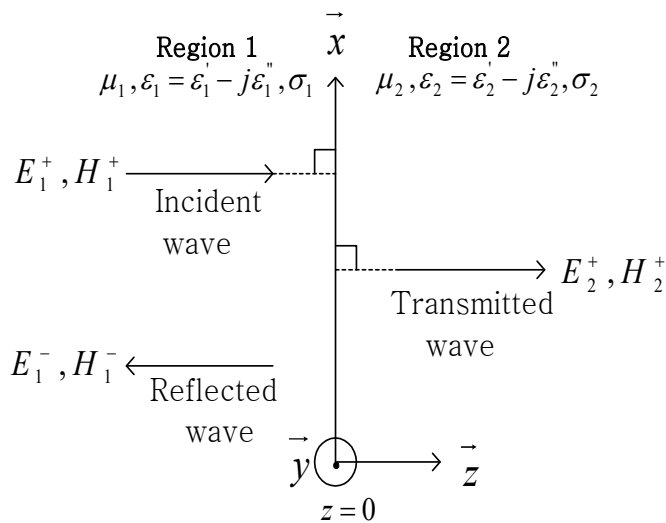


Chapter 12. Plane Wave Reflection and Dispersion

12.1 Reflection of Uniform Plane Wave at Normal Incidence

- A uniform plane wave is incident normally on the boundary between regions composed of two different materials.



- Wave traveling in the

+ $\vec{z}$ direction in region 1:

$$E_{x1}^+ = E_{x10}^+ e^{-\alpha_1 z} \cos(\omega t - \beta_1 z)$$

positive traveling direction
region

phasor $\rightarrow E_{xs1}^+ = E_{x10}^+ e^{-jk_1 z}$ (1)

where $E_{x10}^+ : \text{real}, (jk_1 = \alpha_1 + j\beta_1)$

$$\Rightarrow H_{ys1}^+ = \frac{1}{\eta_1} E_{x10}^+ e^{-jk_1 z} \quad (2)$$

where $\eta_1 = \sqrt{\frac{j\omega\mu_1}{\sigma_1 + j\omega\epsilon_1'}}$: intrinsic impedance

E_{x1}^+, H_{y1}^+ : incident wave perpendicular to the boundary plane (경계

면에 수직으로 입사되는 wave)

- Moving (or transmitted) wave in region 2 toward $(+\vec{z})$ -direction:

$$\begin{cases} E_{xs2}^+ = E_{x20}^+ e^{-jk_2 z} & (3) \\ H_{ys2}^+ = \frac{E_{x20}^+}{\eta_2} e^{-jk_2 z} & (4) \end{cases}$$

$$\leftarrow \eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\epsilon_2'}}$$

- (Special) Boundary condition at $z = 0$:

- i) $\vec{E}$ -field in region 1, 2 must be equal.

Eq. (1) & (3) $\Rightarrow E_{x10}^+ = E_{x20}^+$ (tangential component) $z = 0$ 에서 고려하므로
①, ③식의 진폭이 같게 됨.

- ii) $\vec{H}$ -field in region 1, 2 must be equal.

$$\text{Eq. (2) \& (4)} \Rightarrow \frac{E_{x10}^+}{\eta_1} = \frac{E_{x20}^+}{\eta_2} \Rightarrow \eta_1 = \eta_2$$

$\Rightarrow$ This is very special condition $\Rightarrow$ Not in general.

- Let define reflected wave:

$$\begin{cases} E_{xs1}^- = E_{x10}^- e^{jk_1 z} & (5) \\ H_{ys1}^- = -\frac{E_{x10}^-}{\eta_1} e^{jk_1 z} & (6) \end{cases}$$

• Poynting Vector $\vec{E}_1^- \times \vec{H}_1^-$ 를
만족시키려면 $\vec{H}$ -field는 $-\vec{a}_y$ 이어야 함.

$$\vec{a}_x \times (-\vec{a}_y) = -\vec{a}_z$$

where E_{x10}^- : complex quantity (반사계수에 근거)

- Boundary condition:

i) Total electric field intensity at $z = 0$

$$\begin{aligned}
 E_{xs1} &= E_{xs2} \\
 \Rightarrow E_{xs1}^+ + E_{xs1}^- &= E_{xs2}^+ \\
 \Rightarrow E_{x10}^+ + E_{x10}^- &= E_{x20}^+ \quad (7)
 \end{aligned}$$

ii) Total magnetic field intensity at $z = 0$

$$\begin{aligned}
 H_{ys1} &= H_{ys2} \\
 \Rightarrow H_{ys1}^+ + H_{ys1}^- &= H_{ys2}^+ \\
 \Rightarrow \frac{E_{x10}^+}{\eta_1} - \frac{E_{x10}^-}{\eta_1} &= \frac{E_{x20}^+}{\eta_2} \quad (8) \\
 E_{x20}^+ &= E_{x10}^+ + E_{x10}^- = \frac{\eta_2}{\eta_1} E_{x10}^+ - \frac{\eta_2}{\eta_1} E_{x10}^- \\
 \left(1 + \frac{\eta_2}{\eta_1}\right) E_{x10}^- &= \frac{\eta_2 + \eta_1}{\eta_1} E_{x10}^- = \left(\frac{\eta_2}{\eta_1} - 1\right) E_{x10}^+ = \frac{\eta_2 - \eta_1}{\eta_1} E_{x10}^+ \\
 \therefore E_{x10}^- &= \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_{x10}^+
 \end{aligned}$$

- Reflection coefficient (Γ): ratio of the amplitudes of the reflected and incident electric fields

$$\Gamma = \frac{E_{x10}^-}{E_{x10}^+} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = |\Gamma| e^{j\phi} \quad (9) : \text{complex} \quad (\text{complex인 경우 위상지연 발생})$$

$\downarrow$
 phase shift

where $\eta_i = \sqrt{\frac{j\omega\mu_i}{j\omega\epsilon_i + \sigma_i}}$

$$\begin{aligned}
E_{x20}^+ &= E_{x10}^+ + E_{x10}^- = (1 + \Gamma) E_{x10}^+ \\
&= \left(1 + \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \right) E_{x10}^+ = \frac{2\eta_2}{\eta_2 + \eta_1} E_{x10}^+ \\
&= \tau E_{x10}^+
\end{aligned}$$

- Transmit coefficient (τ): ratio of the amplitudes of the transmitted and incident electric fields

$$\tau = 1 + \Gamma = \frac{E_{x20}^+}{E_{x10}^+} = \frac{2\eta_2}{\eta_2 + \eta_1} = |\tau| e^{j\phi_t} \quad (10)$$

[Ex. 1] $\begin{cases} \text{Region 1: Perfect dielectric} \\ \text{Region 2: Perfect conductor} \end{cases} \Rightarrow \sigma_2 = \infty$

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\epsilon_2}} = 0$$

$$\tau = \frac{2\eta_2}{\eta_1 + \eta_2} = 0$$

$$\therefore \frac{E_{x20}^+}{E_{x10}^+} = 0$$

$$\Rightarrow E_{x20}^+ = 0$$

region 2에서는 $\vec{E}$ -field 가 존재하지 못함.
 $\Rightarrow$ skin depth 의 영향으로 생각할 수도 있음

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1$$

$$\therefore E_{x10}^- = -E_{x10}^+ \Rightarrow \text{All wave is reflected}$$

$$E_{xs1} = E_{xs1}^+ + E_{xs1}^-$$

$$= E_{x10}^+ e^{-j\beta_1 z} - E_{x10}^+ e^{j\beta_1 z} \quad \text{perfect dielectric} \quad jk_1 = \alpha_1 + j\beta_1 = j\beta_1$$

$$= (e^{-j\beta_1 z} - e^{j\beta_1 z}) E_{x10}^+ = -(j2) \frac{e^{j\beta_1 z} - e^{-j\beta_1 z}}{j2} E_{x10}^+$$

$$= -j2 \sin \beta_1 z E_{x10}^+ \quad (11)$$

instantaneous
form $\rightarrow E_{x1}(z, t) = \text{Re} \left[(-j) 2 \sin(\beta_1 z) E_{x10}^+ \{ \cos \omega t + j \sin \omega t \} \right]$

$$= 2 E_{x10}^+ \sin(\beta_1 z) \sin \omega t$$

$$= E_{x10}^+ [\cos(\omega t - \beta_1 z) - \cos(\omega t + \beta_1 z)] \rightarrow \sin A \sin B = \frac{1}{2} \{ \cos(A - B) - \cos(A + B) \}$$

$$= E_{x10}^+ \left[\cos \left\{ \omega \left(t - \frac{z}{v_1} \right) \right\} - \cos \left\{ \omega \left(t + \frac{z}{v_1} \right) \right\} \right]$$

$\rightarrow$ Wave traveling in the $\pm z$ direction

For $\beta_1 z = \frac{2\pi}{\lambda_1} z = m\pi \quad (m = 0, \pm 1, \pm 2, \dots), \quad E_{x1} = 0$

Null locations: $z = m\lambda_1/2$

$\Rightarrow$ Standing wave.

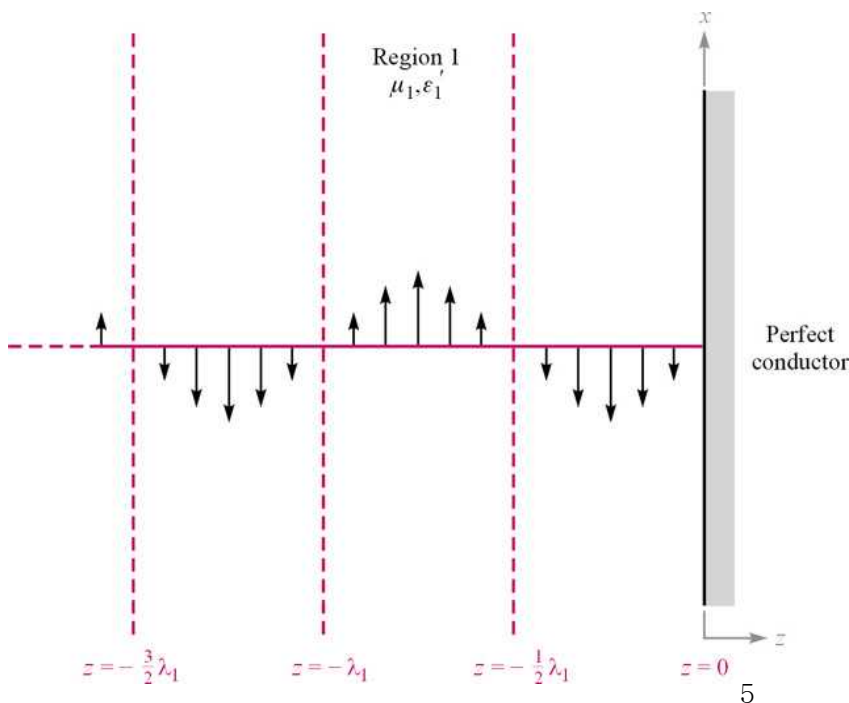
(Wave를 시간에 대한 term과 공간(z)에 대한 term으로 분리. $\beta_1 z = n\pi$ 이면 시간에 관계없이 $E_{x1} = 0$)

• $\omega t = m\pi, -\beta_1 z = m\pi$ 에 의해(서로 다른 두 조건)

$E_{x1} = 0$ 이 될 수 있음

- For $E_{x1} = 0$,

$$\frac{2\pi}{\lambda_1} z = m\pi \quad z = m \frac{\lambda_1}{2}$$



: Instantaneous

values of the total

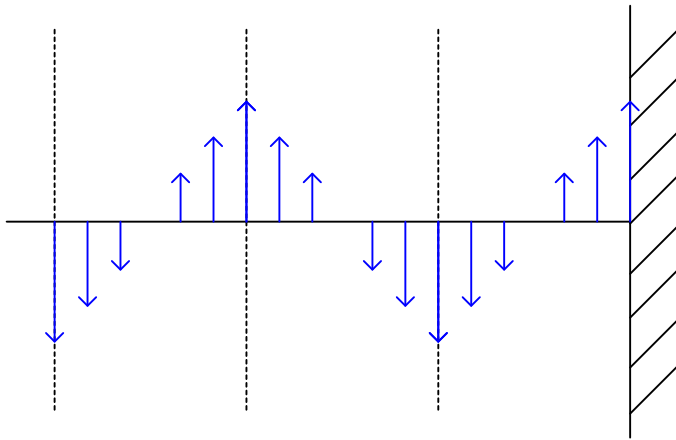
field E_{x1}

$\left(\text{at } \omega t = \frac{\pi}{2} \right)$

Since $E_{xs1}^+ = H_{ys1}^+ \eta_1$, $E_{xs1}^- = -H_{ys1}^- \eta_1$,

$$H_{ys1} = \frac{E_{x10}^+}{\eta_1} (e^{-j\beta_1 z} + e^{j\beta_1 z})$$

instantaneous
form $\rightarrow H_{y1} = 2 \frac{E_{x10}^+}{\eta_1} \cos \beta_1 z \cos \omega t$



$$\begin{aligned} H_{ys1} &= H_{ys1}^+ e^{-j\beta_1 z} + H_{ys1}^- e^{j\beta_1 z} = \frac{E_{xs1}^+}{\eta_1} e^{-j\beta_1 z} - \frac{E_{xs1}^-}{\eta_1} e^{j\beta_1 z} \\ &= \frac{E_{xs1}^+}{\eta_1} (e^{-j\beta_1 z} - \Gamma e^{j\beta_1 z}) \quad \leftarrow \Gamma = -1 \\ &= \frac{E_{xs1}^+}{\eta_1} (e^{-j\beta_1 z} + e^{j\beta_1 z}) \\ &= 2 \frac{E_{xs1}^+}{\eta_1} \frac{e^{j\beta_1 z} + e^{-j\beta_1 z}}{2} = 2 \frac{E_{xs1}^+}{\eta_1} \cos \beta_1 z \end{aligned}$$

$\frac{2\pi}{\lambda} \cdot z$

$\Rightarrow \vec{H}$ is also standing wave
and 90° out of phase with $\vec{E}$.

[Ex. 12.1] Region 1, 2: perfect dielectrics.

$$\Rightarrow \alpha_1 = 0 = \alpha_2$$

$$\eta_1 = 100 [\Omega], \quad \eta_2 = 300 [\Omega]$$

$$E_{x10}^+ = 100 \text{ [V/m]}$$

$$\Rightarrow \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{300 - 100}{300 + 100} = 0.5$$

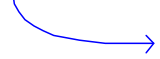
$$E_{x10}^- = 50 \text{ [V/m]}$$

$$H_{y10}^+ = \frac{E_{x10}^+}{\eta_1} = \frac{100}{100} = 1 \quad [\text{A/m}]$$

$$H_{y10}^- = -\frac{E_{x10}^-}{\eta_1} = -\frac{50}{100} = -0.5 \quad [\text{A/m}]$$

$$\therefore \langle S_{li} \rangle = \frac{1}{2} \text{Re} \{ \vec{E}_s \times \vec{H}_s^* \} = \frac{1}{2} \frac{E_{x10}^2}{\eta_1} = \frac{1}{2} E_{x10}^+ H_{y10}^+ = \frac{1}{2} \times 100 \times 1 = 50 \quad [\text{W/m}^2] : \text{incident average power}$$

$$\langle S_{lr} \rangle = -\frac{1}{2} E_{x10}^- H_{y10}^- = -\frac{1}{2} \times 50 \times (-0.5) = 12.5 \quad [\text{W/m}^2] : \text{reflected average power}$$

 $-\vec{z}$ 축으로 진행하므로

• Region 2.

$$E_{x20}^+ = \frac{2\eta_2}{\eta_1 + \eta_2} E_{x10}^+ = \frac{600}{100 + 300} \times 100 = 150 \quad [\text{V/m}]$$

$$H_{y20}^+ = \frac{E_{x20}^+}{\eta_2} = \frac{150}{300} = 0.5 \quad [\text{A/m}]$$

$$\therefore \langle S_2 \rangle = \frac{1}{2} E_{x20}^+ H_{y20}^+ = \frac{1}{2} \times 150 \times 0.5 = 37.5 \quad [\text{W/m}^2] : \text{transmit average power.}$$

$$\therefore \langle S_{li} \rangle = \langle S_{lr} \rangle + \langle S_2 \rangle \quad : \text{입사전력} = \text{반사전력} + \text{전달전력}$$

$$\Rightarrow E_{x10}^+ + E_{x10}^- = E_{x20}^+ \quad : \text{경계면에서 region 1 에서의 } \vec{E}\text{-field의 합.} \\ = \text{region 2 에서의 } \vec{E}\text{-field의 합.}$$

• General rule on the transfer of power through reflection and transmission.

- Incident power density,

$$\langle S_{li} \rangle = \frac{1}{2} \text{Re} \{ E_{xs1}^+ H_{ys1}^{+*} \} = \frac{1}{2} \text{Re} \left\{ E_{x10}^+ \frac{1}{\eta_1^*} E_{x10}^{+*} \right\} = \frac{1}{2} \text{Re} \left\{ \frac{1}{\eta_1^*} \right\} \underline{E_{x10}^+}^2$$

- Reflected power density:

$$\begin{aligned} \langle S_{1r} \rangle &= -\frac{1}{2} \operatorname{Re} \{ E_{x10}^- H_{y10}^{-*} \} = \frac{1}{2} \operatorname{Re} \left\{ \Gamma E_{x10}^+ \frac{1}{\eta_1^*} \Gamma^* E_{x10}^{+*} \right\} \\ &= \frac{1}{2} \operatorname{Re} \left\{ \frac{1}{\eta_1^*} \right\} |E_{x10}^+|^2 |\Gamma|^2 \end{aligned}$$

- General relation between the reflected and incident power

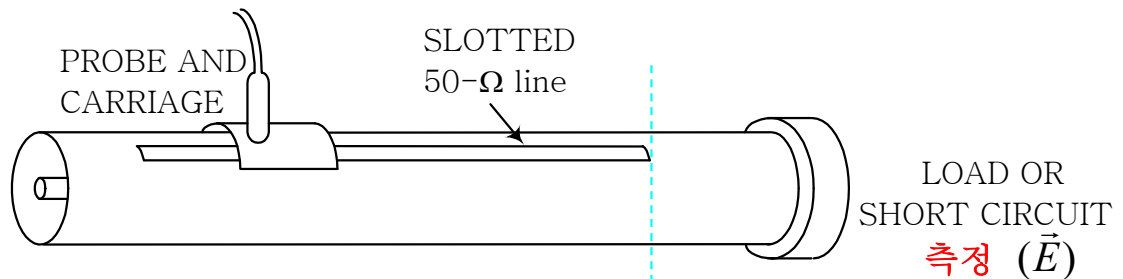
$$\langle S_{1r} \rangle = |\Gamma|^2 \langle S_{1i} \rangle \quad (15)$$

- Transmitted power:

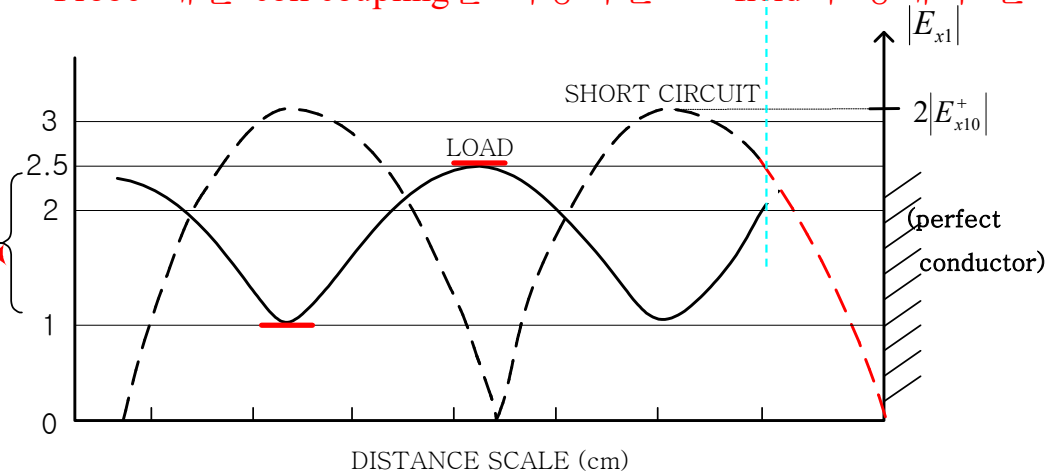
$$\begin{aligned} \langle S_2 \rangle &= \frac{1}{2} \operatorname{Re} \{ E_{x20}^+ H_{y20}^{+*} \} = \frac{1}{2} \operatorname{Re} \left\{ \tau E_{x10}^+ \frac{1}{\eta_2^*} \tau^* E_{x10}^{+*} \right\} = \frac{1}{2} \operatorname{Re} \left\{ \frac{1}{\eta_2^*} \right\} |E_{x10}^+|^2 |\tau|^2 \\ &= \frac{\operatorname{Re} \{ 1/\eta_2^* \}}{\operatorname{Re} \{ 1/\eta_1^* \}} |\tau|^2 \cdot \frac{1}{2} \operatorname{Re} \left\{ \frac{1}{\eta_1^*} \right\} |E_{x10}^+|^2 = \frac{\operatorname{Re} \{ 1/\eta_2^* \}}{\operatorname{Re} \{ 1/\eta_1^* \}} |\tau|^2 \langle S_{1i} \rangle \\ &= \frac{\frac{1}{2} \left(\frac{1}{\eta_2} + \frac{1}{\eta_2^*} \right)}{\frac{1}{2} \left(\frac{1}{\eta_1} + \frac{1}{\eta_1^*} \right)} |\tau|^2 \langle S_{1i} \rangle = \frac{\frac{(\eta_2 + \eta_2^*)}{(\eta_2 \eta_2^*)}}{\frac{(\eta_1 + \eta_1^*)}{(\eta_1 \eta_1^*)}} |\tau|^2 \langle S_{1i} \rangle \\ &= \left| \frac{\eta_1}{\eta_2} \right|^2 \frac{\eta_2 + \eta_2^*}{\eta_1 + \eta_1^*} |\tau|^2 \langle S_{1i} \rangle \quad (16) : \text{Too difficult to calculate} \end{aligned}$$

$$\Leftrightarrow P_{2,av}^+ = (1 - |\Gamma|^2) P_{1,av}^+ \quad (17) : \text{Alternate method.}$$

12.2 Standing Wave Ratio



- Probe를 통해 relative amplitude of electric field intensity
- Probe 대신 coil coupling을 사용하면 $\vec{H}$ -field의 상대적 진폭 측정 가능.



- Perfect short이면 $\Gamma = -1$ 이 되어 standing wave 가 됨.

$$0 \leq |E_{x1}| \leq 2|E_{x10}^+|$$

- 그러나 임의의 load가 달린다면 임의의 신호들은 전달되고 임의의 신호는 반사가 됨.

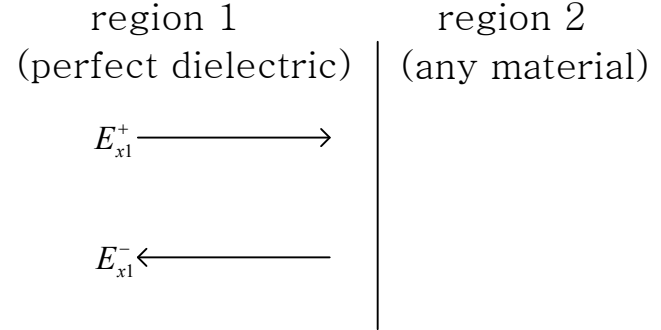
그래서 region 1에서는 traveling wave + standing wave 형태가 됨.

$$\rightarrow |E_{x1}| > 0$$

- Standing wave ratio: ratio of the maximum and minimum amplitudes of electric field intensity

- Total electric field phasor in region 1

$$\begin{aligned} E_{x1T} &= E_{x1}^+ + E_{x1}^- \\ &= E_{x10}^+ e^{-j\beta_1 z} + \Gamma E_{x10}^+ e^{j\beta_1 z} \quad (18) \end{aligned}$$



Reflection coefficient:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = |\Gamma| e^{j\phi}$$

where η_1 : positive real, η_2 : arbitrary complex

- If region 2 is a perfect conductor,

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\epsilon_2}} \cong 0 \quad \Rightarrow \quad \Gamma = -1 = 1 \angle \pi$$

- If η_2 is real and less than η_1 ,

$$\Gamma = |\Gamma| \angle \pi \quad \leftarrow \quad \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -\Delta = |\Delta| \angle \pi$$

- If η_2 is real and greater than η_1 ,

$$\Gamma = |\Gamma| \angle 0$$

- Revised total electric field phasor in region 1:

$$E_{x1T} = \left(e^{-j\beta_1 z} + |\Gamma| e^{j(\beta_1 z + \phi)} \right) E_{x10}^+ \quad (19)$$

$$|E_{x1T}|_{\max} = (1 + |\Gamma|) E_{x10}^+ \quad (20)$$

$$\Rightarrow -\beta_1 z_{\max} = \beta_1 z_{\max} + \phi + 2m\pi \quad (m = 0, \pm 1, \pm 2 \dots) \quad (21)$$

$$-\beta_1 z_{\max} = \frac{\phi}{2} + m\pi$$

$$z_{\max} = -\frac{1}{2\beta_1}(\phi + 2m\pi) \quad (22)$$

← $z_{\max}$: E_{x1T} 를 일으키는 z 좌표값.

- If $\phi = 0 \Rightarrow \Gamma : \text{real positive} \Rightarrow \eta_2 > \eta_1 \text{ \& } (\eta_1, \eta_2 : \text{real})$

$\Rightarrow$ Voltage maximum is located at the boundary plane ($z = 0$)

($\because @ m = 0$)

$$\Delta z_{\max} = -\frac{m\pi}{\beta_1} = -\frac{m\pi}{2\pi/\lambda_1} = -\frac{m\lambda_1}{2} \quad (z_{\max} \text{ point들의 간격})$$

- For the perfect conductor ($\phi = \pi$) ← $\Gamma = -1 = 1 \angle \pi$

$$-\beta_1 z_{\max} = \frac{\phi}{2} + m\pi \Big|_{\phi=\pi} = \frac{\pi}{2} + m\pi$$

(경계면에서 $-\pi/2$ 만큼 떨어진 곳에서 maximum point 발생.

$$-\beta_1 z_{\max} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots \quad (@ m = 0, z_{\max} = -\frac{\pi}{2\beta_1})$$

• Voltage minimum:

$$|E_{x1T}|_{\min} = (1 - |\Gamma|) E_{x10}^+ \quad (23)$$

$$\Rightarrow -\beta_1 z_{\min} = \beta_1 z_{\min} + \phi + \pi + 2m\pi \quad (m = 0, \pm 1, \pm 2, \dots) \quad (24)$$

$$-\beta_1 z_{\min} = \frac{\phi}{2} + m\pi + \frac{\pi}{2}$$

$$z_{\min} = -\frac{1}{2\beta_1} [\phi + (2m+1)\pi] \quad (25)$$

← $z_{\min}$: $E_{xs1,\min}$ 를 일으키는 z 좌표값.

- For the perfect conductor,

$$\Gamma = -1 = 1\angle -\pi \quad \Rightarrow \quad \phi = -\pi$$

$$\Rightarrow \text{Voltage minimum is found at } z = 0 \quad \boxed{z_{\min} = -\frac{1}{2\beta_1} [-\pi + \pi] = 0}$$

$$\Rightarrow \Gamma: \text{real negative} \quad \Rightarrow \eta_2 < \eta_1 \text{ \& } (\eta_1, \eta_2: \text{real})$$

• Revised total electric field:

$$\begin{aligned} E_{xs1} &= (e^{-j\beta_1 z} + |\Gamma| e^{j(\beta_1 z + \phi)}) E_{x10}^+ \\ &= E_{x10}^+ (1 - |\Gamma|) e^{-j\beta_1 z} + E_{x10}^+ |\Gamma| \left(e^{-j\frac{\phi}{2}} e^{-j\beta_1 z} + e^{j\frac{\phi}{2}} e^{j\beta_1 z} \right) e^{j\frac{\phi}{2}} \\ &= (1 - |\Gamma|) E_{x10}^+ e^{-j\beta_1 z} + 2|\Gamma| E_{x10}^+ e^{j\frac{\phi}{2}} \frac{e^{j\frac{\phi}{2}} e^{j\beta_1 z} + e^{-j\frac{\phi}{2}} e^{-j\beta_1 z}}{2} \\ &= (1 - |\Gamma|) E_{x10}^+ e^{-j\beta_1 z} + 2|\Gamma| E_{x10}^+ e^{j\frac{\phi}{2}} \cos\left(\beta_1 z + \frac{\phi}{2}\right) \end{aligned}$$

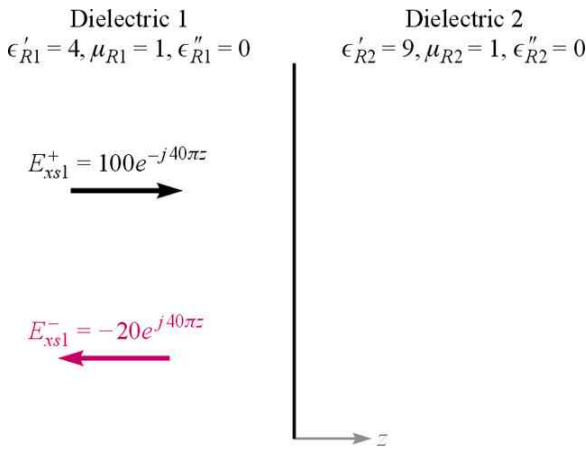
instantaneous
form $\rightarrow E_{x1T}(x, t) = \underbrace{(1 - |\Gamma|) E_{x10}^+ \cos(\omega t - \beta_1 z)}_{\text{traveling wave}}$

계속 진행되는 traveling wave

$$+ \underbrace{2|\Gamma| E_{x10}^+ \cos(\beta_1 z + \phi/2) \cos(\omega t + \phi/2)}_{\text{standing wave}} \quad (26)$$

reflected 되어 standing wave를 일으키는 성분.

[Ex. 12.2] Propagating wave in material 1: 100 [V/m], 3 [GHz]



$$\omega = 2\pi f = 6\pi \times 10^9 \quad [\text{rad/sec}]$$

$$\beta_1 = \omega \sqrt{\mu_1 \epsilon_1} = 6\pi \times 10^9 \frac{\sqrt{4 \times 1}}{3 \times 10^8} = 40\pi \quad [\text{rad/m}]$$

$$\beta_2 = \omega \sqrt{\mu_2 \epsilon_2} = 60\pi \quad [\text{rad/m}]$$

$$\lambda_1 = \frac{2\pi}{\beta_1} = \frac{2\pi}{40\pi} = 0.05 \quad [\text{m}]$$

$$\lambda_2 = \frac{2\pi}{\beta_2} = \frac{2\pi}{60\pi} = 0.0333 \quad [\text{m}]$$

$$\eta_1 = \sqrt{\frac{j\omega\mu_1}{\sigma + j\omega\epsilon_1}} = \sqrt{\frac{\mu_1}{\epsilon_1}} = \frac{1}{\sqrt{\epsilon_{R1}}} \times 120\pi = 60\pi \quad [\Omega] \leftarrow \sigma = \omega\epsilon''$$

$$\eta_2 = \frac{1}{\sqrt{\epsilon_{R2}}} \times 120\pi = 40\pi \quad [\Omega]$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{40\pi - 60\pi}{40\pi + 60\pi} = -0.2 = 0.2 \angle \pi$$

$\Rightarrow \vec{E}_{\min}$ will be at the boundary and repeated at half-wavelength (0.025m) intervals in dielectric 1.

$$E_{xs1,\min} = (1 - |\Gamma|) E_{x10}^+ = 80 \quad [\text{V/m}]$$

$$\Rightarrow E_{xs1,\max} = (1 + |\Gamma|) E_{x10}^+ = 120 \quad [\text{V/m}] \quad \text{at} \quad \left(\begin{array}{l} z = -0.0125, -0.0375, \\ -0.0625, \dots \quad [\text{m}] \end{array} \right.$$

(Region 2에서는 반사파가 없으므로 max, min점들이 없음)

- Standing wave ratio (SWR): ratio of maximum to minimum electric field amplitudes

$$s = \frac{|E_{x1T}|_{\max}}{|E_{x1T}|_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|} \quad (27)$$

$$\text{Since } |\Gamma| \leq 1, \quad s \geq 1$$

$$[\text{Ex.}] \textcircled{1} \quad \Gamma = -0.2 \Rightarrow s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1.2}{0.8} = 1.5 \Rightarrow 1.5:1$$

$$\textcircled{2} \quad |\Gamma| = 1 \Rightarrow \text{The reflected and incident wave amplitude are equal.}$$

$$\Rightarrow \text{All the incident wave energy is reflected.}$$

$$\Rightarrow E_{x1} \text{ is zero which plane is separated } \lambda_1/2.$$

$$E_{x1} \text{ is } 2E_{xs10} \text{ which plane is separated } \lambda_1/2.$$

$$\textcircled{3} \quad \eta_2 = \eta_1 \Rightarrow \Gamma = 0 \Rightarrow \text{No energy reflected.}$$

$$\Rightarrow s = 1$$

$$\textcircled{4} \text{ If 1/2 of incident power is reflected. } \Rightarrow |\Gamma|^2 = 0.5 \Rightarrow |\Gamma| = 0.707$$

$$\Rightarrow s = \frac{1+0.707}{1-0.707} = \frac{1.707}{0.293} = 5.83$$

(SWR은 반사신호의 진폭 변화 비를 의미하므로 반사신호의 진폭 측정에 의해 반사계수를 알 수 있다. $s \rightarrow |\Gamma|$)

Ex.] An uniform plane wave is reflected from an unknown material to the air. The measured electric field is 1.5 m between maxima.

$$z_{\max} = -0.75 \text{ [m]}, s = 5. \quad \eta_u = ?$$

$$\rightarrow \lambda = 1.5 \times 2 = 3 \text{ [m]} \quad \rightarrow \quad f = c / \lambda = (3 \times 10^8) / 3 = 10^8 \text{ [Hz]} \\ \text{or } 100 \text{ [MHz]}$$

$z_{\max} = -0.75 \text{ [m]} \rightarrow$ The 1st minimum occurs at the boundary.

$$|\Gamma| = \frac{s-1}{s+1} = \frac{5-1}{5+1} = \frac{2}{3}$$

$$\Gamma = -\frac{2}{3} = \frac{\eta_u - \eta_0}{\eta_u + \eta_0}$$

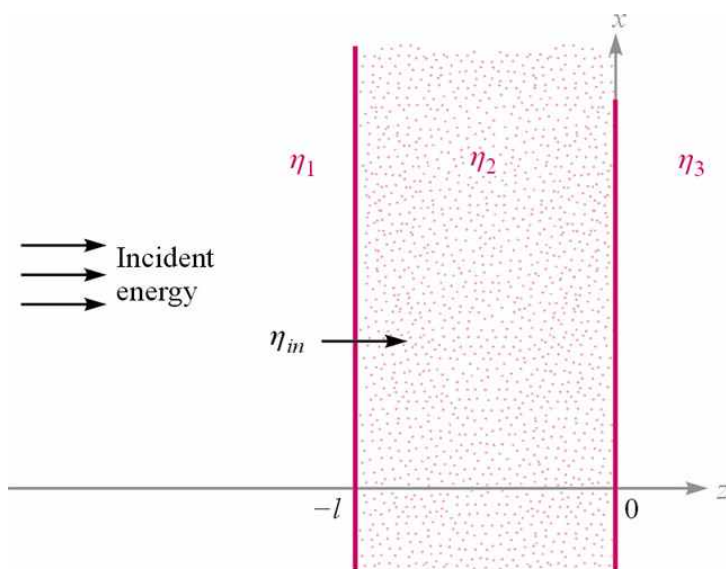
$$\eta_u = \eta_0 / 5 = 377 / 5 = 75.4 \text{ } [\Omega]$$

12.3 Wave Reflection from Multiple Interfaces

- Consider wave reflection from materials that are finite in extent.

$\rightarrow$ Effects of the front and back surfaces.

[Ex.] Light incident on a flat piece of glass. (coated glasses, etc.)



- Let us assume all regions are composed of lossless media.

- Electric fields in region 2:

$$E_{xs2} = E_{x20}^+ e^{-j\beta_2 z} + E_{x20}^- e^{j\beta_2 z} \quad (28a)$$

$$\text{where } \beta_2 = \omega \sqrt{\mu_2 \epsilon_2} = \omega \sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon_{r2}} \\ = \frac{\omega \sqrt{\epsilon_{r2}}}{c}$$

$$E_{x20}^+, E_{x20}^- : \text{ complex}$$

- Magnetic fields in region 2:

$$H_{ys2} = H_{y20}^+ e^{-j\beta_2 z} + H_{y20}^- e^{j\beta_2 z} \quad (28b)$$

Reflection coefficient at the second interface of region 2:

$$\Gamma_{23} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} \quad (29)$$

$$\rightarrow E_{x20}^- = \Gamma_{23} E_{x20}^+ \quad (30)$$

$$H_{y20}^+ = \frac{E_{x20}^+}{\eta_2} \quad (31a)$$

$$H_{y20}^- = -\frac{E_{x20}^-}{\eta_2} = -\frac{\Gamma_{23}}{\eta_2} E_{x20}^+ \quad (31b)$$

- Wave impedance (η_w): **ratio of** total electric field **to** total magnetic field at arbitrary position z in region 2

$$\begin{aligned} \eta_w(z) &= \frac{E_{xs2}}{H_{ys2}} = \frac{E_{x20}^+ e^{-j\beta_2 z} + E_{x20}^- e^{j\beta_2 z}}{H_{y20}^+ e^{-j\beta_2 z} + H_{y20}^- e^{j\beta_2 z}} \\ &= \frac{\cancel{E_{x20}^+} e^{-j\beta_2 z} + \Gamma_{23} \cancel{E_{x20}^+} e^{j\beta_2 z}}{\frac{\cancel{E_{x20}^+}}{\eta_2} e^{-j\beta_2 z} - \frac{\Gamma_{23}}{\eta_2} \cancel{E_{x20}^+} e^{j\beta_2 z}} = \eta_2 \frac{e^{-j\beta_2 z} + \Gamma_{23} e^{j\beta_2 z}}{e^{-j\beta_2 z} - \Gamma_{23} e^{j\beta_2 z}} \\ &= \eta_2 \frac{(\cos \beta_2 z - j \sin \beta_2 z) + \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} (\cos \beta_2 z + j \sin \beta_2 z)}{(\cos \beta_2 z - j \sin \beta_2 z) - \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} (\cos \beta_2 z + j \sin \beta_2 z)} \end{aligned}$$

$$\begin{aligned}
&= \eta_2 \frac{(\eta_3 + \eta_2)(\cos \beta_2 z - j \sin \beta_2 z) + (\eta_3 - \eta_2)(\cos \beta_2 z + j \sin \beta_2 z)}{(\eta_3 + \eta_2)(\cos \beta_2 z - j \sin \beta_2 z) - (\eta_3 - \eta_2)(\cos \beta_2 z + j \sin \beta_2 z)} \\
&= \eta_2 \frac{\cancel{2}\eta_3 \cos \beta_2 z - j\cancel{2}\eta_2 \sin \beta_2 z}{\cancel{2}\eta_2 \cos \beta_2 z - j\cancel{2}\eta_3 \sin \beta_2 z} \\
&= \eta_2 \frac{\eta_3 \cos \beta_2 z - j\eta_2 \sin \beta_2 z}{\eta_2 \cos \beta_2 z - j\eta_3 \sin \beta_2 z} \quad (32) \\
&= \eta_2 \frac{\eta_3 - j\eta_2 \tan \beta_2 z}{\eta_2 - j\eta_3 \tan \beta_2 z}
\end{aligned}$$

- Boundary condition at $z = -l$,

$$\begin{cases} E_{xs1}^+ + E_{xs1}^- = E_{xs2} \end{cases} \quad (33a)$$

$$\begin{cases} H_{ys1}^+ + H_{ys1}^- = H_{ys2} \end{cases} \quad (33b)$$

$$\Rightarrow E_{x10}^+ + E_{x10}^- = \underline{E_{xs2}(z = -l)} \quad \leftarrow \text{from eq. (7), (8) in text}$$

→ region 2 에도 E_{x20}^+, E_{x20}^- 성분이 있으므로 E_{xs2} 로 표시.

$$H_{y10}^+ + H_{y10}^- = H_{ys2}(z = -l) \quad \rightarrow \quad \frac{E_{x10}^+}{\eta_1} - \frac{E_{x10}^-}{\eta_1} = \frac{E_{xs2}(z = -l)}{\eta_w(-l)}$$

where E_{x10}^+, E_{x10}^- : amplitudes of incident and reflected fields in region 1.

- Input impedance (η_{in}) = $\eta_w(-l)$ to the two-interface combination

$$\frac{E_{x10}^+}{\eta_1} - \frac{E_{x10}^-}{\eta_1} = \frac{E_{xs2}(z = -l)}{\eta_{in}} = \frac{E_{x10}^+ + E_{x10}^-}{\eta_{in}}$$

$$E_{x10}^+ \left(\frac{1}{\eta_1} - \frac{1}{\eta_{in}} \right) = E_{x10}^- \left(\frac{1}{\eta_{in}} + \frac{1}{\eta_1} \right)$$

$$E_{x10}^+ \frac{\eta_{in} - \eta_1}{\cancel{\eta_1} \eta_{in}^{\rightarrow}} = E_{x10}^- \frac{\eta_{in} + \eta_1}{\cancel{\eta_1} \eta_{in}^{\rightarrow}}$$

$$\frac{E_{x10}^-}{E_{x10}^+} = \Gamma = \frac{\eta_{in} - \eta_1}{\eta_{in} + \eta_1} \quad (35)$$

$$\text{where } \eta_{in} = \eta_w(z = -l) = \eta_2 \frac{\eta_3 \cos \beta_2 l + j \eta_2 \sin \beta_2 l}{\eta_2 \cos \beta_2 l + j \eta_3 \sin \beta_2 l} \quad (36)$$

- In case of $\eta_{in} = \eta_1$, $\Gamma = 0$

$$\Rightarrow \text{No reflected power } \left(\because P_r = (1 - |\Gamma|^2) P_{in} \right) \Rightarrow \text{Matched}$$

- Suppose that $\eta_3 = \eta_1$ and $\beta_2 l = m\pi$ (m : integer)

$$\beta_2 l = \frac{2\pi}{\lambda_2} l = m\pi \quad \left(\text{or } l = \frac{m\lambda_2}{2} \quad (37) \right)$$

$$\eta_{in} = \eta_2 \frac{\eta_3 \cos \beta_2 l + j \eta_2 \sin \beta_2 l}{\eta_2 \cos \beta_2 l + j \eta_3 \sin \beta_2 l} \Big|_{\beta_2 l = m\pi}$$

$$= \eta_2 \frac{\eta_3}{\eta_2} = \eta_3$$

$$= \eta_1 \Rightarrow \text{Matched.}$$

Ex.] Antenna inside of the aircraft.

- (Refractive) Index:

$$n = \sqrt{\epsilon_r} \quad (38)$$

For lossless media ($\epsilon_r'' = 0$ and $\mu_r = 1$),

$$\beta = k = \omega \sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon_r} = \frac{n\omega}{c} \quad (39)$$

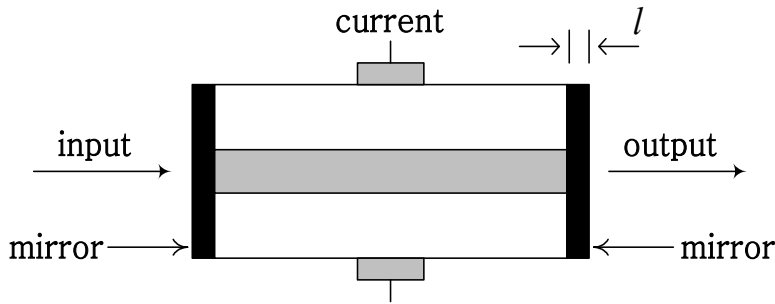
$$\eta = \frac{1}{\sqrt{\epsilon_r}} \sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{\eta_0}{n} \quad (40)$$

Phase velocity and wavelength in a material of index n :

$$v_p = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r}} = \frac{1}{\sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon_r}} = \frac{c}{n} \quad (41)$$

$$\lambda = \frac{v_p}{f} = \frac{c}{nf} = \frac{\lambda_0}{n} \quad (42)$$

Ex.] Fabry-Perot interferometer: optic filter which transmit specific



Fabry-Perot Amplifier

wavelength satisfying

the condition. $\lambda = \lambda_0 / n = 2l / m$

$$\lambda_{m-1} - \lambda_m = \Delta\lambda_f = \frac{2l}{m-1} - \frac{2l}{m} = \frac{2l}{m(m-1)} \approx \frac{2l}{m^2}$$

where m : number of half-wavelength in region 2 or

$$m = 2l / \lambda = 2nl / \lambda_0$$

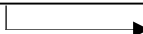
λ_0 : desired free-space wavelength for transmission.

Free-space wavelength in region 2:

$$\Delta\lambda_f = \frac{2l}{m^2} = 2l \frac{\lambda^2}{4l^2} = \frac{\lambda^2}{2l} \quad (43a)$$

Free spectral range: wavelength in free space

$$\Delta\lambda_{f0} = n\Delta\lambda_f = n \frac{\lambda_0^2}{2l} = n \frac{\lambda_0^2 / n^2}{2l} = \frac{\lambda_0^2}{2nl} \quad (43b)$$

[Ex. 12.4] $\Delta\lambda_s = 50 \text{ nm}$, $\lambda_0 = 600 \text{ nm}$, $(\lambda = 600 \pm 50 \text{ nm})$
 Red part of the visible spectrum.

Febry-Perot filter: $n = 1.45$

Required range of glass thickness?

$\Rightarrow$ Optical spectral width $\Delta\lambda_{f0} > \Delta\lambda_s$ ($\because$ 원하는 대역폭의 신호가 모두 통과 되어야 하므로)

$$\frac{\lambda_0^2}{2l} = \Delta\lambda_f > \Delta\lambda_s$$

$$\therefore l < \frac{\lambda_0^2}{2\Delta\lambda_s} = \frac{\left(\frac{600}{\sqrt{1.45}} \right)^2 \text{ nm}^2}{2 \times 50 \text{ nm}} = \frac{600^2}{2 \times 1.45 \times 50} = 2.5 \times 10^3 \text{ nm} = 2.5 \mu\text{m}$$

///

• Another matching condition:

For $\beta_2 l = (2m-1) \frac{\pi}{2}$ (m : integer)

$$\beta_2 l = \frac{2\pi}{\lambda_2} l = (2m-1) \frac{\pi}{2} \quad (44)$$

$$\Rightarrow l = (2m-1) \frac{\lambda_2}{4} : \text{odd multiple of a quarter wavelength}$$

$$\eta_{in} = \eta_2 \frac{\eta_3 \cos \beta_2 l + j\eta_2 \sin \beta_2 l}{\eta_2 \cos \beta_2 l + j\eta_3 \sin \beta_2 l} \Bigg|_{l=(2m-1)\frac{\lambda_2}{4}}$$

$$= \eta_2 \frac{j\eta_2}{j\eta_3} = \frac{\eta_2^2}{\eta_3} \quad (45) \quad \leftarrow \quad \eta_{in} = \eta_1 \quad (\text{for matching})$$

$$\eta_2 = \sqrt{\eta_{in} \eta_3} = \sqrt{\eta_1 \eta_3} \quad (46) : \text{quarter wave matching.}$$

[Ex. 12.5] Coating glass. Transmission wavelength from air to glass: 570 [nm]

$$n_3 = 1.45$$

Required index for the coating and its minimum thickness?

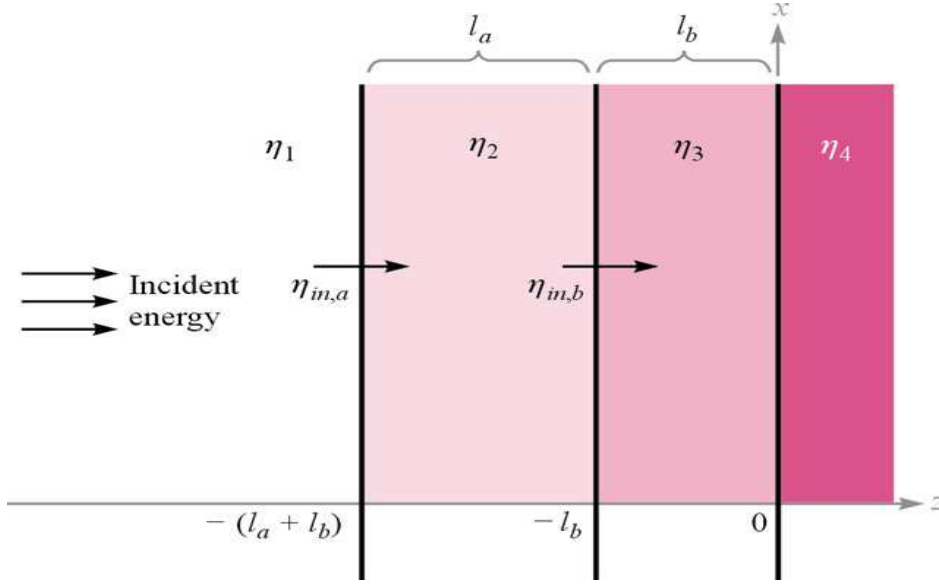
$$\Rightarrow \eta_1 = 377 \Omega \quad (\text{free space}), \quad \eta_3 = \frac{377}{1.45} = 260 \Omega \quad (\text{glass})$$

$$\eta_2 = \sqrt{\eta_1 \eta_3} = \sqrt{377 \times 260} = 313 \Omega$$

$$\therefore \lambda_2 = \frac{570}{1.45} = 475 \text{ nm}$$

$$l = \frac{\lambda_2}{4} = 119 \text{ nm} = 0.119 \mu\text{m} : \text{coating width}$$

- Multiple-interface situation



$$\text{Step 1) } \eta_{in,b} = \eta_3 \frac{\eta_4 \cos \beta_3 l_b + j \eta_3 \sin \beta_3 l_b}{\eta_3 \cos \beta_3 l_b + j \eta_4 \sin \beta_3 l_b} \quad (47)$$

$$\text{Step 2) } \eta_{in,a} = \eta_2 \frac{\eta_{in,b} \cos \beta_2 l_a + j \eta_2 \sin \beta_2 l_a}{\eta_2 \cos \beta_2 l_a + j \eta_{in,b} \sin \beta_2 l_a} \quad (48)$$

$$\text{Step 3) } \Gamma = \frac{\eta_{in,a} - \eta_1}{\eta_{in,a} + \eta_1}$$

$$\text{Step 4) Reflected power: } P_r = |\Gamma|^2 P_{in}$$

$$\text{Transmitted power at layer 4: } P_t = (1 - |\Gamma|^2) P_{in}$$