# Optical design for excellent contrast ratio in a reflective horizontal-switching liquid crystal cell 

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#### Abstract

In this paper, we propose an optical configuration for a reflective liquid crystal (LC) cell with a single polarizer that can show an excellent contrast ratio by effectively eliminating phase dispersion. The proposed configuration consists of a half-wave retarder, two $A$-plates, a quarter-wave LC cell and a reflector; the configuration was designed on a Poincaré sphere with the trigonometric method. From the calculation, we confirm that this configuration can show a high contrast ratio as compared with the conventional configuration due to the excellent dark state.


## 1. Introduction

As many people are familiar with the information-oriented society, further progress in high display quality is needed in making the display device an information interface. In particular, new display devices should have a small weight, high resolution and low power consumption to meet the demand for free exchange of information at any time and any place. Reflective liquid crystal displays (LCDs) are considered as suitable display devices because of their small weight and low power consumption. On the other hand, reflective LCDs have a problem in that they are not sufficiently bright. Further, we use the single polarizer mode because it can provide high brightness in comparison with the double polarizer mode [1-5]. However, the former has a low contrast ratio in comparison with that of the latter because of light leakage in the dark state. To reduce the leakage, a previous paper proposed an optical configuration that consists of a polarizer, a half-wave retarder and a quarter-wave LC cell in addition to a reflector [6, 7]. The previous single polarizer LC cell used could reduce the light leakage by eliminating phase dispersion in the dark state. Therefore, it showed a comparatively good contrast ratio by compensating the phase dispersion at the designed wavelength, for example, at the green wavelength. However, light leakage
in the conventional optical configuration continues to exist at the red and blue wavelengths because the phase dispersion at these wavelengths cannot be completely eliminated.

Therefore, in this paper, we propose an optical configuration for the reflective LCD with a single polarizer that can provide an excellent contrast ratio by eliminating phase dispersion along the entire visual wavelength range by applying two additional $A$-plates in the dark state.

## 2. The optical principle for compensating phase dispersion of light in the dark state

### 2.1. Light leakage in the conventional reflective LC cell in the dark state

In the reflective LC cell, the most important condition for obtaining a high contrast ratio is that the polarization state in the entire visible wavelength of the travelling light in front of the reflector should be a circular polarization state, whose position is $S_{3}$ on the Poincaré sphere, in the dark state [8-11].

Figure 1 shows the conventional optical configuration of the reflective LC mode with a single polarizer and an optical principle for removing the phase dispersion along the visual light on the Poincaré sphere in the dark state. The conventional


Figure 1. Optical configuration of the conventional reflective LC cell and polarization states: (a) optical configuration and (b) polarization path on the Poincaré sphere.
reflective LC cell consists of a polarizer, a half-wave retarder and a quarter-wave LC cell in addition to a reflector as shown in figure $1(a)$. A previous paper reported an optical configuration that could successfully compensate the phase dispersion in the visible wavelength range during the passage of light through the half-wave retarder and the quarter-wave LC cell so that light in all wavelengths could be circularly polarized in front of the reflector. In figure $1(a)$, the axis of the polarizer is set to $0^{\circ}$ and the optic axes of the half-wave retarder and quarterwave LC cell are set to $\theta$ and $2 \theta+45^{\circ}$, respectively. Further, the optimized value of $\theta$ is $15^{\circ}$ [7]. Figure $1(b)$ shows the polarization state of light passing through the reflective LC cell in the previous study on the Poincaré sphere in red ( $\mathbf{\square}$ ), green $(\boldsymbol{\wedge})$ and blue $(\mathbf{\Delta})$ wavelength light. Positions $A, B$ and $C$ represent the position of the polarizer, the optic axis of the halfwave retarder and the optic axis of the quarter-wave retarder. Positions $D_{\mathrm{R}}, D_{\mathrm{G}}$ and $D_{\mathrm{B}}$ represent the polarization positions of red, green and blue wavelength lights after passing through the half-wave retarder. The circular path $L_{1}$ represents the polarization path of light produced by the half-wave retarder, which is centred at $B$. Further, circular paths $L_{2 \mathrm{R}}, L_{2 \mathrm{G}}$ and $L_{2 \mathrm{~B}}$ represent the polarization paths of the red, green and blue wavelength lights passing through the quarter-wave LC layer; these paths are centred at $C$ that is determined by the optic axis of the LC layer. After passing through the half-wave retarder, phase dispersion occurs because of the different phase retardations of the red, green and blue wavelengths. However, optical phase dispersion can be effectively reduced by passing the lights through the quarter-wave LC cell. In the bright state,


Figure 2. Ideal polarization positions for the perfect dark and the bright state for a reflective LC cell with a single polarizer.
the slow axis of the LC cell rotates to $2 \theta+90^{\circ}$ from $2 \theta+45^{\circ}$ [7] that has a polarization position $G$. In this configuration, light experiences half-wave retardation by double passing through the LC cell with a centring position $G$ that is the opposite position to position $D_{\mathrm{G}}$. Therefore, the polarization of light will rotate by $180^{\circ}$ with the centring position $D_{\mathrm{G}}$. Finally, they can go back to the input polarization position $A$ by passing through the half-wave retarder again so that we can show a good bright state.

However, the optical path of light passing through the quarter-wave LC cell and proceeding towards $S_{3}$ can vary for lights with different wavelengths, as shown in figure $1(b)$. After passing through the half-wave retarder and the quarterwave LC cell, the polarization state of the green wavelength light arrives alone at $S_{3}$ in front of the reflector. In contrast, the polarization state of light in red and blue wavelengths can be deviated to the goal position $S_{3}$, and this causes light leakage in the dark state. Therefore, in order to obtain a perfect dark state, we need to gather the polarization states of the lights in the entire visible wavelength in front of the reflector at the circular polarization position $S_{3}$.

### 2.2. Optical principle of the proposed LC cell in the dark state

Figure 2 shows the ideal polarization positions of light in front of the LC cell that is represented by a solid line on the circular path $L_{2}$, which is determined by the LC cell and centred at $C$. In order to achieve an excellent dark and bright state, the most important optical design that we have to perform is to put the polarization positions of the three wavelengths on the circular path $L_{2}$ before the lights pass through the LC cell, as shown in figure $2 . F_{\mathrm{R}}, F_{\mathrm{G}}$ and $F_{\mathrm{B}}$ represent the ideal polarization positions of red, green and blue wavelengths before passing through the quarter-wave LC cell. $D_{\mathrm{G}}$ and $F_{\mathrm{G}}$ are at the same position. From this configuration, all polarization positions of light in the visual wavelength range can gather on the target position $S_{3}$ by passing through the LC cell so that we can get an excellent dark state. Therefore, we need another process to put the polarization positions of


Figure 3. Optical configuration of the proposed reflective LC cell and the optical principle for moving the polarization positions of the three wavelengths to the circular path $L_{2}$ in two steps; $(a)$ optical configuration, (b) first step and $(c)$ second step to achieve excellent dark and bright states on the Poincaré sphere.
the light in the entire visible wavelength range on the circular path $L_{2}$ shown as dotted arrows on the Poincaré sphere in figure 2.

The required process can be achieved in two steps by using two additional $A$-plates. Figure 3(a) shows the optical structure of the proposed reflective LC cell. Figures $3(b)$ and (c) show the optical principle underlying the application of the proposed reflective LC cell for moving the polarization positions of the three wavelengths to the circular path $L_{2}$ in two steps before the lights pass through the LC cell.

The first step for moving the polarization positions of the three wavelengths to the circular path $L_{2}$ is performed by $A$-plate 1 , as shown in figure $3(b)$. The polarization position of the light passing through the polarizer is $A\left(=0^{\circ}\right)$. By passing through the half-wave retarder, the polarization states of the red, green and blue wavelength lights proceed to positions $D_{\mathrm{R}}$, $D_{\mathrm{G}}$ and $D_{\mathrm{B}}$ through a circular path $L_{1}$ that is centred at $B$. As for the half-wave retarder, in particular, we set the polarization position of the red wavelength light to $D_{\mathrm{R}}$ by controlling the retardation of the half-wave retarder, as shown in figure $3(b)$. Besides, we set the angle of the fast axis of $A$-plate 1 as $2 \theta$. Therefore, the position of the fast axis of $A$-plate 1 is the same as the polarization state of the red wavelength light passing through the half-wave retarder. Therefore, the polarization state does not move anywhere on the Poincaré sphere, and therefore the polarization position of the red wavelength light always occurs on the circular path $L_{2}$. In contrast, we can position the polarization state of a light in the visible wavelength range on the circular path $L_{2}$ by calculating the
optimized retardation value of $A$-plate 1 . Positions $E_{\mathrm{R}}, E_{\mathrm{G}}, E_{\mathrm{B}}$ on the circular path $L_{2}$ represent the polarization positions of the red, green and blue wavelength lights after passing through $A$-plate 1. Arcs $D_{\mathrm{G}} E_{\mathrm{G}}$ and $D_{\mathrm{B}} E_{\mathrm{B}}$ represent the polarization paths of the green and blue wavelength lights after their passage through $A$-plate 1 , respectively. Therefore, we can obtain the polarization position of the visible wavelength light in front of the reflector at $S_{3}$.

By using $A$-plate 2, we can adjust the polarization states of light as the target polarization states as shown in figure 2. Figure 3(c) shows the second step for moving the polarizations of light in front of $A$-plate 1 to the target polarization by using $A$-plate 2. By controlling the retardation of $A$-plate 2 , we can adjust the phase dispersion of the quarter-wave LC cell so that we can finally achieve an excellent achromatic black state.

### 2.3. Optimization of optical films

We used ML-0249 for the quarter-wave LC cell $\left(\Delta n d_{\text {red }}=\right.$ $131 \cdot 16 \mathrm{~nm}, \Delta n d_{\text {green }}=136 \cdot 5 \mathrm{~nm}$ and $\left.\Delta n d_{\text {blue }}=148 \cdot 15 \mathrm{~nm}\right)$. In practice, we do not change the material properties of the LC , which implies that the polarization positions of the red, green and blue wavelength lights $F_{\mathrm{R}}, F_{\mathrm{G}}$ and $F_{\mathrm{B}}$ are fixed because they depend on the LC material. In order to simplify the calculation, we assumed normal dispersion of $A$-plate 2.

The starting point for optimization is to determine the optimized retardation of $A$-plate 2 , which can be made by the condition that $F_{\mathrm{R}}, F_{\mathrm{G}}$ and $F_{\mathrm{B}}$ should move to $E_{\mathrm{R}}, E_{\mathrm{G}}$ and $E_{\mathrm{B}}$, respectively. This condition induces a simple equation as given below:

$$
\begin{equation*}
\Gamma_{A 2}^{\mathrm{R}}=\Gamma_{\mathrm{LC}}^{\mathrm{R}}-\Gamma_{\mathrm{LC}}^{\mathrm{G}}, \tag{1}
\end{equation*}
$$

where $\Gamma=2 \pi \Delta n d / \lambda . \Gamma_{A 2}^{\mathrm{R}}$ is the phase retardation of the red wavelength light in $A$-plate 2 . $\Gamma_{\mathrm{LC}}^{\mathrm{R}}$ and $\Gamma_{\mathrm{LC}}^{\mathrm{G}}$ represent the phase retardations of the quarter-wave LC cell for red and green wavelengths, respectively. From the calculated result $\Gamma_{A 2}^{\mathrm{R}}$, we can design phase retardations of green and blue wavelength lights in $A$-plate $2 \Gamma_{A 2}^{\mathrm{G}}$ and $\Gamma_{A 2}^{\mathrm{B}}$, so as to be satisfied with the normal and practical phase dispersion.

Then we can obtain the optimized optical retardations of the half-wave retarder and $A$-plate 1 simultaneously. For designing the half-wave retarder the condition $\overparen{D R}_{\mathrm{R}}=$ $\overparen{E_{\mathrm{R}} E_{\mathrm{G}}}, \overparen{D}_{\mathrm{R}} D_{\mathrm{B}}=\overparen{E_{\mathrm{R}} E_{\mathrm{B}}}$, should be satisfied; $\overparen{D \mathrm{R}}^{\mathrm{D}} \mathrm{G}$ and $\overparen{E_{\mathrm{R}} E_{\mathrm{G}}}$ are the radii of the polarization paths of the green wavelength light and $\overparen{D R}_{\mathrm{R}} D_{\mathrm{B}}$ and $\overparen{E} \mathrm{R} E_{\mathrm{B}}$ are the radii of the polarization paths of the blue wavelength light due to the positive $A$-plate 1 .
 calculated using the equations given below:

$$
\begin{align*}
& {\overparen{D_{\mathrm{R}} D_{\mathrm{G}}}=\overparen{E}_{\mathrm{R}} E_{\mathrm{G}}=\Gamma_{A 2}^{\mathrm{R}}-\Gamma_{A 2}^{\mathrm{G}},}_{\overparen{D}_{\mathrm{R}} D_{\mathrm{B}}}=\overparen{E},_{\mathrm{R} E_{\mathrm{G}}}=\Gamma_{A 2}^{\mathrm{R}}-\Gamma_{A 2}^{\mathrm{B}} .
\end{align*}
$$

From this, we can calculate the optimized polarization positions of light on the circular path $L_{1}$.

Figure 4 shows the calculation of the retardation of green and blue wavelength lights caused by the half-wave retarder. From the triangles $B D_{\mathrm{R}} D_{\mathrm{G}}$ and $B D_{\mathrm{R}} D_{\mathrm{B}}$, we can obtain the


Figure 4. Spherical triangles on the Poincaré sphere for determining the retardation of the green and blue wavelengths for the half-wave retarder and the positive $A$-plate 1 .
retardation of green and blue wavelength lights, respectively, by using the following simple trigonometric equation:

$$
\begin{align*}
& \Gamma_{\lambda / 2}^{\mathrm{G}}=\pi+\angle D_{\mathrm{R}} B D_{\mathrm{G}}, \quad \Gamma_{\lambda / 2}^{\mathrm{B}}=\pi+\angle D_{\mathrm{R}} B D_{\mathrm{B}},  \tag{3}\\
& \angle D_{\mathrm{R}} B D_{\mathrm{G}}=\cos ^{-1}\left[\left(\cos {\overparen{D_{\mathrm{R}} D_{\mathrm{G}}}}\right.\right. \\
& \left.\left.\quad-\cos \overparen{B D_{\mathrm{G}}} \cos \overparen{B D_{\mathrm{R}}}\right) / \sin \overparen{B D_{\mathrm{G}}} \sin \overparen{B D_{\mathrm{R}}}\right], \tag{4}
\end{align*}
$$

$$
\begin{align*}
& \angle D_{\mathrm{R}} B D_{\mathrm{B}}=\cos ^{-1}\left[\left(\cos \overparen{D}_{\mathrm{R}} D_{\mathrm{B}}\right.\right. \\
& \left.\left.\quad-\cos \overparen{B D_{\mathrm{B}}} \cos \overparen{B D_{\mathrm{R}}}\right) / \sin \overparen{B D_{\mathrm{B}}} \sin \overparen{B D_{\mathrm{R}}}\right] \tag{5}
\end{align*}
$$

where $\Gamma_{\lambda / 2}^{\mathrm{G}}$ and $\Gamma_{\lambda / 2}^{\mathrm{B}}$ represent the calculated retardations of green and blue wavelength lights by the half-wave retarder, respectively.

The optimized retardation value of positive $A$-plate 1 can also be obtained by using the following simple trigonometric equation that employs $\angle B D_{\mathrm{R}} D_{\mathrm{G}}$ and $\angle B D_{\mathrm{R}} D_{\mathrm{B}}$ :
$\Gamma_{A 1}^{\mathrm{G}}=\pi / 2-\angle B D_{\mathrm{R}} D_{\mathrm{G}}, \quad \Gamma_{A 1}^{\mathrm{B}}=\pi / 2-\angle B D_{\mathrm{R}} D_{\mathrm{B}}$,
$\angle B D_{\mathrm{R}} D_{\mathrm{G}}=\cos ^{-1}\left[\left(\cos \overparen{B D_{\mathrm{G}}}\right.\right.$

$$
\begin{equation*}
-\cos {\overparen{D R} D_{\mathrm{G}}}^{\left.\left.\cos \overparen{B D_{\mathrm{R}}}\right) / \sin \overparen{D \mathrm{R}} \sin \overparen{B D_{\mathrm{R}}}\right], ~ ; ~} \tag{7}
\end{equation*}
$$

$$
\begin{align*}
& \angle B D_{\mathrm{R}} D_{\mathrm{B}}=\cos ^{-1}\left[\left(\cos \overparen{B D_{\mathrm{B}}}\right.\right. \\
& \quad-\cos {\left.\overparen{D_{\mathrm{R}} D_{\mathrm{B}}} \cos \overparen{B D_{\mathrm{R}}}\right) / \sin {\overparen{D_{\mathrm{R}} D_{\mathrm{B}}}}^{\left.\sin \overparen{B D_{\mathrm{R}}}\right] .}}^{2} . \tag{8}
\end{align*}
$$

$\Gamma_{A 1}^{\mathrm{G}}$ and $\Gamma_{A 1}^{\mathrm{B}}$ represent the calculated retardations of green and blue wavelength lights by $A$-plate 1 , respectively.

## 3. Result

By using the above-mentioned equations and Cauchy's formula [12,13], we could obtain the optimized retardation values for the half-wave retarder, the positive $A$-plate 1 and the positive $A$-plate 2 . We verified the improved dark state and the bright state of the reflective LC cell by using the helpful LC software DiMOS (developed by Autronic-Melchers,


Figure 5. Comparison of the calculated reflectance between the conventional and the proposed LC cells in the dark state at $\theta=15^{\circ}$.



Figure 6. Calculated dispersion of the refractive indices for $(a)$ the half-wave retarder and (b) $A$-plate 1 at $\theta=20^{\circ}$ and $\theta=30^{\circ}$.

Germany) instead of performing experiments because each optimized film requires to be supported by a very long time by material companies.

Figure 5 shows a comparison of the calculated reflection spectra between the conventional and the proposed reflective LC cells in the dark state when $\theta=15^{\circ}$. From this figure, we confirm that light leakage still occurs for the blue wavelength because we found that no optimized condition exists for the blue light.

Figure 6 shows the calculated optimized phase dispersion for the half-wave retarder and $A$-plate 1 when $\theta=20^{\circ}$ and $30^{\circ}$, respectively. In figure $6(a)$, we can observe the abnormal

Table 1. Calculated retardations of the half-wave retarder, $A$-plate 1 , and $A$-plate $2\left(\theta=20^{\circ}\right)$.

|  | $\Delta n(\lambda) / \Delta n(546 \mathrm{~nm})$ |  |  |  | $\Delta n$ |
| :---: | :--- | :--- | :--- | :--- | :--- |
|  | 400 nm | 436 nm | 633 nm | $\Delta n$ <br> $(546 \mathrm{~nm})$ | $d$ <br> $(\mu \mathrm{~m})$ |
| Half-wave <br> retarder | 1.168 | 1.058 | 0.998 | 0.0063 | 50 |
| A-plate 1 | 4.710 | 2.906 | 0.879 | 0.0017 | 10 |
| A-plate 2 | 1.130 | 1.085 | 0.961 | 0.0028 | 10 |


(a)


Figure 7. Comparison of the calculated reflectance between the conventional and the proposed reflective LC cells: (a) dark state and (b) bright state.
dispersion property of the half-wave retarder when $\theta=30^{\circ}$. Instead, we can observe the normal dispersion characteristics in both $A$-plate 1 and the half-wave retarder if we use the condition $\theta=20^{\circ}$. Table 1 shows the calculated optimized retardations for $A$-plate $1, A$-plate 2 and the half-wave retarder for each wavelength for a single polarizer reflective LC cell using ML-0249.

Figure 7 shows a comparison of the calculated reflection spectra between the conventional and the proposed reflective LC cells in the dark state and the bright state. From this figure, we confirm that an excellent dark state can be achieved
by using the proposed optical configuration for $\theta=20^{\circ}$; the proposed configuration can permit a high contrast ratio that is greater than twice the calculated conventional optical configuration.

## 4. Conclusion

In conclusion, we have proposed an optical configuration for the reflective LC cell with a single polarizer that provides an excellent contrast ratio. By applying and optimizing two $A$-plates and a half-wave retarder, we have simultaneously achieved an excellent contrast ratio in comparison with the conventional reflective LC cell by effectively eliminating phase dispersion.

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