac{1}{2}L \left(\frac{\partial I}{\partial t} + \frac{\partial \Delta I}{\partial t} \right) - \frac{1}{2}R(I + \Delta I)$$

$$= -\left(RI + L \frac{\partial I}{\partial t} + \frac{1}{2}L \frac{\partial \Delta I}{\partial t} + \frac{1}{2}R\Delta I \right) \quad (2)$$

$$\text{Applying } \Delta I = \frac{\partial I}{\partial z} \Delta z \text{ and } \Delta V = \frac{\partial V}{\partial z} \Delta z \quad (3)$$

$$\frac{\partial V}{\partial z} = -\left\{ \left(RI + L \frac{\partial I}{\partial t} \right) + \frac{1}{2}L \frac{\partial}{\partial t} \left(\frac{\partial I}{\partial z} \Delta z \right) + \frac{1}{2}R \frac{\partial I}{\partial z} \Delta z \right\}$$

$$= -\left\{ \left(RI + L \frac{\partial I}{\partial t} \right) + \frac{\Delta z}{2} \frac{\partial}{\partial z} \left(RI + L \frac{\partial I}{\partial t} \right) \right\} = -\left(1 + \frac{\Delta z}{2} \frac{\partial}{\partial z} \right) \left(RI + L \frac{\partial I}{\partial t} \right) \quad (4)$$

As $\Delta z \rightarrow 0$,

$$\frac{\partial V}{\partial z} = - \left(RI + L \frac{\partial I}{\partial t} \right) \quad (5)$$

- KCL at the mid-point using the symmetric property

$$\begin{aligned} I &= I_g + I_c + (I + \Delta I) \quad \leftarrow Q = CV \text{ \& } I = \frac{dQ}{dt} \\ &= G\Delta z \left(V + \frac{\Delta V}{2} \right) + C\Delta z \frac{\partial}{\partial t} \left(V + \frac{\Delta V}{2} \right) + (I + \Delta I) \end{aligned} \quad (6)$$

$$\begin{aligned} (I + \Delta I) - I &= -G\Delta z \left(V + \frac{\Delta V}{2} \right) - C\Delta z \frac{\partial}{\partial t} \left(V + \frac{\Delta V}{2} \right) \\ \frac{\Delta I}{\Delta z} &= -G \left(V + \frac{\Delta V}{2} \right) - C \frac{\partial}{\partial t} \left(V + \frac{\Delta V}{2} \right) \quad \leftarrow \Delta V = \frac{\partial V}{\partial z} \Delta z \\ &= -G \left(V + \frac{\Delta z}{2} \frac{\partial V}{\partial z} \right) - C \frac{\partial}{\partial t} \left(V + \frac{\Delta z}{2} \frac{\partial V}{\partial z} \right) \\ &= - \left\{ GV \left(1 + \frac{\Delta z}{2} \frac{\partial}{\partial z} \right) + C \frac{\partial V}{\partial t} \left(1 + \frac{\Delta z}{2} \frac{\partial}{\partial z} \right) \right\} \\ &= - \left(1 + \frac{\Delta z}{2} \frac{\partial}{\partial z} \right) \left(GV + C \frac{\partial V}{\partial t} \right) \end{aligned} \quad (7)$$

As $\Delta z \rightarrow 0$

$$\frac{\partial I}{\partial z} = - \left(GV + C \frac{\partial V}{\partial t} \right) \quad (8)$$

- Telegraphist's equations: Eqs. (5) and (8)

$$\frac{\partial V}{\partial z} = - \left(RI + L \frac{\partial I}{\partial t} \right) \quad (5)$$

$$\frac{\partial I}{\partial z} = - \left(GV + C \frac{\partial V}{\partial t} \right) \quad (8)$$

- $\partial(\text{Eq. (5)})/\partial z$ and $\partial(\text{Eq. (8)})/\partial t$:

$$\frac{\partial^2 V}{\partial z^2} = -R \frac{\partial I}{\partial z} - L \frac{\partial^2 I}{\partial t \partial z} \quad (9)$$

$$\frac{\partial^2 I}{\partial z \partial t} = -G \frac{\partial V}{\partial t} - C \frac{\partial^2 V}{\partial t^2} \quad (10)$$

- Eq. (9) $\leftarrow$ Eqs. (8) and (10):

$$\begin{aligned} \frac{\partial^2 V}{\partial z^2} &= R \left(GV + C \frac{\partial V}{\partial t} \right) + L \left(G \frac{\partial V}{\partial t} + C \frac{\partial^2 V}{\partial t^2} \right) \\ &= LC \frac{\partial^2 V}{\partial t^2} + (LG + RC) \frac{\partial V}{\partial t} + RGV \end{aligned} \quad (11)$$

- $\partial(\text{Eq. (5)})/\partial t$ and $\partial(\text{Eq. (8)})/\partial z$:

$$\frac{\partial^2 V}{\partial z \partial t} = -R \frac{\partial I}{\partial t} - L \frac{\partial^2 I}{\partial t^2} \quad (*)$$

$$\frac{\partial^2 I}{\partial z^2} = -G \frac{\partial V}{\partial z} - C \frac{\partial^2 V}{\partial t \partial z} \quad (**)$$

- Eq. (**) $\leftarrow$ Eqs. (5) and (*):

$$\begin{aligned} \frac{\partial^2 I}{\partial z^2} &= G \left(RI + L \frac{\partial I}{\partial t} \right) + C \left(R \frac{\partial I}{\partial t} + L \frac{\partial^2 I}{\partial t^2} \right) \\ &= LC \frac{\partial^2 I}{\partial t^2} + (LG + RC) \frac{\partial I}{\partial t} + RGI \end{aligned} \quad (12)$$

- **General wave equations**: Eqs. (11) and (12)

(V 와 I 의 방정식이 공간과 시간에 대한 함수로 표현됨)

10.3 Lossless propagation

- Lossless propagation: (input) power isn't dissipated or otherwise deviated as the wave travels down the transmission line.

$$\rightarrow R = G = 0$$

- Lossless general wave equation:

$$\frac{\partial^2 V}{\partial z^2} = LC \frac{\partial^2 V}{\partial t^2} \quad (13) \leftarrow \text{Eq.(11)}$$

$\rightarrow$ General solution:

$$V(z, t) = f_1\left(t - \frac{z}{v}\right) + f_2\left(t + \frac{z}{v}\right) = V^+ + V^- \quad (14)$$

where v : wave velocity

f_1, f_2 : forward and backward z -directional travel functions

V^+, V^- : forward and backward voltage wave components

Ex.] As t is $\uparrow$, z must be $\uparrow$ for $f_1(0)$.

$\rightarrow$ Positive (or forward) z -direction traveling

As t is $\uparrow$, z must be $\downarrow$ for $f_2(0)$.

$\rightarrow$ Negative (or backward) z -direction traveling

- From Eq. (14),

$$\frac{\partial f_1}{\partial z} = \frac{\partial f_1}{\partial(t - z/v)} \frac{\partial(t - z/v)}{\partial z} = -\frac{1}{v} f_1' \quad (15)$$

$$\frac{\partial f_1}{\partial t} = \frac{\partial f_1}{\partial(t - z/v)} \frac{\partial(t - z/v)}{\partial t} = f_1' \quad (16)$$

$$\frac{\partial^2 f_1}{\partial z^2} = \frac{\partial}{\partial z} \left(\frac{\partial f_1}{\partial z} \right) = \frac{1}{v^2} f_1'' \quad \text{and} \quad \frac{\partial^2 f_1}{\partial t^2} = \frac{\partial}{\partial t} \left(\frac{\partial f_1}{\partial t} \right) = f_1'' \quad (17)$$

- Eq. (13) ← Eq. (17):

$$\frac{1}{v^2} f_1'' = LC f_1'' \quad (18)$$

$$v = \frac{1}{\sqrt{LC}} : \text{wave velocity} \quad (19) \leftarrow \text{page 2}$$

- Lossless voltage and current equations along the transmission line:

$$\frac{\partial V}{\partial z} = -L \frac{\partial I}{\partial t} \quad (20) \leftarrow \text{Eq. (5)}$$

$$\frac{\partial I}{\partial z} = -C \frac{\partial V}{\partial t} \quad (21) \leftarrow \text{Eq. (8)}$$

- Eq. (20) ← Eq. (14):

$$\begin{aligned} \frac{\partial I}{\partial t} &= -\frac{1}{L} \frac{\partial V}{\partial z} \quad \leftarrow \text{Eq. (14)} \\ &= -\frac{1}{L} \frac{\partial}{\partial z} \left[f_1 \left(t - \frac{z}{v} \right) + f_2 \left(t + \frac{z}{v} \right) \right] = \frac{1}{Lv} (f_1' - f_2') \quad (22) \end{aligned}$$

- Integrate Eq. (22) over time:

$$I(z, t) = \int \frac{\partial I}{\partial t} dt = \frac{1}{Lv} \left[f_1 \left(t - \frac{z}{v} \right) - f_2 \left(t + \frac{z}{v} \right) \right] = I^+ + I^- \quad (23) \Leftrightarrow I = \frac{V}{Z}$$

where Lv : characteristic impedance of lossless line

➔ $(f_1 + f_2)$ 가 voltage wave이고, $(f_1 - f_2)$ 가 current wave이므로

- Characteristic impedance (Z_0): a ratio of voltage to current waves

$$Z_0 = Lv = L \sqrt{\frac{1}{LC}} = \sqrt{\frac{L}{C}} \quad (24)$$

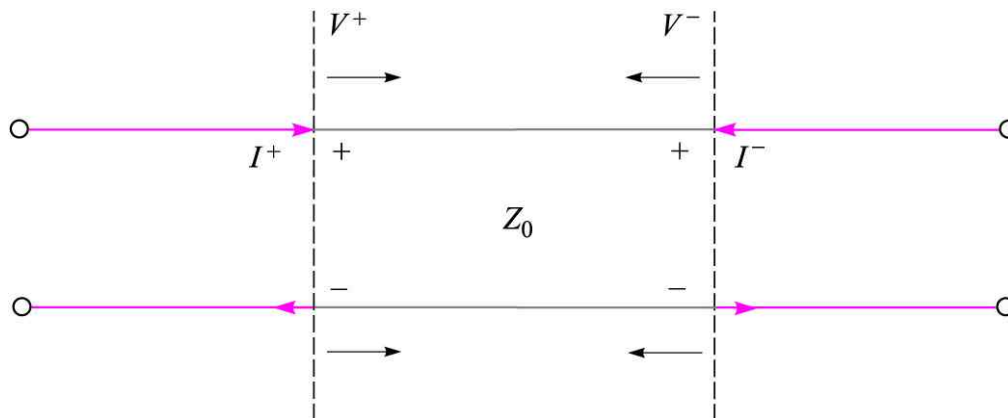
- By comparing Eqs. (14) and (23),

$$V^+ = Z_0 I^+ \quad (25a)$$

$$V^- = -Z_0 I^- \quad (25b)$$

where minus (-) means backward propagating.

- Current directions in waves having positive and negative voltages:



10.4 Lossless Propagation of Sinusoidal Voltages

- Any transmitted signal can be composed into a discrete or continuous summation of sinusoids.
 ➔ Fourier transformation (time domain ↔ frequency domain)
- Sinusoidal voltage function:

$$V(z, t) = f_1\left(t - \frac{z}{v}\right) + f_2\left(t + \frac{z}{v}\right) = V^+ + V^- \quad (14)$$

Consider a specific example:

$$f = \omega/2\pi, f_1 = f_2 = V_0 \cos(\omega t + \phi), t \rightarrow t \pm z/v_p$$

$$\Rightarrow V(z, t) = |V_0| \cos[\omega(t \pm z/v_p) + \phi] = |V_0| \cos[\omega t \pm \beta z + \phi] \quad (26)$$

For sine wave, $\phi = \pi/2 \rightarrow V(z, t) = |V_0| \sin[\omega t \pm \beta z]$

In case of $\phi = 0$,

$$V_f(z, t) = |V_0| \cos(\omega t - \beta z) \quad (\text{forward } z \text{ propagation}) \quad (27a)$$

$$V_b(z, t) = |V_0| \cos(\omega t + \beta z) \quad (\text{backward } z \text{ propagation}) \quad (27b)$$

➔ Real instantaneous forms of transmission-line voltage
 V_f or $V_b = f(\omega t, \beta z)$

where $|V_0|$: value of V at $z = 0$ and $t = 0$

ω : angular frequency [rad/sec]

β : spatial frequency [rad/m]

- Phase constant (β):

$$\beta \equiv \frac{\omega}{v_p} \quad (28) \quad : [\text{rad/sec}]/[\text{m/sec}] = [\text{rad/m}]$$

- Instantaneous voltage in case of $t = 0$,

$$V_f(z, 0) = V_b(z, 0) = |V_0| \cos(\beta z) \quad (29)$$

➔ Simple periodic function with distance λ

Requirement of wavelength (λ): $\beta\lambda = 2\pi$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega/v_p} = \frac{v_p}{f} \quad (30)$$

10.5 Complex Analysis of Sinusoidal Waves

- Expressing sinusoidal waves as complex function:
usability of evaluation and visualization of signal phase

- Euler identity: $e^{\pm jx} = \cos(x) \pm j\sin(x)$ (32)

- ➔ $\cos(x) = \text{Re}[e^{\pm jx}] = (e^{jx} + e^{-jx}) / 2 = e^{jx}/2 + c.c.$ (33a)

- $\sin(x) = \pm \text{Im}[e^{\pm jx}] = (e^{jx} - e^{-jx}) / 2j = e^{jx}/2j - c.c.$ (33b)

where $j = (-1)^{0.5}$

$c.c.$: complex conjugate of the preceding term

$$V(z, t) = |V_0| \cos[\omega(t \pm z / v_p) + \phi] = |V_0| \cos[\omega t \pm \beta z + \phi] \quad (26)$$

$$= \frac{1}{2} \underbrace{|V_0| e^{j\phi}}_{V_0} e^{\pm j\beta z} e^{j\omega t} + c.c. \quad (34)$$

where $V_0 (= |V_0| e^{j\phi})$: complex amplitude of the wave

- Complex instantaneous voltage:

$$V_c(z, t) = V_0 e^{\pm j\beta z} e^{j\omega t} \quad (35)$$

- Phasor voltage:

$$V_s(z) = V_0 e^{\pm j\beta z} \quad (36)$$

➔ sinusoidal steady-state condition ($\neq f(t)$)

- Real instantaneous voltage:

$$V(z, t) = |V_0| \cos[\omega t \pm \beta z + \phi] = \text{Re}[V_c(z, t)] = \frac{1}{2} V_c + c.c. \quad (37a)$$

- Phasor voltage:

$$V(z, t) = |V_0| \cos[\omega t \pm \beta z + \phi] = \text{Re}[V_s(z) e^{j\omega t}] = \frac{1}{2} V_s e^{j\omega t} + c.c. \quad (37b)$$

(sinusoidal instantaneous voltage: $V(z, t) = \text{Re}[V_s(z) e^{j\omega t}]$)

[Ex. 10.1] Two voltage waves having equal frequencies and amplitudes propagate in opposite directions on a lossless transmission line.

→ Phasor form:

$$V_{sT}(z) = V_0 e^{-j\beta z} + V_0 e^{+j\beta z} = 2V_0 \cos(\beta z)$$

Real instantaneous form:

$$\begin{aligned} V(z, t) &= \text{Re}[(V_0 e^{-j\beta z} + V_0 e^{+j\beta z}) e^{j\omega t}] = \text{Re}\left[2 \frac{V_0 (e^{-j\beta z} + e^{+j\beta z})}{2} e^{j\omega t}\right] \\ &= \text{Re}[2V_0 \cos(\beta z) e^{j\omega t}] = 2V_0 \cos(\beta z) \cos(\omega t) \end{aligned}$$

$$\rightarrow V(z, t) = f(z, t)$$

10.6 Transmission Line Equations and Their Solutions in Phasor Form

• Real instantaneous voltage of general wave equations:

$$\frac{\partial^2 V}{\partial z^2} = LC \frac{\partial^2 V}{\partial t^2} + (LG + RC) \frac{\partial V}{\partial t} + RGV \quad (11)(38)$$

$$\begin{aligned} \frac{\partial V(z, t)}{\partial t} &= \frac{\partial}{\partial t} \text{Re}[V_0 e^{j\beta z} e^{j\omega t}] = \frac{\partial}{\partial t} \text{Re}[V_0 e^{j(\omega t + \beta z)}] = \frac{\partial}{\partial t} [V_0 \cos(\omega t + \beta z)] \\ &= -\omega V_0 \sin(\omega t + \beta z) = j\omega V_0 \cdot \{j \sin(\omega t + \beta z)\} \\ &= \text{Re}[j\omega V_0 e^{j(\omega t + \beta z)}] = \text{Re}[j\omega V_0 e^{j\beta z} e^{j\omega t}] = \text{Re}[j\omega V_s e^{j\omega t}] \end{aligned}$$

$$\therefore \frac{\partial}{\partial t} \underset{\text{instantaneous}}{\overset{\text{phasor}}{\leftrightarrow}} j\omega$$

• Phasor voltage of the general wave equations:

$$\frac{d^2 V_s}{dz^2} = -\omega^2 LC V_s + j\omega(LG + RC) V_s + RGV_s \quad (39)$$

$$= \underbrace{(R + j\omega L)}_Z \underbrace{(G + j\omega C)}_Y V_s = \gamma^2 V_s \quad (40)$$

where Z , Y : net series impedance and shunt admittance of transmission line as per-unit-distance

- Propagation constant:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{ZY} = \alpha + j\beta \quad (41)$$

- General solution of Eq. (40):

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \quad (42a)$$

- Phasor current of general wave equations:

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z} \quad (42b)$$

- Real instantaneous current:

$$I(z, t) = |I_0| \cos(\omega t \pm \beta z + \xi) = \frac{1}{2} \left(\underbrace{|I_0| e^{j\xi}}_{I_0} \right) e^{\pm j\beta z} e^{j\omega t} + c.c. = \frac{1}{2} I_s(z) e^{j\omega t} + c.c. \quad (43)$$

- Eqs. (5) and (8) $\leftarrow$ Eqs. (37b) and (43)

$$\frac{\partial V}{\partial z} = - \left(RI + L \frac{\partial I}{\partial t} \right) \Rightarrow \frac{dV_s}{dz} = - (R + j\omega L) I_s = -ZI_s \quad (44a)$$

$$\frac{\partial I}{\partial z} = - \left(GV + C \frac{\partial V}{\partial t} \right) \Rightarrow \frac{dI_s}{dz} = - (G + j\omega C) V_s = -YI_s \quad (44b)$$

- Eqs. (44a) and (44b) $\leftarrow$ Eqs. (42a) and (42b)

$$-\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{+\gamma z} = -Z(I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}) \quad (45)$$

- Line characteristic impedance:

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \frac{Z}{\gamma} = \frac{Z}{\sqrt{ZY}} = \sqrt{\frac{Z}{Y}} \quad (46) \quad \leftarrow (40)$$

$$= \sqrt{\frac{R + j\omega L}{G + j\omega C}} = |Z_0| e^{j\theta} \quad (47)$$

[Ex. 10.2] Lossless line length $l = 0.8$ [m], $f = 600$ [MHz],

Line parameters: $L = 0.25$ [μ H/m], $C = 100$ [pF/m]

➔ Because of lossless line, $R = G = 0$.

Characteristic impedance:

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.25 \times 10^{-6}}{100 \times 10^{-12}}} = 50 \text{ } [\Omega]$$

Since $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} = j\omega\sqrt{LC}$,

$$\alpha = 0$$

$$\beta = 2\pi \times (600 \times 10^6) \times \sqrt{(0.25 \times 10^{-6})(100 \times 10^{-12})} = 18.85 \text{ } [\text{rad/m}]$$

$$v_p = \frac{\omega}{\beta} = \frac{2\pi(600 \times 10^6)}{18.85} = 2 \times 10^8 \text{ } [\text{m/s}]$$

10.7 Low-Loss Propagation

- Eq. (42a) ← Eq. (41):

$$V_s(z) = V_0^+ e^{-\alpha z} e^{-j\beta z} + V_0^- e^{\alpha z} e^{j\beta z} \quad (48)$$

- Real instantaneous voltage:

$$V(z, t) = \text{Re}[V_s(z) e^{j\omega t}] = V_0^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_0^- e^{\alpha z} \cos(\omega t + \beta z) \quad (49)$$

Forward-propagating wave

Backward-propagating wave

Attenuation coefficient: α [Np/m]

Phase constant: β ($= \omega / v_p$) [rad/m]

- Lossless case: $R = G = 0$, $\alpha = 0$, $\gamma = j\beta = j\omega(LC)^{0.5}$,
 $v_p = 1 / (LC)^{0.5}$

- Low loss case: $R \ll \omega L$, $G \ll \omega C$ (but $R \neq 0$ and $G \neq 0$)

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (41)$$

$$= \sqrt{j\omega L \left(1 + \frac{R}{j\omega L}\right)} \sqrt{j\omega C \left(1 + \frac{G}{j\omega C}\right)} = j\omega \sqrt{LC} \sqrt{1 + \frac{R}{j\omega L}} \sqrt{1 + \frac{G}{j\omega C}} \quad (50)$$

$$\leftarrow \sqrt{1+x} \cong 1 + \frac{x}{2} - \frac{x^2}{8} + \dots \quad (x \ll 1) \quad (51)$$

$$\gamma \cong j\omega \sqrt{LC} \left[\left(1 + \frac{R}{j2\omega L} + \frac{R^2}{8\omega^2 L^2} + \dots\right) \left(1 + \frac{G}{j2\omega C} + \frac{G^2}{8\omega^2 C^2} + \dots\right) \right] \quad (52)$$

Since $R \ll \omega L$ and $G \ll \omega C$, RG^2 , R^2G and R^2G^2 can be negligible,

$$\begin{aligned}\gamma = \alpha + j\beta &\cong j\omega\sqrt{LC} \left[\left(1 + \frac{R}{j2\omega L} + \frac{R^2}{8\omega^2 L^2} \right) \left(1 + \frac{G}{j2\omega C} + \frac{G^2}{8\omega^2 C^2} \right) \right] \\ &\cong j\omega\sqrt{LC} \left[1 + \frac{1}{j2\omega} \left(\frac{R}{L} + \frac{G}{C} \right) + \frac{1}{8\omega^2} \left(\frac{R^2}{L^2} - \frac{2RG}{LC} + \frac{G^2}{C^2} \right) \right] \quad (53)\end{aligned}$$

$$\alpha = j\omega\sqrt{LC} \left[\frac{1}{j2\omega} \left(\frac{R}{L} + \frac{G}{C} \right) \right] = \frac{1}{2} \left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}} \right) \quad (54a)$$

$$\begin{aligned}\beta &= \omega\sqrt{LC} \left[1 + \frac{1}{8\omega^2} \left(\frac{R^2}{L^2} - \frac{2RG}{LC} + \frac{G^2}{C^2} \right) \right] = \omega\sqrt{LC} \left[1 + \frac{1}{8} \left(\frac{R}{\omega L} - \frac{G}{\omega C} \right)^2 \right] \\ &= \omega\sqrt{LC} \left[1 + \frac{1}{8} \left(\frac{G}{\omega C} - \frac{R}{\omega L} \right)^2 \right] \quad (54b)\end{aligned}$$

- Phase velocity: $v_p = \omega/\beta$

Signal velocity: $v_g = d\omega/d\beta \quad \leftarrow$ signal distortion

➔ If signal is distortionless, $\beta = \omega(LC)^{0.5}$ ($\leftarrow R = G = 0$)

- Characteristic impedance in case of low-loss line:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{j\omega L(1 + \frac{R}{j\omega L})}{j\omega C(1 + \frac{G}{j\omega C})}} \cong \sqrt{\frac{L}{C}} \left[\frac{1 + \frac{R}{j2\omega L} + \frac{R^2}{8\omega^2 L^2}}{1 + \frac{G}{j2\omega C} + \frac{G^2}{8\omega^2 C^2}} \right] \quad (55)$$

$$\cong \sqrt{\frac{L}{C}} \left[\left(1 + \frac{R}{j2\omega L} + \frac{R^2}{8\omega^2 L^2} \right) \left(1 - \frac{G}{j2\omega C} - \frac{G^2}{8\omega^2 C^2} \right) \right] \leftarrow \frac{1}{1+x} \cong 1-x$$

$$\cong \sqrt{\frac{L}{C}} \left[1 + \frac{1}{2\omega^2} \left\{ \frac{1}{4} \left(\frac{R^2}{L^2} - \frac{G^2}{C^2} \right) - \frac{RG}{LC} \right\} + \frac{j}{2\omega} \left(\frac{G}{C} - \frac{R}{L} \right) \right] \quad (56)$$

[Ex. 10.3] Transmission line with $G = 0$, $R \neq 0$ and $R \ll \omega L$

Characteristic impedance:

$$Z_0 = \sqrt{\frac{L}{C}} \left[1 + \frac{1}{2\omega^2} \left\{ \frac{1}{4} \left(\frac{R^2}{L^2} - \frac{G^2}{C^2} \right) - \frac{RG}{2LC} \right\} + \frac{j}{2\omega} \left(\frac{G}{C} - \frac{R}{L} \right) \right] \bigg|_{G=0, R \ll \omega L}$$

$$\cong \sqrt{\frac{L}{C}} \left[1 + \frac{j}{2\omega} \left(-\frac{R}{L} \right) \right] \cong |Z_0| e^{j\theta}$$

$$\text{where } |Z_0| \cong \sqrt{L/C} \quad \text{and} \quad \theta = \tan^{-1}(-R / 2\omega L)$$

10.8 Power Transmission and The Use of Decibels in Loss Characterization

- Consider the forward-propagating voltage and current term with some phase shifting.

$$P(z, t) = V(z, t)I(z, t) = |V_0| |I_0| e^{-2\alpha z} \cos(\omega t - \beta z) \cos(\omega t - \beta z + \theta) \quad (57)$$

- Time-averaged power $\langle P \rangle$:

$$\langle P \rangle = \frac{1}{T} \int_0^T |V_0| |I_0| e^{-2\alpha z} \cos(\omega t - \beta z) \cos(\omega t - \beta z + \theta) dt \quad (58)$$

$$= \frac{1}{2T} \int_0^T |V_0| |I_0| e^{-2\alpha z} [\cos(2\omega t - 2\beta z + \theta) + \cos(\theta)] dt \quad (59)$$

$$= \frac{1}{2} |V_0| |I_0| e^{-2\alpha z} \cos(\theta) = \frac{1}{2} \frac{|V_0|^2}{|Z_0|} e^{-2\alpha z} \cos(\theta) \quad [\text{W}], \quad (60)$$

where $T = 2\pi/\omega$

- Another method to obtain the instantaneous power

phasor voltage: $V_s(z) = V_0 e^{-\alpha z} e^{-j\beta z}$ (61)

phasor current: $I_s(z) = I_0 e^{-\alpha z} e^{-j\beta z} = \frac{V_0}{Z_0} e^{-\alpha z} e^{-j\beta z}$ (62)

where $Z_0 = |Z_0| e^{j\theta}$

$\langle P \rangle = \frac{1}{2} \text{Re} \{ V_s I_s^* \}$ (63) : time averaged power

$$= \frac{1}{2} \text{Re} \left\{ V_0 e^{-\alpha z} e^{-j\beta z} \frac{V_0^*}{|Z_0| e^{-j\theta}} e^{-\alpha z} e^{+j\beta z} \right\}$$

$$= \frac{1}{2} \text{Re} \left\{ \frac{V_0 V_0^*}{|Z_0|} e^{-2\alpha z} e^{j\theta} \right\} = \frac{1}{2} \frac{V_0 V_0^*}{|Z_0|} e^{-2\alpha z} \cos \theta \quad (64)(60)$$

$$\langle P(z) \rangle = \frac{1}{2} \frac{V_0 V_0^*}{|Z_0|} \cos \theta \cdot e^{-2\alpha z} = \frac{1}{2} \frac{V_0 V_0^*}{|Z_0|} e^{-2\alpha z} \cos \theta \Big|_{z=0} e^{-2\alpha z} = \langle P(0) \rangle e^{-2\alpha z} \quad (65)$$

➔ $V(z, t)$ and $I(z, t) = f(e^{-\alpha z})$

$\langle P \rangle = f(e^{-2\alpha z})$

- Power decreasing ratio:

$$\frac{\langle P(z) \rangle}{\langle P(0) \rangle} = e^{-2\alpha z} = 10^{-\kappa \alpha z} \quad (66)$$

Setting $\alpha z = 1$

$$e^{-2} = 10^{-\kappa} \Rightarrow \kappa = \log_{10}(e^2) = 0.869 \quad (67)$$

$$\text{Power loss (dB)} = 10 \log_{10} \left[\frac{\langle P(0) \rangle}{\langle P(z) \rangle} \right] = 10 \log_{10}(10^{\kappa \alpha z}) = 10(\kappa \alpha z)$$

$$= 8.69 \alpha z \quad (68)$$

$$\begin{aligned} \text{Power Loss (dB)} &= 10 \log_{10} \left[\frac{\left. \frac{1}{2} \frac{|V_0|^2}{|Z_0|} e^{-2\alpha z} \cos \theta \right|_{z=0}}{\frac{1}{2} \frac{|V_0|^2}{|Z_0|} e^{-2\alpha z} \cos \theta} \right] \\ &= 20 \log_{10} \left[\frac{|V_0(0)|}{|V_0(z)|} \right] \quad (69) \end{aligned}$$

$$\text{where } |V_0(z)| = |V_0(0)| e^{-\alpha z}$$

[Ex. 10.4] Insertion loss of line with a 20 m length: 2.0 dB

a) Power fraction: $\frac{\langle P(20) \rangle}{\langle P(0) \rangle} = 10^{-0.2} = 0.63$

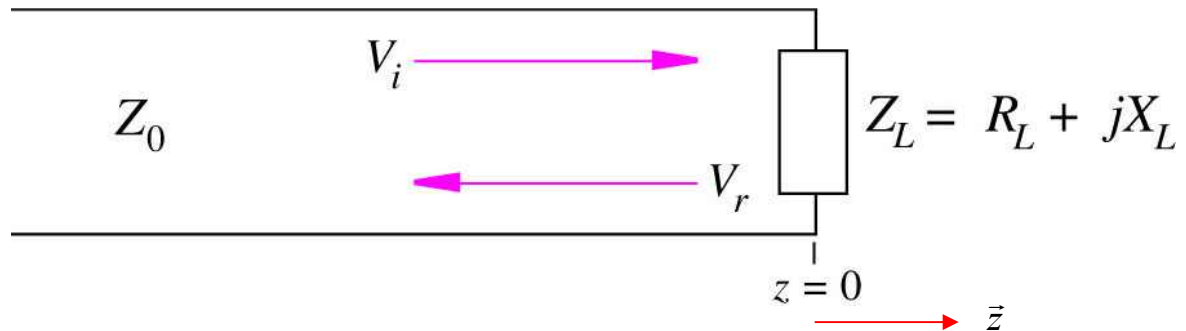
b) Power fraction at the midpoint: $10^{-0.1} = 0.79$

c) Exponential attenuation coefficient:

$$\alpha = \frac{2.0 \text{ dB}}{(8.69 \text{ dB/Np})(20 \text{ m})} = 0.012 [\text{Np/m}]$$

10.9 Wave Reflection at Discontinuities

- Voltage wave reflection from a complex load impedance



Characteristic impedance of line: Z_0

Load impedance at $z = 0$: $Z_L = R_L + jX_L$

- (Phase formed) Incident voltage wave on load:

$$V_i(z) = V_{0i} e^{-\alpha z} e^{-j\beta z} \quad (70a)$$

Reflected voltage wave from load:

$$V_r(z) = V_{0r} e^{+\alpha z} e^{+j\beta z} \quad (70b)$$

Phasor voltage at load ($@z = 0$):

$$V_L = V_{0i} + V_{0r} \quad (71)$$

- Phasor current through load:

$$I_L = I_{0i} + I_{0r} = \frac{1}{Z_0} [V_{0i} - V_{0r}] = \frac{V_L}{Z_L} = \frac{1}{Z_L} [V_{0i} + V_{0r}] \quad (72)$$

- Reflection coefficient (Γ):

$$\Gamma \equiv \frac{V_{0r}}{V_{0i}}$$

$$\frac{V_{0i}}{Z_0} \left(1 - \frac{V_{0r}}{V_{0i}} \right) = \frac{V_{0i}}{Z_L} \left(1 + \frac{V_{0r}}{V_{0i}} \right) \quad \leftarrow (72)$$

$$\frac{Z_L}{Z_0} (1 - \Gamma) = (1 + \Gamma) \quad \Gamma \left(1 + \frac{Z_L}{Z_0} \right) = \frac{Z_L}{Z_0} - 1$$

$$\Gamma = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1} = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\phi_r} \quad (73)$$

- With Eq. (71) and (73)

$$V_L = V_{0i} + V_{0r} = V_{0i} + \Gamma V_{0i} \quad (74)$$

- Transmission coefficient (τ):

$$\tau \equiv \frac{V_L}{V_{0i}} = \frac{V_{0i} + \Gamma V_{0i}}{V_{0i}} = 1 + \Gamma = 1 + \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{2Z_L}{Z_L + Z_0} = |\tau| e^{j\phi_t} \quad (75)$$

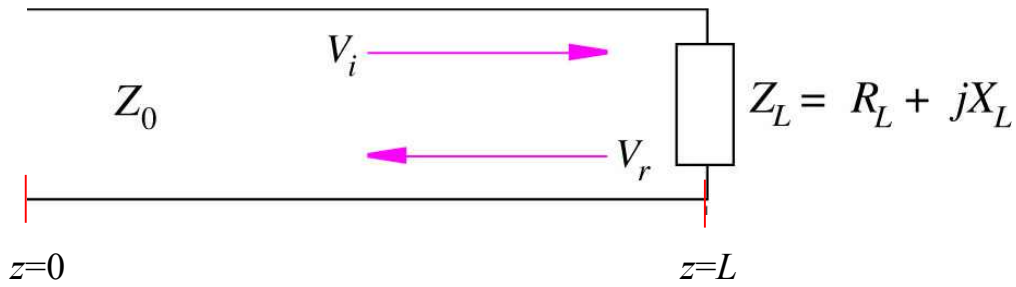
- Circuit design goal: No reflection at load

$$\Rightarrow \Gamma = 0 \quad \Rightarrow Z_L = Z_0$$

$$\Rightarrow (\text{Impedance}) \text{ Matching!!}$$

- Incident power with Eq. (64), $z = L$ and line input $z = 0$:

$$\langle P_i \rangle = \frac{1}{2} \text{Re} \left\{ \frac{V_0 V_0^*}{|Z_0|} e^{-2\alpha L} e^{j\theta} \right\} = \frac{1}{2} \frac{|V_0|^2}{|Z_0|} e^{-2\alpha L} \cos \theta \quad (76a)$$



Reflected power:

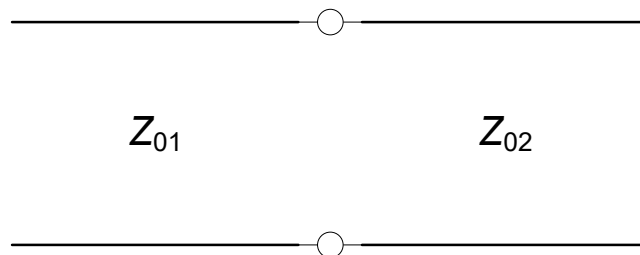
$$\begin{aligned} \langle P_r \rangle &= \frac{1}{2} \operatorname{Re} \left\{ \frac{V_{0r} V_{0r}^*}{|Z_0|} e^{-2\alpha L} e^{j\theta} \right\} = \frac{1}{2} \operatorname{Re} \left\{ \frac{(\Gamma V_0)(\Gamma^* V_0^*)}{|Z_0|} e^{-2\alpha L} e^{j\theta} \right\} \\ &= |\Gamma|^2 \frac{1}{2} \frac{|V_0|^2}{|Z_0|} e^{-2\alpha L} \cos \theta = \langle P_i \rangle |\Gamma|^2 \end{aligned} \quad (76b)$$

Reflected and transmitted power fractions at load:

$$\frac{\langle P_r \rangle}{\langle P_i \rangle} = \Gamma \Gamma^* = |\Gamma|^2 \quad (77a)$$

$$\frac{\langle P_t \rangle}{\langle P_i \rangle} = 1 - |\Gamma|^2 \quad (77b)$$

- Reflection coefficient at the connection of two semi-infinite lines:



$$\Gamma = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} \quad (78)$$

[Ex. 10.5] Lossless line characteristic impedance: 50Ω

Load impedance: $Z_L = 50 - j75 \Omega$

Incident Power: 100 mW

➔ Reflection coefficient and transmitted power:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - j75 - 50}{50 - j75 + 50} = \frac{-j75}{100 - j75} = 0.36 - j0.48 = 0.6e^{-j0.93}$$

$$\langle P_t \rangle = (1 - |\Gamma|^2) \langle P_i \rangle = [1 - (0.6)^2](100) = 64 \text{ mW}$$

[Ex. 10.6] Two lossy lines are jointed end-to-end.

Line 1: $l_1 = 10 \text{ m}$, loss rate = 0.2 dB/m

Line 2: $l_2 = 15 \text{ m}$, loss rate = 0.1 dB/m

Reflection coefficient at the junction: $\Gamma = 0.3$

$\langle P_i \rangle = 100 \text{ mW}$

➔ (a) dB loss at joint:

$$L_j \text{ (dB)} = 10 \log_{10} \left(\frac{1}{1 - |\Gamma|^2} \right) = 10 \log_{10} \left(\frac{1}{1 - 0.09} \right) = 0.41 \text{ dB}$$

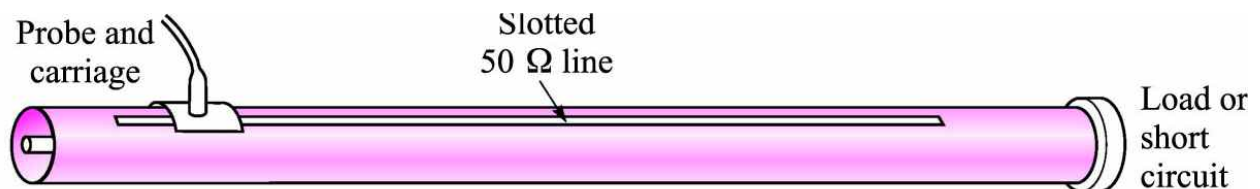
Total loss of link:

$$L_t \text{ (dB)} = (0.2)(10) + 0.41 + (0.1)(15) = 3.91 \text{ dB}$$

(b) Output power: $P_{\text{out}} = 100 \times 10^{-0.391} = 41 \text{ mW}$

10.10 Voltage Standing Wave Ratio

- Coaxial slotted line
 - Ability to measure voltage amplitudes according to the position.
 - Ruler
 - Measuring items
 - 1) Maximum & minimum voltages
 - 2) Distance between the maximum (or minimum) and the next maximum (or minimum) voltage point



- Voltage standing wave ratio (VSWR):

A ratio of maximum voltage amplitude to minimum voltage amplitude
- Case study
 - 1) Case 1: The slotted line is terminated by a matched impedance.
 - No reflected wave voltage
 - Same voltage amplitude in spite of different instantaneous voltage
 - 2) Case 2: The slotted line is terminated by an open or short circuit.
 - Reflected wave voltage
 - Different voltage amplitude

- Derivation of the (total) voltage at the specific point

- Load position: $z = 0$
- All measuring point $z < 0$
- Input voltage wave amplitude: V_0
- Line impedance: Z_0 (real), load impedance: Z_L (complex)
- Total phasor voltage:

$$V_{sT}(z) = V_0 e^{-j\beta z} + \Gamma V_0 e^{j\beta z} \quad (79)$$

- Complex reflection coefficient

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\phi} \quad (80)$$

$$\text{Ex.]} Z_L = 0 \text{ or } Z_L \text{ (for real value)} < Z_0 \rightarrow \phi = \pi$$

$$Z_L \text{ (for real value)} > Z_0 \rightarrow \phi = 0$$

$$\begin{aligned} V_{sT}(z) &= V_0 e^{-j\beta z} + \Gamma V_0 e^{j\beta z} = V_0 \left(e^{-j\beta z} + |\Gamma| e^{j(\beta z + \phi)} \right) \\ &= V_0 e^{j\phi/2} \left(e^{-j\beta z} e^{-j\phi/2} + |\Gamma| e^{j\beta z} e^{j\phi/2} \right) \quad (81) \\ &= \color{red}{V_0 (1 - |\Gamma|) e^{-j\beta z}} + V_0 e^{j\phi/2} \left(e^{-j\beta z} e^{-j\phi/2} + |\Gamma| e^{j\beta z} e^{j\phi/2} \right) - \color{red}{V_0 (1 - |\Gamma|) e^{-j\beta z}} \\ &= V_0 (1 - |\Gamma|) e^{-j\beta z} + \color{blue}{V_0 e^{-j\beta z}} + V_0 |\Gamma| e^{j(\beta z + \phi)} - \color{blue}{V_0 e^{-j\beta z}} + V_0 |\Gamma| e^{-j\beta z} \\ &= V_0 (1 - |\Gamma|) e^{-j\beta z} + V_0 |\Gamma| e^{j(\beta z + \phi)} + V_0 |\Gamma| e^{-j\beta z} \\ &= V_0 (1 - |\Gamma|) e^{-j\beta z} + V_0 |\Gamma| e^{j\phi/2} \left(e^{j\beta z} e^{j\phi/2} + e^{-j\beta z} e^{-j\phi/2} \right) \quad (82) \\ &= V_0 (1 - |\Gamma|) e^{-j\beta z} + 2V_0 |\Gamma| e^{j\phi/2} \cos(\beta z + \phi / 2) \quad (83) \end{aligned}$$

- Real instantaneous form:

$$\begin{aligned}
 V(z, t) &= \text{Re}[V_{sT}(z)e^{j\omega t}] \\
 &= \underbrace{V_0(1-|\Gamma|)\cos(\omega t - \beta z)}_{\text{traveling wave}} + \underbrace{2|\Gamma|V_0\cos(\beta z + \phi/2)\cos(\omega t + \phi/2)}_{\text{standing wave}} \quad (84)
 \end{aligned}$$

Standing wave: the reflected or back-propagated wave interferes with an equivalent portion of the incident wave to form a standing wave.

- Phasor-formed incident voltage with Eq. (79) and (80):

$$V_{sT}(z) = V_0(e^{-j\beta z} + |\Gamma|e^{j(\beta z + \phi)}) \quad (85)$$

Locations of minimum voltage amplitude:

$$-\beta z_{\min} = \beta z_{\min} + \phi + \pi + 2m\pi \quad (m = 0, 1, 2, \dots)$$

$$z_{\min} = -\frac{1}{2\beta}\{\phi + (2m + 1)\pi\} \quad (86)$$

Minimum voltage amplitude:

$$V_{sT}(z_{\min}) = V_0(1 - |\Gamma|) \quad (87)$$

In Eq. (84),

$$\omega t - \beta z_{\min} = \omega t - \beta \times \left(-\frac{1}{2\beta}\right)\{\phi + (2m + 1)\pi\} = \omega t + \frac{\phi}{2} + \frac{1}{2}(2m + 1)\pi$$

$$\beta z_{\min} + \frac{\phi}{2} = \beta \times \left(-\frac{1}{2\beta}\right)\{\phi + (2m + 1)\pi\} + \frac{\phi}{2} = -\frac{1}{2}(2m + 1)\pi$$

Instantaneous voltage at the minimum point from Eq. (84)

$$\begin{aligned}
 V(z_{\min}, t) &= \underbrace{V_0(1-|\Gamma|)\cos(\omega t - \beta z)}_{\text{traveling wave}} + \underbrace{2|\Gamma|V_0\cos(\beta z + \phi/2)\cos(\omega t + \phi/2)}_{\text{standing wave}} \bigg|_{z=z_{\min}} \quad (84) \\
 &= V_0(1-|\Gamma|)\cos\left\{\omega t + \frac{\phi}{2} + \frac{1}{2}(2m+1)\pi\right\} + 2|\Gamma|V_0\cos\{(2m+1)\pi/2\}\cos(\omega t + \phi/2) \\
 &= V_0(1-|\Gamma|)\{\cos(\omega t + \phi/2)\cos\{(2m+1)\pi/2\} - \sin(\omega t + \phi/2)\sin\{(2m+1)\pi/2\}\} \\
 &\quad + 2|\Gamma|V_0\cos\{(2m+1)\pi/2\}\cos(\omega t + \phi/2) \\
 &= \pm V_0(1-|\Gamma|)\sin(\omega t + \phi/2) \quad (88)
 \end{aligned}$$

• Locations of maximum voltage amplitude:

$$\begin{aligned}
 -\beta z_{\max} &= \beta z_{\max} + \phi + 2m\pi \quad (n = 0, \pm 1, \pm 2, \dots) \\
 z_{\max} &= -\frac{1}{2\beta}(\phi + 2m\pi) \quad (89)
 \end{aligned}$$

Maximum voltage amplitude:

$$V_{sT}(z_{\max}) = V_0(1 + |\Gamma|) \quad (90)$$

In Eq. (84),

$$\begin{aligned}
 \omega t - \beta z_{\max} &= \omega t - \beta \times \left(-\frac{1}{2\beta}\right)(\phi + 2m\pi) = \omega t + \frac{\phi}{2} + m\pi \\
 \beta z_{\max} + \frac{\phi}{2} &= \beta \times \left(-\frac{1}{2\beta}\right)(\phi + 2m\pi) + \frac{\phi}{2} = -m\pi
 \end{aligned}$$

Instantaneous voltage at the maximum point from Eq. (84)

$$\begin{aligned}
 V(z_{\max}, t) &= \underbrace{V_0(1-|\Gamma|)\cos(\omega t - \beta z)}_{\text{traveling wave}} + \underbrace{2|\Gamma|V_0\cos(\beta z + \phi/2)\cos(\omega t + \phi/2)}_{\text{standing wave}} \bigg|_{z=z_{\max}} \quad (84) \\
 &= V_0(1-|\Gamma|)\left[\cos\left\{\omega t + \frac{\phi}{2} + m\pi\right\} + 2|\Gamma|V_0\cos(-m\pi)\cos\left(\omega t + \frac{\phi}{2}\right)\right] \\
 &= V_0(1-|\Gamma|)\left[\cos\left(\omega t + \frac{\phi}{2}\right)\cos(m\pi) + \sin\left(\omega t + \frac{\phi}{2}\right)\sin(m\pi)\right] + 2|\Gamma|V_0\cos\left(\omega t + \frac{\phi}{2}\right)\cos(m\pi) \\
 &= V_0(1+|\Gamma|)\cos\left(\omega t + \frac{\phi}{2}\right)\cos(m\pi) = \pm V_0(1+|\Gamma|)\cos\left(\omega t + \frac{\phi}{2}\right) \quad (91)
 \end{aligned}$$

* Case studies

$$Z_L > Z_0 \ \& \ (Z_L, Z_0 : \text{real}) \Rightarrow \Gamma : \text{real positive} \Rightarrow \phi = 0$$

$\Rightarrow$ Voltage maximum is at load.

$$\begin{aligned}
 \Rightarrow \text{Voltage maxima are found at } z_{\max} &= -m\pi / \beta = -m\lambda / 2 \\
 &\stackrel{\text{or}}{=} 0, -\frac{\lambda}{2}, -\lambda, \dots
 \end{aligned}$$

$$Z_L = 0 \Rightarrow \Gamma = -1 \Rightarrow \phi = \pi$$

$\Rightarrow$ Voltage maxima are found at

$$\begin{aligned}
 z_{\max} &= -\frac{1}{2\beta}(\phi + 2m\pi) \bigg|_{\phi=\pi} = -\frac{\pi}{2\beta}, -\frac{3\pi}{2\beta}, -\frac{5\pi}{2\beta}, \dots \leftarrow \frac{1}{\beta} = \frac{\lambda}{2\pi} \\
 &\stackrel{\text{or}}{=} -\frac{\lambda}{4}, -\frac{3\lambda}{4}, \dots
 \end{aligned}$$

Distance between the minima: $\lambda/2$

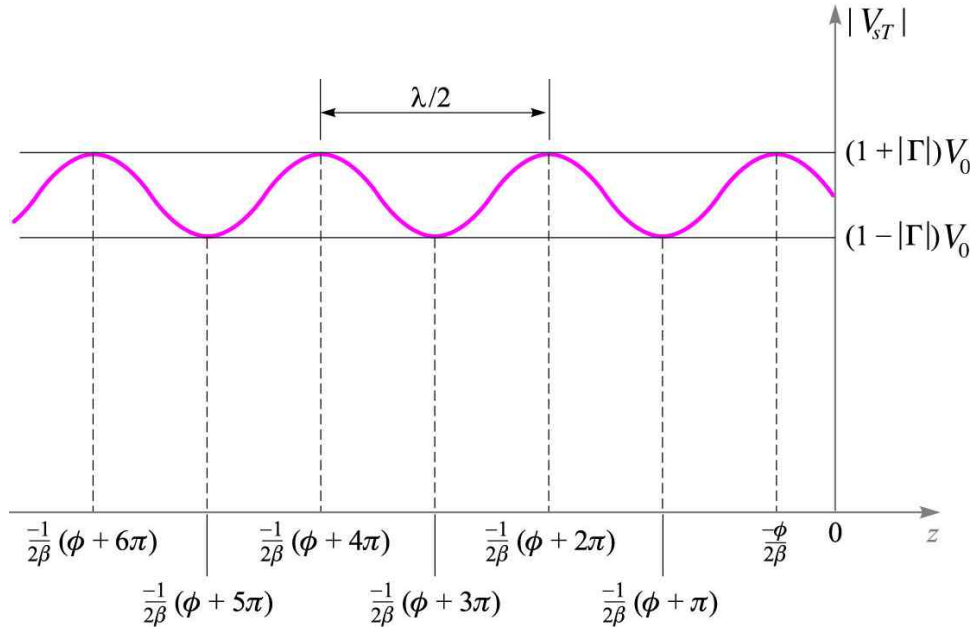
$$\begin{aligned}
 z_{\min} &= -\frac{1}{2\beta}\{\phi + (2m+1)\pi\} \quad (86) \leftarrow \frac{1}{\beta} = \frac{\lambda}{2\pi} \\
 &= -\frac{\phi}{2\beta} - \frac{(2m+1)\pi}{2\beta} = -\frac{\phi}{2\beta} - \frac{(2m+1)\lambda}{4}
 \end{aligned}$$

$$Z_L = 0 \Rightarrow \Gamma = -1 \Rightarrow \phi = \pi$$

$\Rightarrow$ The 1st voltage minimum is found at

$$z_{\min} = -\frac{1}{2\beta} \left\{ \phi + (2m+1)\pi \right\} \Big|_{\phi=\pi, m=-1} = 0$$

$$Z_L < Z_0 \Rightarrow \phi = \pi \Rightarrow z_{\min} = -\frac{1}{2\beta} \left\{ \phi + (2m+1)\pi \right\} \Big|_{\phi=\pi, m=-1} = 0$$



• Voltage standing wave ratio (VSWR):

$$S \equiv \frac{V_{sT}(z_{\max})}{V_{sT}(z_{\min})} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad (92)$$

[Ex. 10.7] 50 Ω slotted line measurement

- VSWR (s) = 5, successive voltage maximum spacing = 15 cm
- Distance of the 1st maximum from load = 7.5 cm

→ $\lambda/2 = 0.15, \lambda = 0.3$

$$f = c / \lambda = (3 \times 10^8) / 0.3 = 1 \times 10^9 = 1 \text{ GHz}$$

7.5 cm $\leftrightarrow \lambda/4 \leftrightarrow$ Voltage minimum occurs at the load.

$$\leftrightarrow Z_L < Z_0$$

$$\leftrightarrow \phi = \pi$$

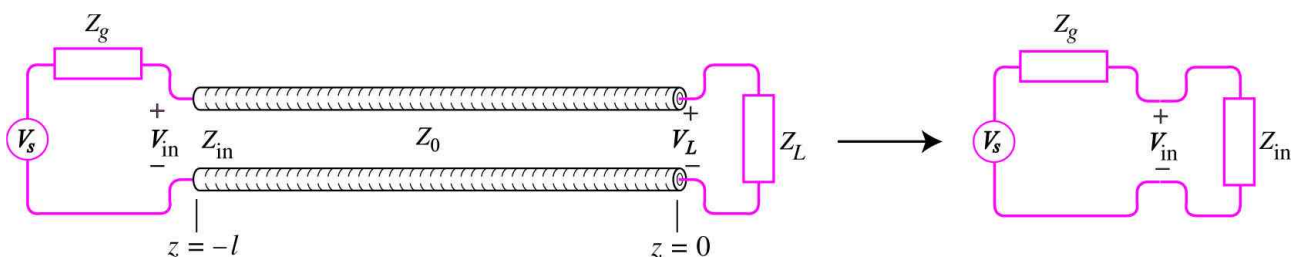
$$|\Gamma| = \frac{s-1}{s+1} = \frac{5-1}{5+1} = \frac{2}{3}$$

$$\Gamma = -\frac{2}{3} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_L = \frac{1}{5} Z_0 = 10 \text{ } \Omega$$

10.11 Transmission Lines of Finite Length

- Finite-length transmission line configuration and its equivalent circuit



- Total voltage on line

$$V_{sT}(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \quad (93)$$

Where V_0^+ : sum of all individual forward waves

V_0^- : sum of all individual backward waves

- Total current in line:

$$I_{sT}(z) = I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z} \quad (94)$$

- Wave impedance $Z_w(z)$: a ratio of total phasor voltage to total phasor current at $z = -l$

$$Z_w(z) \equiv \frac{V_{sT}(z)}{I_{sT}(z)} = \frac{V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}}{I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z}} \quad (95) \quad \leftarrow V_0^- = \Gamma V_0^+, I_0^+ = V_0^+ / Z_0, I_0^- = -V_0^- / Z_0$$

$$= \frac{V_0^+ e^{-j\beta z} + \Gamma V_0^+ e^{j\beta z}}{(V_0^+ e^{-j\beta z} - \Gamma V_0^+ e^{j\beta z}) / Z_0} = Z_0 \left[\frac{e^{-j\beta z} + \Gamma e^{j\beta z}}{e^{-j\beta z} - \Gamma e^{j\beta z}} \right] \quad (96) \quad \leftarrow \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= Z_0 \left[\frac{(Z_L + Z_0)e^{-j\beta z} + (Z_L - Z_0)e^{j\beta z}}{(Z_L + Z_0)e^{-j\beta z} - (Z_L - Z_0)e^{j\beta z}} \right]$$

$$= Z_0 \left[\frac{Z_L(e^{j\beta z} + e^{-j\beta z}) - Z_0(e^{j\beta z} - e^{-j\beta z})}{Z_0(e^{j\beta z} + e^{-j\beta z}) - Z_L(e^{j\beta z} - e^{-j\beta z})} \right]$$

$$= Z_0 \left[\frac{Z_L(e^{j\beta z} + e^{-j\beta z}) / 2 - jZ_0(e^{j\beta z} - e^{-j\beta z}) / j2}{Z_0(e^{j\beta z} + e^{-j\beta z}) / 2 - jZ_L(e^{j\beta z} - e^{-j\beta z}) / j2} \right]$$

$$= Z_0 \left[\frac{Z_L \cos(\beta z) - jZ_0 \sin(\beta z)}{Z_0 \cos(\beta z) - jZ_L \sin(\beta z)} \right] \quad (97) \quad \leftarrow z = -l$$

$$= Z_0 \left[\frac{Z_L \cos(\beta l) + jZ_0 \sin(\beta l)}{Z_0 \cos(\beta l) + jZ_L \sin(\beta l)} \right] \quad (98)$$

$$= Z_0 \left[\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right]$$

- In case of $\beta l = \frac{2\pi}{\lambda} \frac{m\lambda}{2} = m\pi \quad (m = 0, 1, 2, \dots)$,

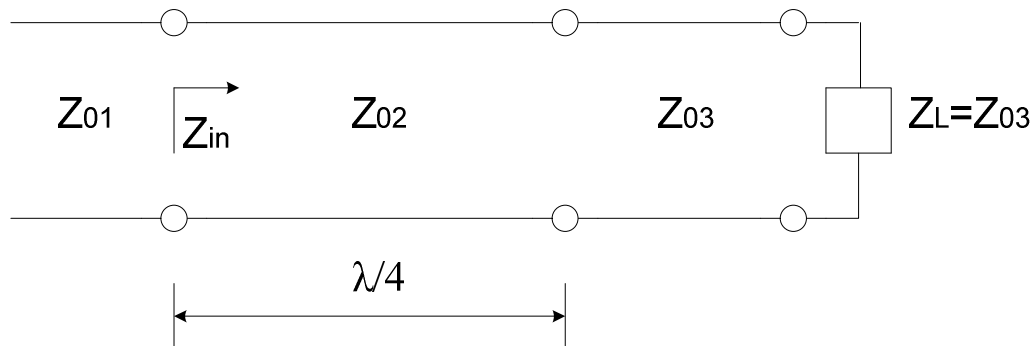
$$Z_{in}(l = m\lambda/2) = Z_L \quad (99)$$

➔ 부하(load)에서 $m\lambda/2$ 떨어진 지점에서의 입력 임피던스는 부하 임피던스와 같음.

- In case $\beta l = \frac{2\pi}{\lambda} (2m+1) \frac{\lambda}{4} = (2m+1) \frac{\pi}{2} \quad (m = 0, 1, 2, \dots)$,

$$Z_{in}(l = \lambda/4) = \frac{Z_0^2}{Z_L} \quad (100)$$

- Joining two lines having different characteristic impedances



Input impedance to line 2:

$$Z_{in} = Z_{02} \frac{Z_{03} \cos(\beta_2 l) + jZ_{02} \sin(\beta_2 l)}{Z_{02} \cos(\beta_2 l) + jZ_{03} \sin(\beta_2 l)} \quad (101) \quad \leftarrow l = \frac{\lambda}{4}, \beta l = \frac{2\pi}{\lambda} \frac{\lambda}{4} = \frac{\pi}{2}$$

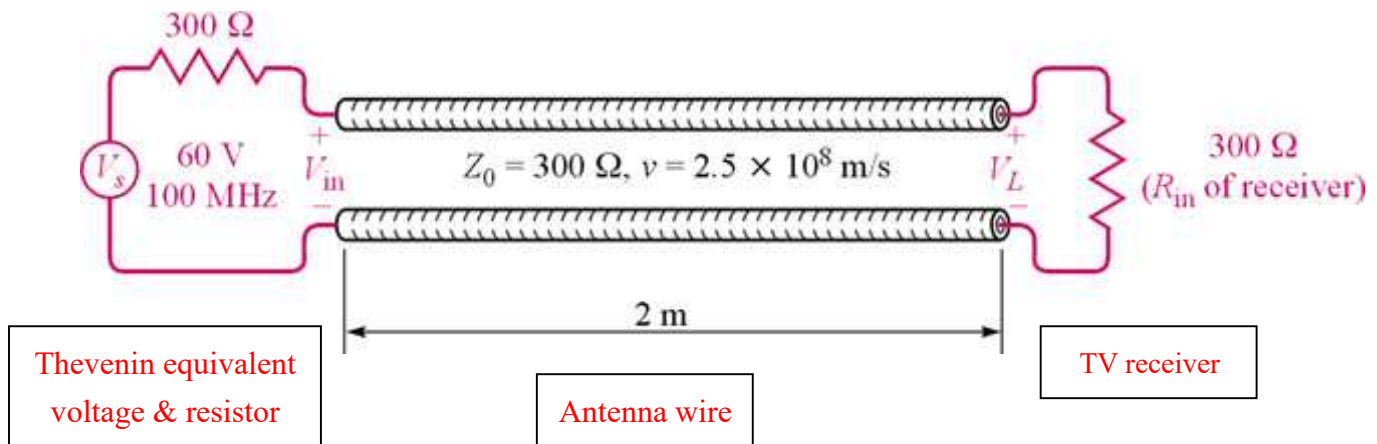
$$= \frac{Z_{02}^2}{Z_{03}} \quad (102)$$

Quarter-wave matching:

$$Z_{02} = \sqrt{Z_{01} Z_{03}} \quad (@ l = (2m+1)\lambda/4) \quad (103)$$

10.12 Some Transmission Line Examples

[Ex. 1]



$$\bullet R_s = R_C = R_L \Rightarrow \text{Matched} \Rightarrow \Gamma = 0 \Rightarrow S = 1$$

$$\bullet \lambda = \frac{v}{f} = \frac{2.5 \times 10^8}{100 \times 10^6} = 2.5 \text{ [m]} \quad (\text{wavelength})$$

$$\bullet \beta = \frac{2\pi}{\lambda} = \frac{2\pi}{2.5} = 0.8\pi \text{ [rad/m]} \quad (\text{phase constant})$$

$$\alpha = 0 \quad (\text{lossless assumption})$$

$$\bullet \beta l = 0.8\pi \times 2 = 1.6\pi \text{ [rad]} \quad (\text{electric length})$$

$$\Leftrightarrow \frac{1.6\pi}{2\pi} \times 360^\circ = 288^\circ \quad \Leftrightarrow \frac{1.6\pi}{2\pi} \times \lambda = 0.8\lambda$$

$$\bullet V_s = 60 \cos 2\pi 10^8 t \text{ [V]}$$

$$V_{\text{in}} = \frac{300}{300 + 300} V_s = 30 \cos 2\pi 10^8 t \text{ [V]}$$

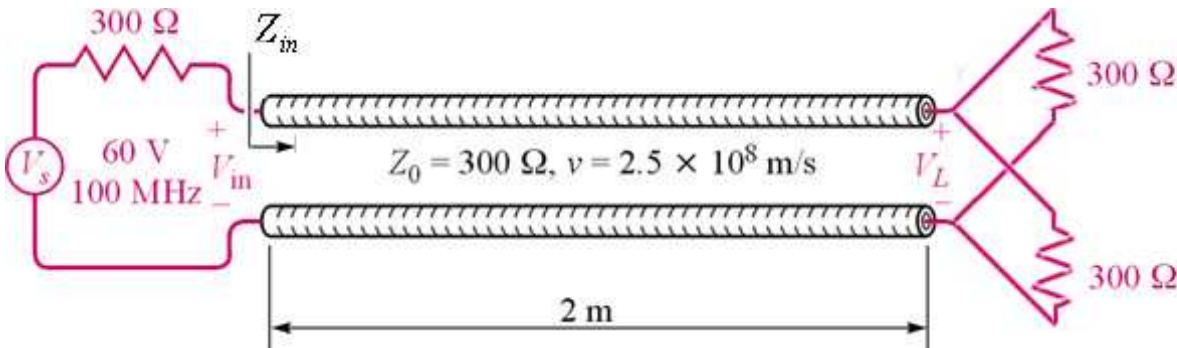
$$V_L = 30 \cos(\omega t - \beta l) = 30 \cos(2\pi 10^8 t - 1.6\pi) \quad [\text{V}]$$

$$I_{\text{in}} = \frac{V_{\text{in}}}{Z_{\text{in}}} = \frac{30 \cos 2\pi 10^8 t}{300} = 0.1 \cos 2\pi 10^8 t \quad [\text{A}]$$

$$I_L = \frac{V_L}{Z_L} = 0.1 \cos(2\pi 10^8 t - 1.6\pi) \quad [\text{A}]$$

$$P_{\text{in}} = P_L = \frac{1}{2} \times 30 \times 0.1 = 1.5 \quad [\text{W}] \leftarrow P_L = 30 \cos(2\pi 10^8 t) \times 0.1 \cos(2\pi 10^8 t) \\ = 3 \cos^2(2\pi 10^8 t) = \frac{3 + 3 \cos(2\pi \times 2 \times 10^8 t)}{2}$$

[Ex. 2]



$$Z_{L1} = Z_{L2} = 300 \, [\Omega], \quad Z_L = Z_{L1} // Z_{L2} = 150 \, [\Omega]$$

$$\Gamma = \frac{150 - 300}{150 + 300} = -\frac{1}{3} = \frac{1}{3} \angle \pi, \quad s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{4/3}{2/3} = 2$$

$$Z_{\text{in}} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} = 300 \frac{150 + j300 \tan 288^\circ}{300 + j150 \tan 288^\circ} = 300 \frac{150 - j923.3}{300 - j461.7}$$

$$= 300 \frac{935.1 \angle -80.77^\circ}{550.6 \angle -56.99^\circ} = 509.5 \angle -23.78^\circ$$

$$= 466.2 \angle -23.78^\circ - j205.4 \quad \rightarrow \text{capacitive} \Rightarrow \text{store electric field energy}$$

$$I_{s,\text{in}} = \frac{V_{s,\text{in}}}{Z_s + Z_{\text{in}}} = \frac{60 \angle 0^\circ}{300 + (466.2 - j205.4)} = \frac{60 \angle 0^\circ}{766.2 - j205.4}$$

$$= \frac{60 \angle 0^\circ}{793.3 \angle -15^\circ} = 0.0756 \angle 15^\circ \quad [\text{A}]$$

$$P_{\text{in}} = \frac{1}{2} |I_{s,\text{in}}|^2 \cdot \text{Re}(Z_{\text{in}}) = \frac{1}{2} \times 0.0756^2 \times 466.2 = 1.332 \quad [\text{W}]$$

$$= P_L \quad \because \quad \text{lossless transmission line}$$

$$< 1.5 \text{ [W]} \quad (\text{in previous example result})$$

$$P_L \text{ is divided equally between two receivers.} \Rightarrow 0.666 \text{ [W / receiver]}$$

Voltage across the receiver:

$$\frac{1}{2} P_L = \frac{1}{2} \left(|V_{s,L}|^2 / R_L \right)$$

$$0.666 = \frac{1}{2} \frac{|V_{s,L}|^2}{300}$$

$$|V_{s,L}| = \sqrt{0.666 \times 600} = 19.99 \quad [\text{V}]$$

$$< 30 \text{ [V]} \quad (\text{in previous example result})$$

• Voltage maxima:

$$-\beta z_{\text{max}} = \frac{\phi}{2} + n\pi \quad (n = 0, \pm 1, \pm 2, \dots) \quad \text{cf.)} \quad -\beta z_{\text{min}} = \frac{\phi}{2} + n\pi + \frac{\pi}{2}$$

$$\Gamma = |\Gamma| e^{j\phi} \quad \rightarrow \quad \because \quad \Gamma = -\frac{1}{3} = \frac{1}{3} \angle \pi$$

When $\beta = 0.8\pi$, $\phi = \pi$, $\leftarrow$ region 2가 conductor이거나 $Z_L < Z_0$ 인 경우이므로

$$z_{\text{max}} = -\frac{1}{0.8\pi} \left[\frac{\pi}{2} + n\pi \right] = -0.625 - 1.25n$$

$$= -0.625 \quad \text{or} \quad -1.875 \text{ m}$$

$$z_{\min} = z_{\max} \pm \frac{\lambda}{4} \quad \leftarrow \quad \text{Fig 11.6}$$

$$= 0 \quad \text{or} \quad -1.25 \text{ [m]}$$

General conclusion: voltage minimum at load if $Z_L < Z_0$
in case Z_L and Z_0 are real
voltage maximum at load if $Z_L > Z_0$
in case Z_L and Z_0 are real

$$\bullet V_{s,\text{in}} = I_{s,\text{in}} Z_{\text{in}} = (0.0756 \angle 15^\circ) \times (509.5 \angle -23.78^\circ)$$

$$= 38.52 \angle -8.78^\circ$$

$$\text{cf.) } \left(\begin{array}{l} |V_{s,L}| = 19.99 \text{ V} \approx 20 \text{ V} \quad \because Z_L < Z_0 \text{ 이므로.} \\ V_{s,\text{max}} = S \times V_{s,\text{min}} = 2 \times 20 = 40 \text{ V} \\ V_{s,\text{in}} \text{ 은 } 40 \text{ V 에 근접함 } \because L = 2 \text{ m 이고 } \lambda = 2.5 \text{ m 이므로} \\ (3/4)\lambda = 1.875 \text{ m 로 } L \text{ 은 } (3/4)\lambda \text{ 근처이므로} \end{array} \right.$$

Near voltage maximum location

• Voltage at an arbitrary point on the line:

$$V_s = (e^{-j\beta z} + \Gamma e^{j\beta z}) V_0^+ \quad (104)$$

At $z = -l$, ($z = -2 \text{ m}$)

$$V_{s,\text{in}} = (e^{j\beta l} + \Gamma e^{-j\beta l}) V_0^+ \quad (105)$$

$$V_0^+ = \frac{V_{s,\text{in}}}{e^{j\beta l} + \Gamma e^{-j\beta l}} = \frac{38.52 \angle -8.78^\circ}{e^{j1.6\pi} - (1/3)e^{-j1.6\pi}} = \frac{38.52 \angle -8.78^\circ}{(0.309 - j0.957) - (0.103 + j0.317)}$$

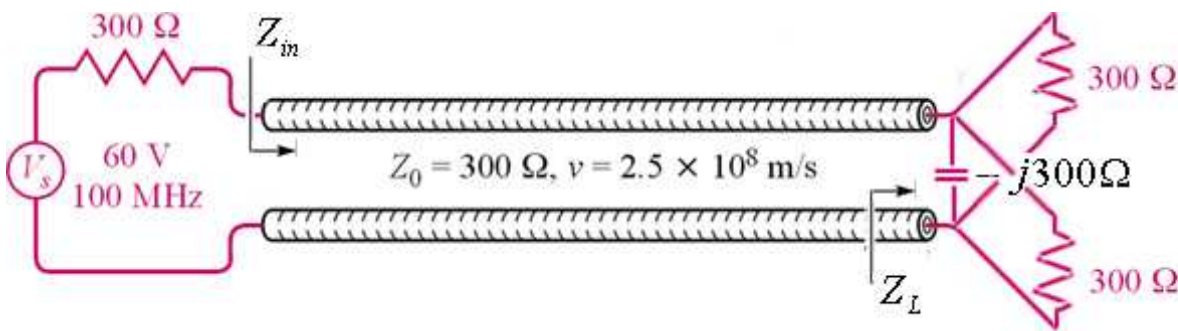
$$= \frac{38.52 \angle -8.78^\circ}{0.206 - j1.268} = \frac{38.52 \angle -8.78^\circ}{1.285 \angle -80.77^\circ} = 29.98 \angle 71.99^\circ$$

At $z = 0$,

$$V_{s,L} = (1 + \Gamma)V_0^+ = \frac{2}{3} \times 29.98 \angle 71.99^\circ = 19.99 \angle 71.99^\circ = \underline{19.99 \angle -288.1^\circ}$$

: Same result with the previous

[Ex. 10.8]



$$Z_L = \frac{150(-j300)}{150 - j300} = \frac{-j300}{1 - j2} = -j \frac{300}{5} (1 + j2) = 120 - j60 \quad \Omega$$

$$\Gamma = \frac{(120 - j60) - 300}{(120 - j60) + 300} = \frac{-180 - j60}{420 - j60} = \frac{189.7 \angle -161.6^\circ}{424.3 \angle -8.13^\circ} = 0.447 \angle -153.47^\circ$$

$$s = \frac{1 + 0.447}{1 - 0.447} = 2.62$$

⇒ Standing wave ratio is higher than example 2.

$$\begin{aligned} Z_{in} &= 300 \frac{(120 - j60) + j300 \tan 288^\circ}{300 + j(120 - j60) \tan 288^\circ} = 300 \frac{(120 - j60) + j300 \times (-3.08)}{300 + j(120 - j60)(-3.08)} \\ &= 300 \frac{120 - j984}{115.2 - j369.6} = 300 \frac{991.3 \angle -83^\circ}{387.1 \angle -72.7^\circ} \end{aligned}$$

$$= 768.3 \angle -10.3^\circ = 755.9 - j137.4$$

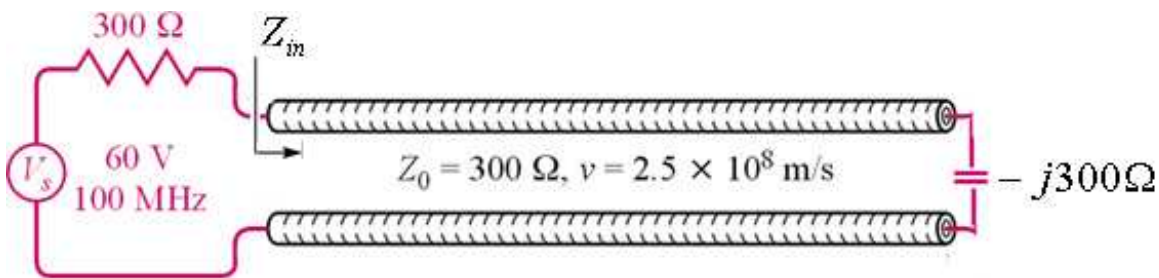
$$I_{s,\text{in}} = \frac{V_{Th}}{Z_{Th} + Z_{\text{in}}} = \frac{60}{300 + 755.9 - j137.4} = \frac{60}{1065 \angle -7.41^\circ} = 0.0563 \angle 7.41^\circ \text{ A}$$

$$P_{\text{in}} = \frac{1}{2} \times 0.0563^2 \times 755.9 = 1.198 \text{ W}$$

$$= P_L \quad (\because \text{Transmission line is lossless.})$$

➔ Each load dissipates about 0.6 W and less than the previous Ex.

[Ex. 10.9]



$$Z_L = -j300$$

$$\Gamma = \frac{-j300 - 300}{-j300 + 300} = \frac{\sqrt{2} \angle -135^\circ}{\sqrt{2} \angle -45^\circ} = 1 \angle -90^\circ$$

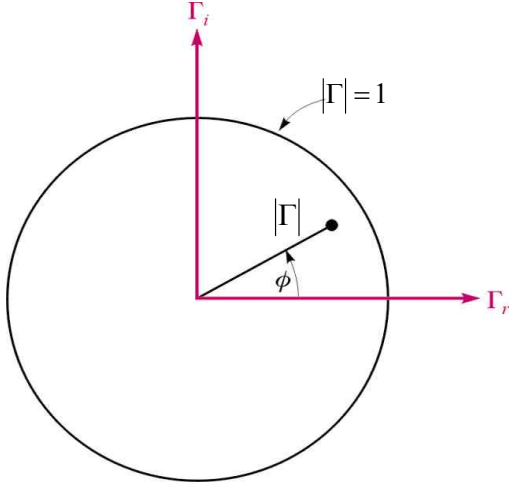
$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \infty$$

$$Z_{\text{in}} = 300 \frac{(-j300) + j300 \tan 288^\circ}{300 + j(-j300) \tan 288^\circ} = 300 \frac{-j1224}{300 - 924} = -j588.5$$

$$\Rightarrow \text{No average power can be delivered.} \quad \left(\because \frac{1}{2} I_{s,\text{in}}^2 \cdot \underbrace{R_{\text{in}}}_{0} \right)$$

10.13 Graphic Methods: The Smith Chart

- Polar coordinates of the Smith chart: **Reflection coefficient plane**



$$\Gamma = |\Gamma|e^{j\phi}$$

$$\text{where } 0 \leq |\Gamma| \leq 1$$

- Basic relationship

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (106)$$

Normalized load impedance

$$z_L = r + jx = \frac{Z_L}{Z_0} = \frac{R_L + jX_L}{Z_0}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(Z_L/Z_0) - 1}{(Z_L/Z_0) + 1} = \frac{z_L - 1}{z_L + 1} = \Gamma_r + j\Gamma_i \quad (108)$$

$$z_L = \frac{1 + \Gamma}{1 - \Gamma} \quad (107)$$

$$= \frac{(1 + \Gamma_r) + j\Gamma_i}{(1 - \Gamma_r) - j\Gamma_i} = \frac{[(1 + \Gamma_r) + j\Gamma_i][(1 - \Gamma_r) + j\Gamma_i]}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

$$= \frac{[(1 - \Gamma_r^2) - \Gamma_i^2] + j[\Gamma_i(1 - \Gamma_r) + \Gamma_i(1 + \Gamma_r)]}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

$$= \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2} + j \frac{2\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2} = r + jx \quad (109)$$

$$\therefore r = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2} \quad (110)$$

$$x = \frac{2\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2} \quad (111)$$

1) Real part

$$\frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2} = r$$

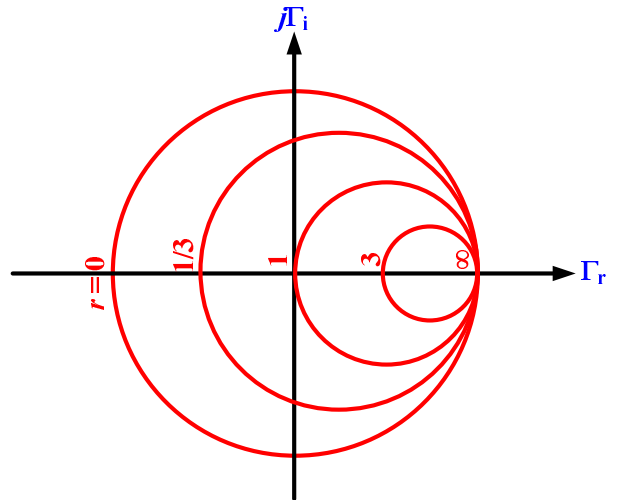
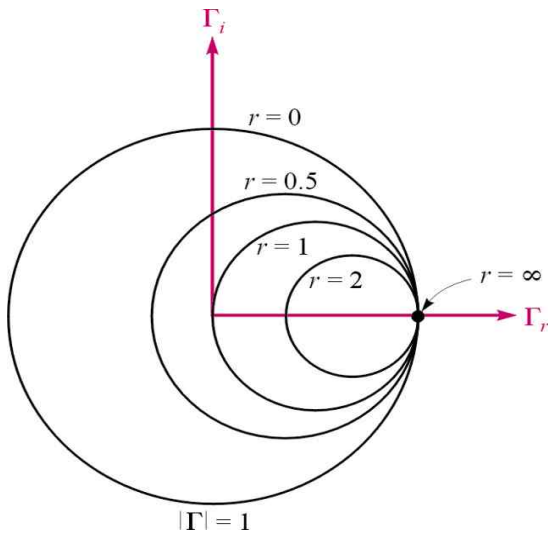
$$(1 - \Gamma_r^2) - \Gamma_i^2 = r(1 - \Gamma_r)^2 + r\Gamma_i^2$$

$$\Gamma_r^2(r + 1) - 2r\Gamma_r + \Gamma_i^2(r + 1) = 1 - r$$

$$\Gamma_r^2 - \frac{2r}{1+r}\Gamma_r + \Gamma_i^2 = \frac{1-r}{1+r}, \quad \Gamma_r^2 - \frac{2r}{1+r}\Gamma_r + \left(\frac{r}{r+1}\right)^2 + \Gamma_i^2 = \frac{1-r}{1+r} + \left(\frac{r}{r+1}\right)^2$$

$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \frac{1-r}{1+r} + \left(\frac{r}{r+1}\right)^2 = \frac{1}{(r+1)^2} \quad (112)$$

Constant r - circles: radius - $\frac{1}{r+1}$, center - $\left(\frac{r}{r+1}, 0\right)$

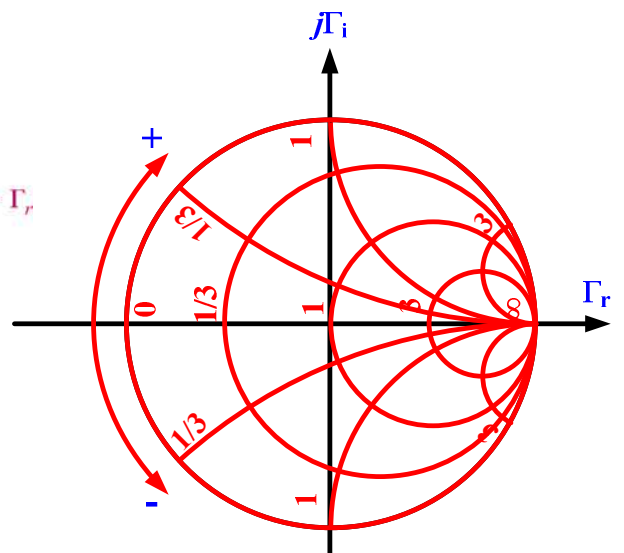
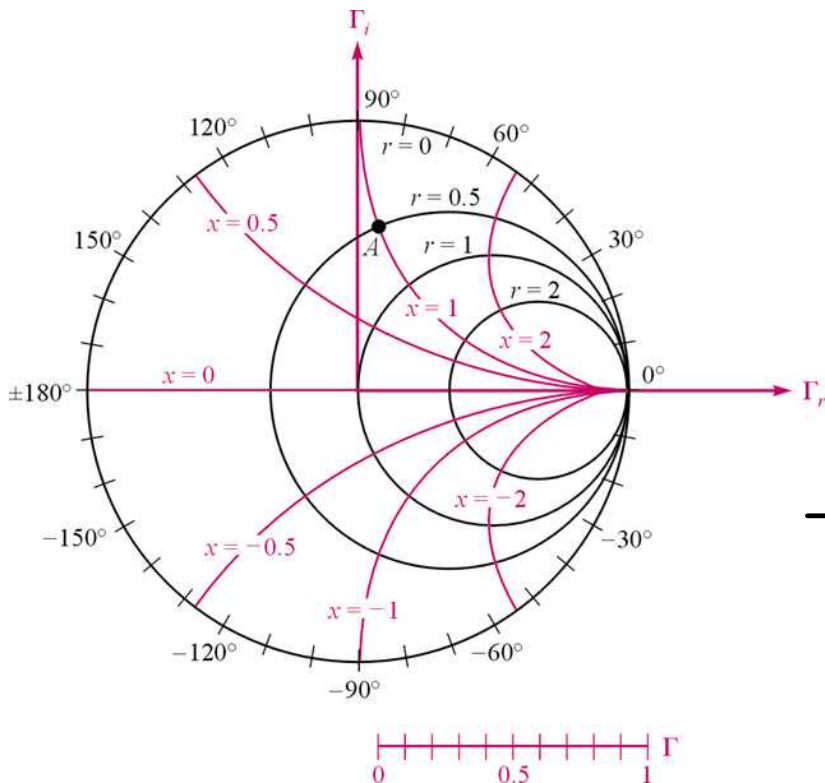
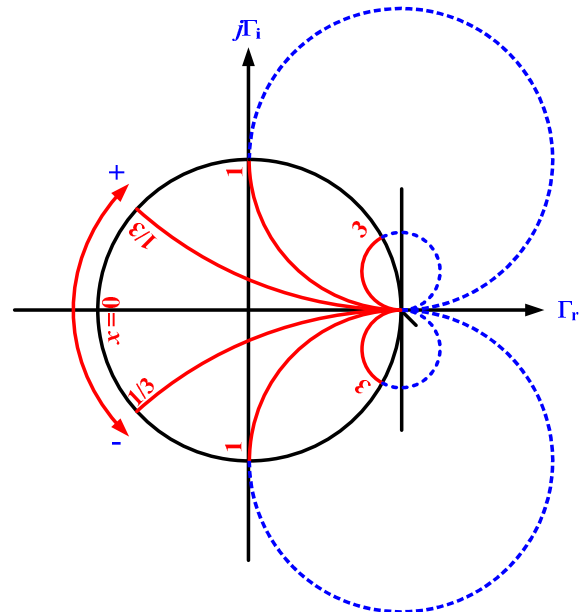
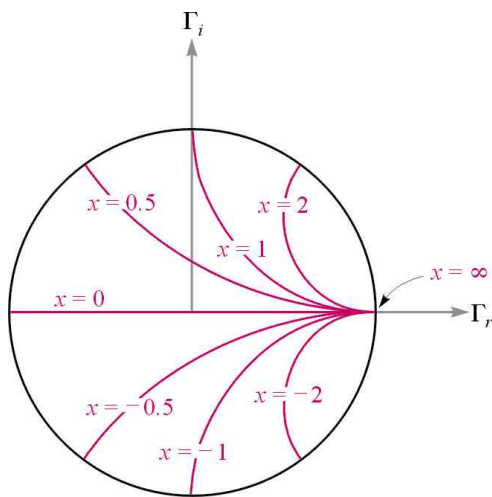


2) Imaginary part

$$\frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2} = x \quad (\Gamma_r - 1)^2 + \Gamma_i^2 - \frac{2\Gamma_i}{x} = 0$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \quad (113)$$

constant x - circles: radius - $\left|\frac{1}{x}\right|$, center - $\left(1, \frac{1}{x}\right)$



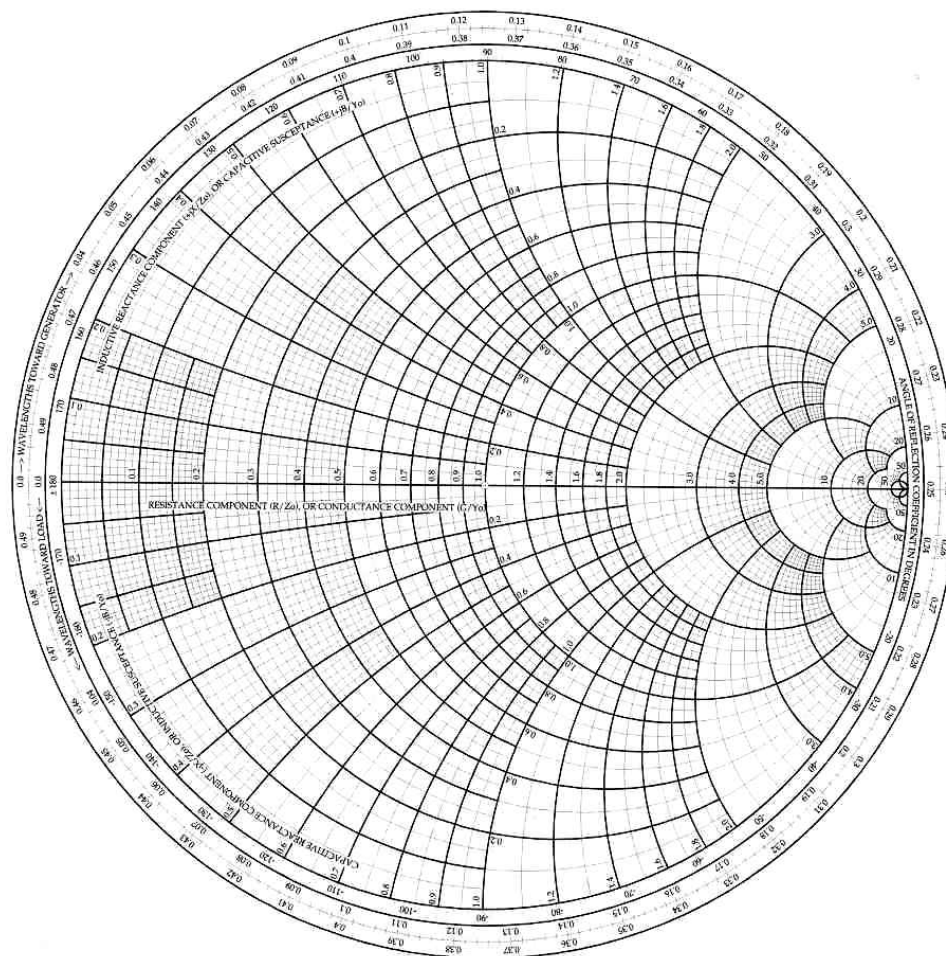
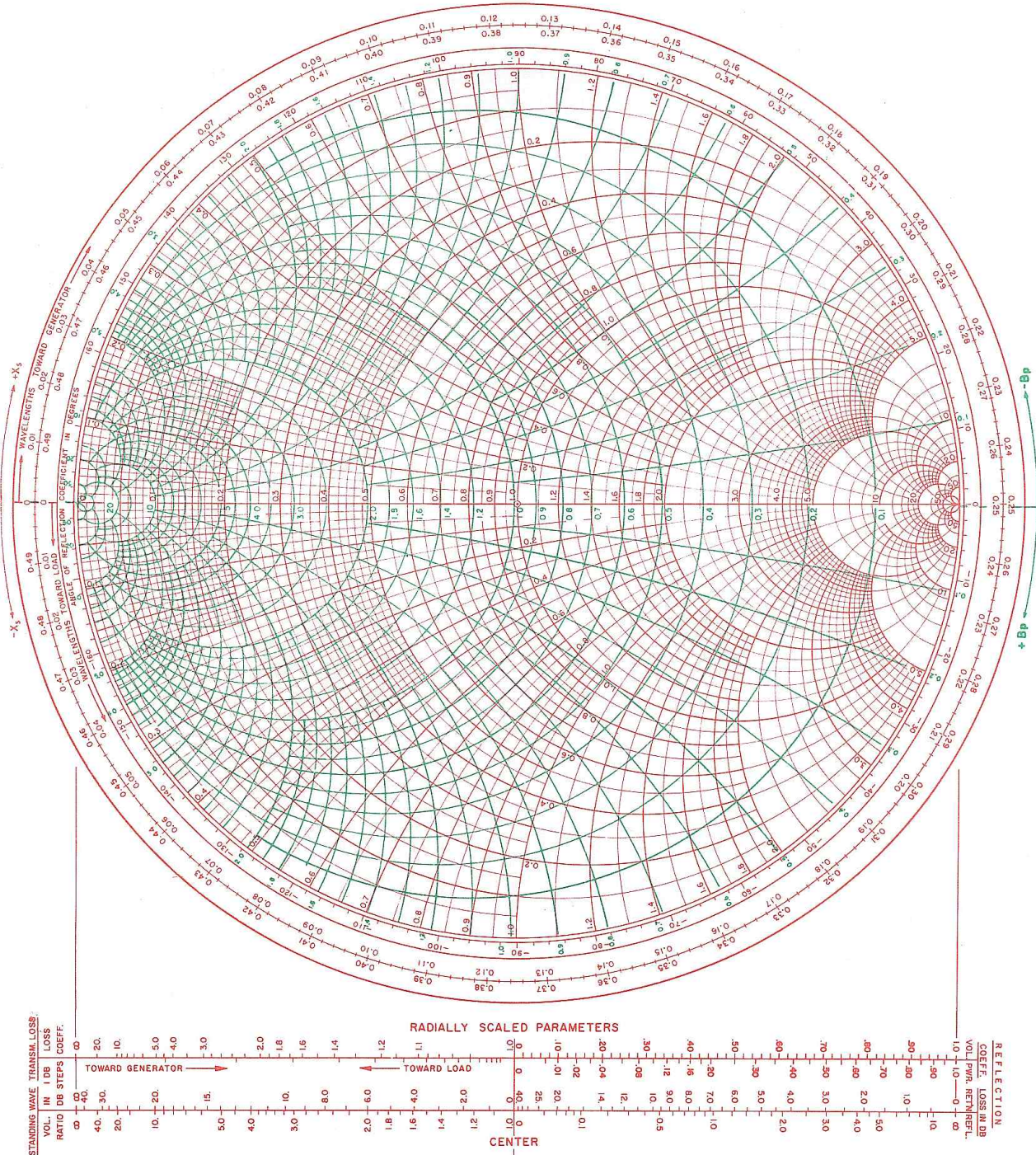


Figure 10.13 A photographic reduction of one version of a useful Smith chart.

- Normalized Impedance **and** Admittance Coordinates

NAME	TITLE	DWG. NO.
SMITH CHART FORM ZY-01-N	ANALOG INSTRUMENTS COMPANY, NEW PROVIDENCE, N.J. 07974	DATE

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



[Ex.] $Z_L = 25 + j50 \rightarrow z_L = 0.5 + j1$

From the Smith chart, $|\Gamma| = 0.62$, $\phi = 83^\circ$

• Second scale on the circumference

$$V_s = V_0^+(e^{-j\beta z} + \Gamma e^{j\beta z})$$

$$I_s = \frac{V_0^+}{Z_0}(e^{-j\beta z} - \Gamma e^{j\beta z})$$

$$\begin{aligned} z_{\text{in}} = \frac{Z_{\text{in}}}{Z_0} &= \frac{1}{Z_0} \cdot \frac{V_0^+(e^{-j\beta z} + \Gamma e^{j\beta z})}{(V_0^+/Z_0)(e^{-j\beta z} - \Gamma e^{j\beta z})} = \frac{e^{-j\beta z} + \Gamma e^{j\beta z}}{e^{-j\beta z} - \Gamma e^{j\beta z}} \leftarrow z = -l \\ &= \frac{e^{j\beta l} + \Gamma e^{-j\beta l}}{e^{j\beta l} - \Gamma e^{-j\beta l}} = \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}} = \frac{1 + |\Gamma|e^{j(\phi - 2\beta l)}}{1 - |\Gamma|e^{j(\phi - 2\beta l)}} \quad (114) \end{aligned}$$

If $l = 0$,

$$z_{\text{in}} = \frac{1 + \Gamma}{1 - \Gamma} = z_L$$

If $z = -l$

$$z_{\text{in}} = \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}} \quad : \quad \leftarrow (\Gamma e^{-j2\beta l} \leftarrow \Gamma)$$

(위 두 식을 비교하면, Γ 대신에 $\Gamma e^{-j2\beta l}$ 을 대입한 것과 동일. 부하에서 l 만큼 떨어진 지점에서의 입력 임피던스는 반사계수 평면에서 부하에서의 반사계수 Γ 를 전기각 $-2\beta l$ 만큼 신호원 방향으로 회전시켜 측정하는 것과 같음)

- ∴ As we proceed from the load z_L to the input impedance z_{in} , we move toward the generator a distance l and move through a clockwise angle of $2\beta l$ on the Smith chart.
- Since the magnitude of Γ stays constant, the movement toward the source is made along a constant-radius circle
(∴ 전송선로가 **lossless** 이므로)

(l 만큼의 물리적 위치 변화에 $2\beta l$ 만큼의 위상 변화를 가져옴. 따라서 부하에서 $l = \lambda/2$ 떨어진 지점은 $2\beta l = 2\pi$ 가 되어 부하와 동일한 임피던스를 갖게 됨. 동일지점. ⇒ 외곽 테두리에 반과장을 명시.)

[Ex. 10.10]

$$Z_L = 25 + j50 \quad \Omega$$

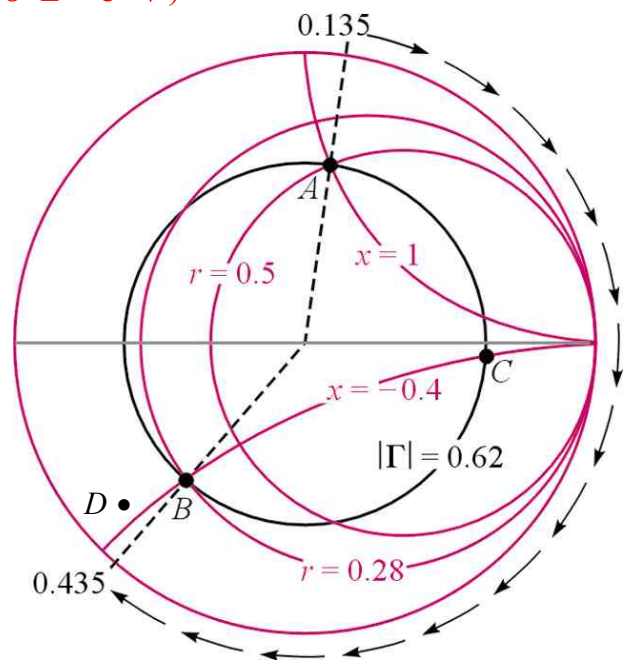
$$z_L = 0.5 + j1 \rightarrow \text{point A}$$

$\lambda = 2$ m 일 때 load로부터 60 cm 떨어진 곳의 입력 임피던스:

$$\frac{l}{\lambda} = \frac{0.6}{2} = 0.3\lambda$$

$$0.135\lambda + 0.3\lambda = 0.435\lambda \rightarrow \text{point B}$$

$$\therefore z_{in} = 0.28 - j0.4, \quad Z_{in} = 14 - j20$$

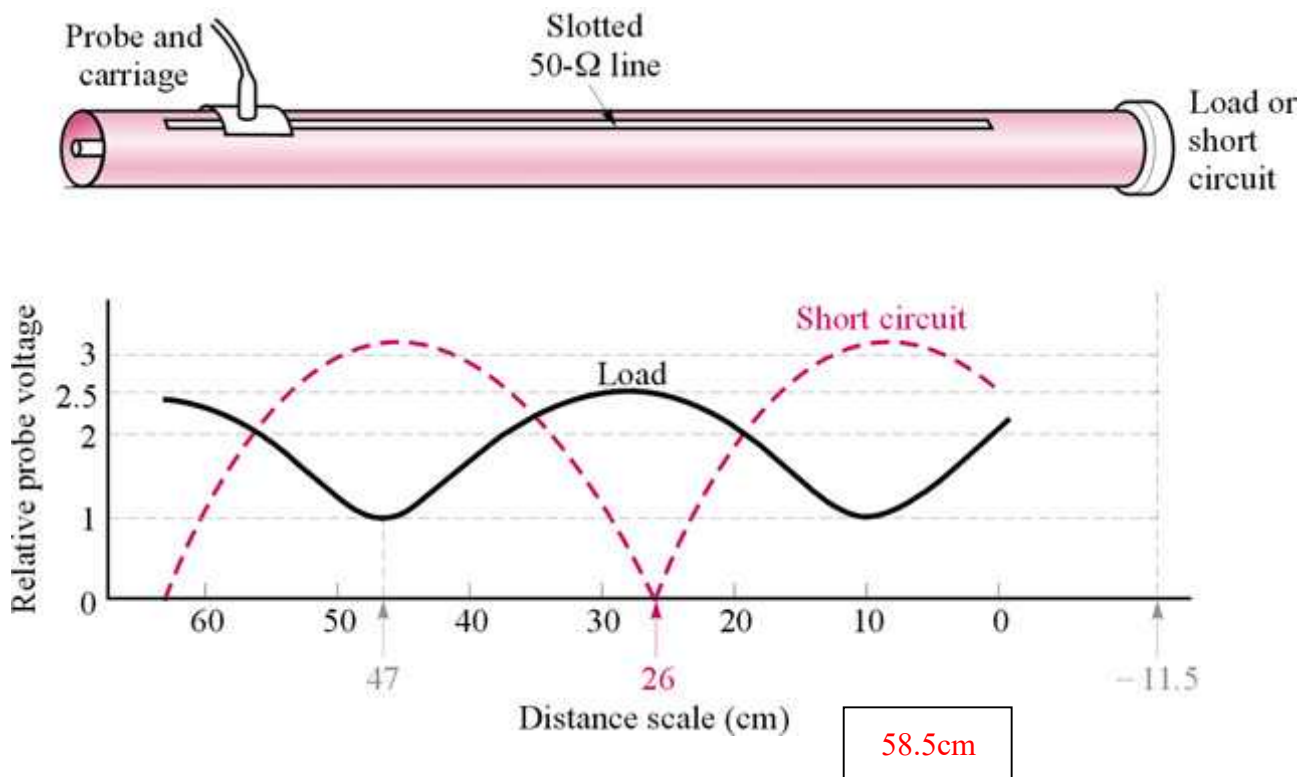


- Voltage maxima: @wtg = 0.25 wtg : **w**avelength **t**oward **g**enerator
∴ point C (∵ $R_L > Z_0$, pure resistance)
- Voltage minima: @ wtg = 0 or 0.5
∴ point D (∵ $R_L < Z_0$, pure resistance)

[Ex.] The determination of load impedance

- ① $s = 2.5$, operating freq. = 400 MHz $\Rightarrow \lambda = 75$ cm
- ② Carriage location for minimum voltage: 47 cm
- ③ Replace load with a short circuit
- ④ Repeat step ②. Minimum at 26cm.
- ⑤ Short circuit locates one half-wavelength away and this is

load location : $26 - 37.5 = -11.5$ cm



- ⑥ The minimum is $47.0 - (-11.5) = 58.5$ cm from the load.
(load를 달고 시험할 때 minimum point 가 load로부터 58.5 cm 이격)
- ⑦ The maximum is $58.5 - \left(\frac{3}{4} \times 75\right) = 58.5 - 56.25 = 2.25$ cm from the load.

(47 cm 지점(= load로부터 58.5 cm 지점)에서 $\lambda/2$ 이격되면 또 minimum. 다시 $\lambda/4$ 이격하면 maximum.)

⑧ So $2.25 \text{ cm} \rightarrow \frac{2.25}{75} = 0.03\lambda$

⑨ $|\Gamma| = \frac{s-1}{s+1} = \frac{1.5}{3.5} = 0.4258$

⑩ $z_L = 2.1 + j0.8 \rightarrow Z_L = 105 + j40$

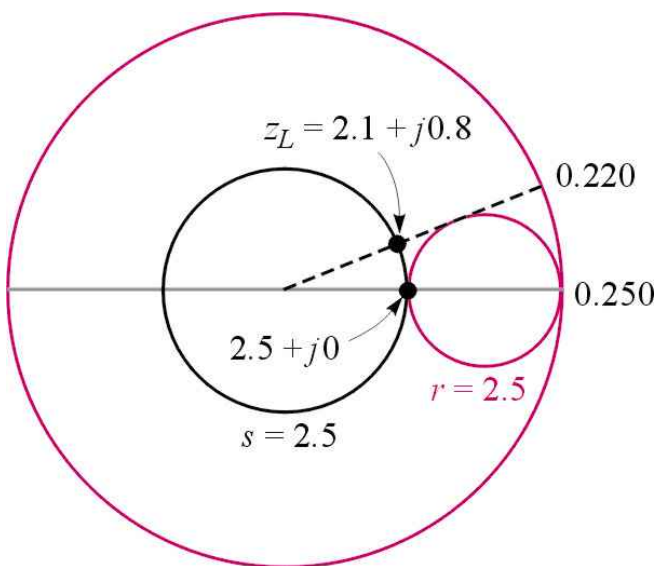
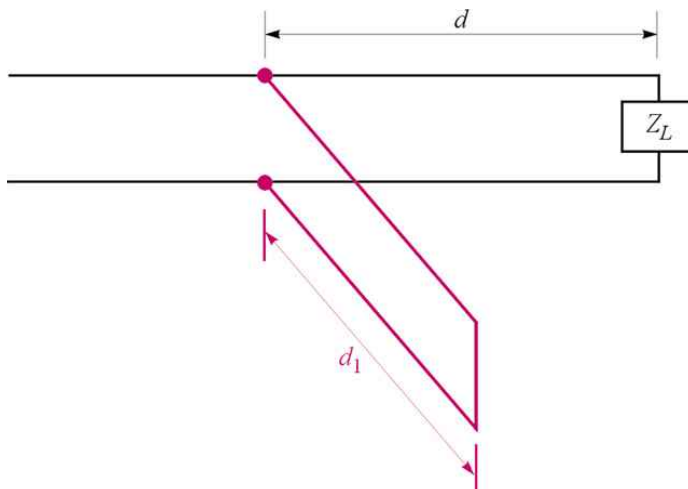


Figure 10.16

If $z_{in} = 2.5 + j0$ on a line 0.03 wavelength long, then
 $z_L = 2.1 + j0.8$

[Ex.] Match the above load to the 50Ω by placing a short-circuited stub of length d_1 at a distance d from the load.



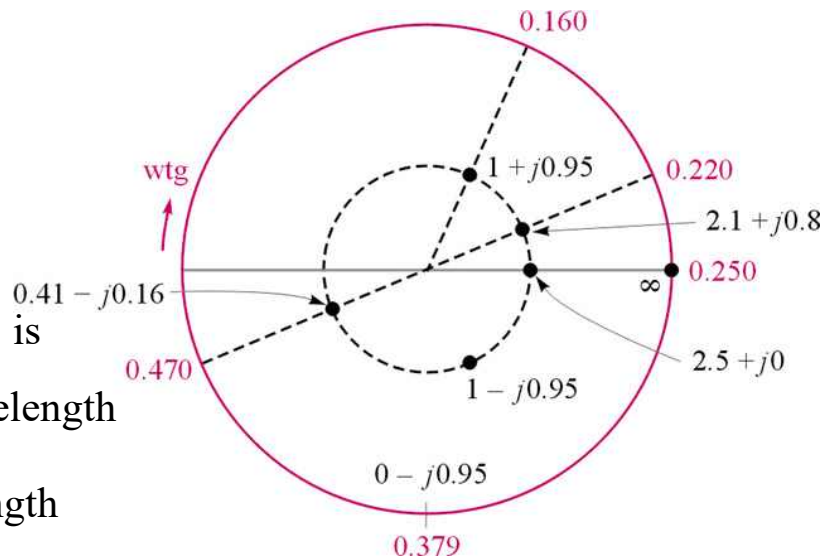
- Adding one-quarter wavelength ($\lambda/4$) to Z_{in} . (z_{in})

$$z_{in}' = \frac{z_{in} + j \tan \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}}{1 + j z_{in} \tan \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}} = \frac{1}{z_{in}} = y_{in}$$

(load 임피던스를 180° rotation시키면 load admittance 와 동일)

Figure 10.18

A normalized load $z_L = 2.1 + j0.8$ is matched by placing a 0.129-wavelength short-circuited stub 0.19 wavelength from the load.



- Target: $y_{in} = 1 + j0 \rightarrow z_{in} = 1$

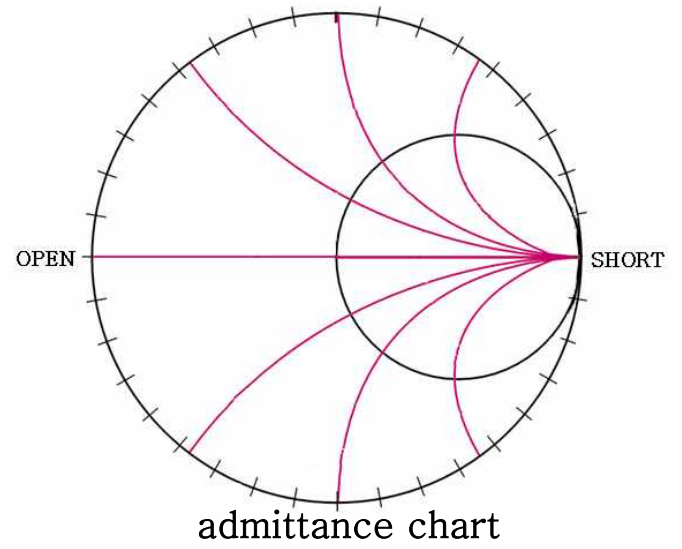
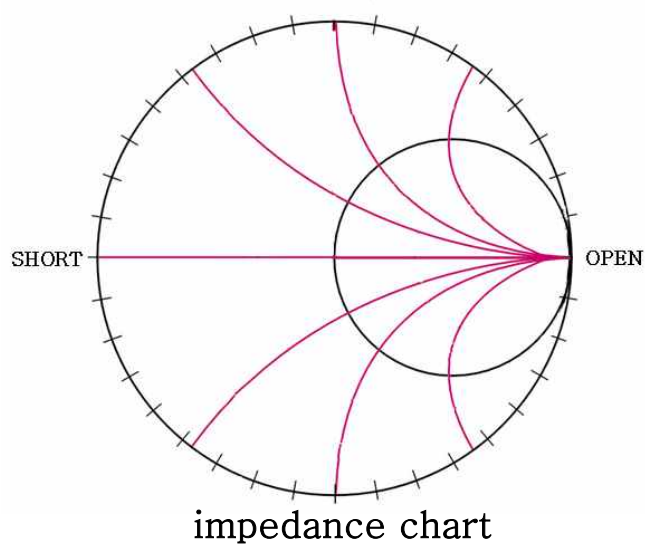
- $y_L = 0.41 - j0.16$

- Moving toward generator by $0.16\lambda + (0.5 - 0.47)\lambda = 0.19\lambda$

$$\Rightarrow d = 0.19 \times 75 \text{ cm} = 14.25 \text{ cm}$$

$$\Rightarrow y_{in}' = 1 + j0.95$$

- $y_{in}'' = 0 - j0.95 \rightarrow (0.379 - 0.25)\lambda = 0.129\lambda$ short stub.



$$0.129\lambda \rightarrow d_1 = 0.129 \times 75 \text{ cm} = 9.675 \text{ cm}$$

$$\therefore y_{in} = y'_{in} + y''_{in} = (1 + j0.95) + (-0.95\lambda) = 1 \quad (\text{Matching})$$