Chapter 7. Power divider and directional coupler

•Power dividers and directional couplers: Passive (or active) microwave circuits used for power division or power combining.

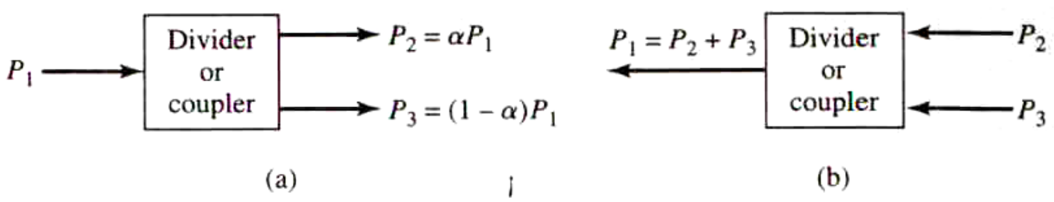


Figure 7.1 Power division and combining. (a) Power division. (b) Power combining.

√ Divided into two (or more paths) signals of lesser power
Ex.] 3-port networks: T-junction, Wilkinson divider, etc.
4-port networks: directional couplers, hybrids, etc.

√ Equal-division (-3 dB) or unequal power division

√ 90° (quadrature) or a 180° phase shift between the output ports.

7.1 BASIC PROPERTIES OF DIVIDERS

•Three-Port Networks (T-Junctions)

√ Simplest type of power divider

√ Three-port network with two inputs and one output (or with one input and two outputs)

√ Scattering matrix of an arbitrary three-port network:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \qquad (7.1)$$

√ In case of passive and no anisotropic materials, [S] matrix must be reciprocal and symmetric ($S_{ij}= S_{ji}$)

√ If all ports are matched and lossless network, then S_{ii} = 0.

√ Revised scattering matrix:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} \qquad (7.2)$$

√ By the unitary property of the scattering parameter

($\Leftrightarrow$ energy conservation: [S]* [S]^t =[U]),

$$\begin{bmatrix} 0 & S_{12}^* & S_{13}^* \\ S_{12}^* & 0 & S_{23}^* \\ S_{13}^* & S_{23}^* & 0 \end{bmatrix}\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|S_{12}|^2+|S_{13}|^2=1 \qquad (7.3a)$$

$$|S_{12}|^2+|S_{23}|^2=1 \qquad (7.3b)$$

$$|S_{13}|^2+|S_{23}|^2=1 \qquad (7.3c)$$

$$S_{13}^*S_{23}=0 \qquad (7.3d)$$

$$S_{23}^*S_{12}=0 \qquad (7.3e)$$

$$S_{12}^*S_{13}=0 \qquad (7.3f)$$

√ Equations (7.3*d*-*f*) can't be satisfied simultaneously.

➔At least two of the three parameters S_{12}, S_{13}, S_{23} must be zero.

➔Inconsistent with one of equations (7.3*a*-*c*) that implying a three-port network cannot be lossless, reciprocal, and matched at all ports.

√ If the three-port network is nonreciprocal, then $S_{ij} \neq S_{ji}$ and the conditions of input matching at all ports and energy conservation can be satisfied. ➔ Circulator

√ The [S] matrix of a <u>matched three-port network</u> has the following form: (← Nonreciprocal)

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix} \qquad (7.4)$$

If network is lossless, [S] must be unitary ([S]*[S]^t =[U]),

$$\begin{bmatrix} 0 & S_{12}^* & S_{13}^* \\ S_{21}^* & 0 & S_{23}^* \\ S_{31}^* & S_{32}^* & 0 \end{bmatrix}\begin{bmatrix} 0 & S_{21} & S_{31} \\ S_{12} & 0 & S_{32} \\ S_{13} & S_{23} & 0 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_{13}^*S_{23}=0, \quad S_{12}^*S_{32}=0 \qquad (7.5a)\ \ \text{X}$$

$$S_{23}^*S_{13}=0, \quad S_{21}^*S_{31}=0 \qquad (7.5b)\ \ \text{X}$$

$$S_{32}^*S_{12}=0, \quad S_{12}^*S_{13}=0 \qquad (7.5c)\ \ \text{X}$$

$$|S_{12}|^2+|S_{13}|^2=1 \qquad (7.5d)$$

$$|S_{21}|^2+|S_{23}|^2=1 \qquad (7.5e)$$

$$|S_{31}|^2+|S_{32}|^2=1 \qquad (7.5f)$$

These equations can be satisfied in one of two ways. Either

$$S_{12}=S_{23}=S_{31}=0, \quad |S_{21}|=|S_{32}|=|S_{13}|=1, \qquad (7.6a)$$

or $$S_{21}=S_{32}=S_{13}=0, \quad |S_{12}|=|S_{23}|=|S_{31}|=1, \qquad (7.6b)$$

➔ $S_{ij} \neq S_{ji}$ for $i \neq j$, : Nonreciprocal.

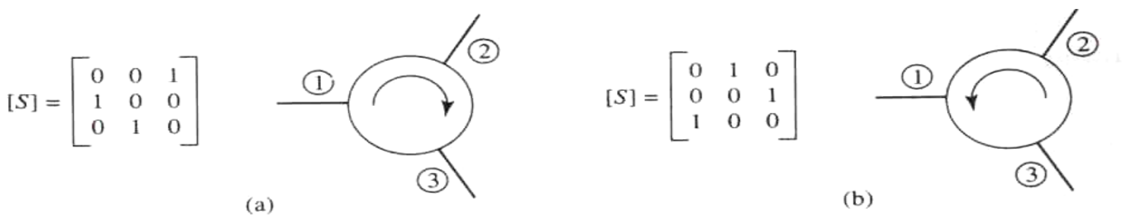


FIGURE 7.2 The two types of circulators and their [S] matrices. (The phase references for the ports are arbitrary.) (a) <u>Clockwise</u> circulation. (b) <u>Counterclockwise</u> circulation.

√ Alternatively, <u>a lossless and reciprocal three-port network</u> can be physically realized if only two of its ports are matched. If ports 1 and 2 are these matched ports,

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} \qquad (7.7)$$

To be lossless, the following unitary conditions must be satisfied:

$$\begin{bmatrix} 0 & S_{12}^* & S_{13}^* \\ S_{12}^* & 0 & S_{23}^* \\ S_{13}^* & S_{23}^* & S_{33}^* \end{bmatrix}\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_{13}^*S_{23}=0 \qquad (7.8a)$$

$$S_{12}^*S_{13}+S_{23}^*S_{33}=0 \qquad (7.8b)$$

$$S_{23}^*S_{12}+S_{33}^*S_{13}=0 \qquad (7.8c)$$

$$|S_{12}|^2+|S_{13}|^2=1 \qquad (7.8d)$$

$$|S_{12}|^2+|S_{23}|^2=1 \qquad (7.8e)$$

$$|S_{13}|^2+|S_{23}|^2+|S_{33}|^2=1 \qquad (7.8f)$$

➔ $|S_{13}|=|S_{23}|$ due to Eq.$(7.8d\text{ - }e)$,

$S_{13}=S_{23}=0$ (←Eq.(7.8*a*)), and $|S_{12}|=|S_{33}|=1$ (←Eq.(7.8*d*) and .(7.8*f*))

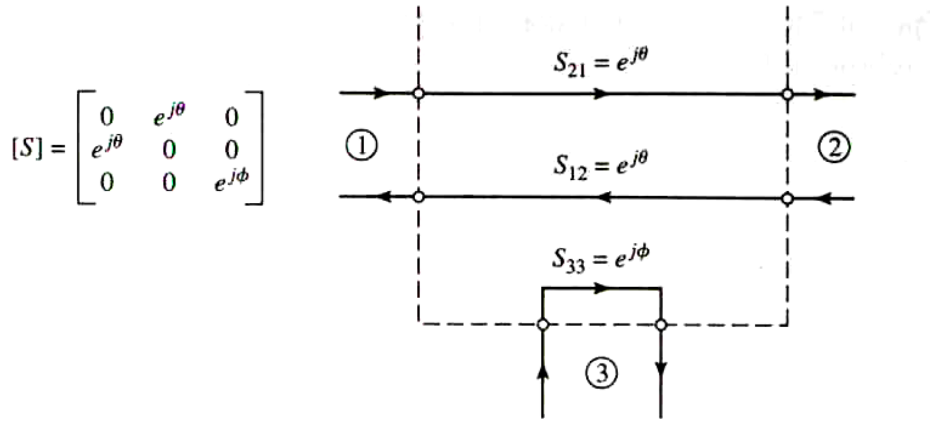


FIGURE 7.3 A reciprocal, lossless three-port network matched at ports 1 and 2.

•FOUR-PORT Networks (Directional Couplers)

√ The [S] matrix of a reciprocal four-port network is matched at all ports:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix} \qquad (7.9)$$

√ From the unitarity, or energy conservation, consider the multiplication of row 1 (of 1st matrix) and column 2 (of 2nd matrix), and the multiplication of row 4 (of 1st matrix) and column 3(of 2nd matrix):

$$\begin{bmatrix} 0 & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & S_{24}^* \\ S_{13}^* & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & S_{24}^* & S_{34}^* & 0 \end{bmatrix}\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S_{13}^*S_{23}+S_{14}^*S_{24}=0, \qquad (7.10a)$$

$$S_{14}^*S_{13}+S_{24}^*S_{23}=0, \qquad (7.10b)$$

√ [Eq.(7.10b)×S_{13}^* - Eq.(7.10a) × S_{24}^*]:

$$S_{14}^*(|S_{13}|^2-|S_{24}|^2)=0. \qquad (7.11)$$

√ Multiplication of row 1 (of 1st matrix) and column 3 (of 2nd matrix), and the multiplication of row 4 (of 1st matrix) and row 2 (of 2nd matrix):

$$S_{12}^*S_{23}+S_{14}^*S_{34}=0 \qquad (7.12a)$$

$$S_{14}^*S_{12}+S_{34}^*S_{23}=0 \qquad (7.12b)$$

√ [Eq.(7.12a)× S_{12} - Eq.(7.12b) × S_{34}]:

$$S_{23}(|S_{12}|^2-|S_{34}|^2)=0 \qquad (7.13)$$

√ One way for (7.11) and (7.13) to be satisfied is $S_{14} = S_{23} = 0$.

➔ (Isolation of) Directional coupler

√ Self-products of the rows of the unitary [S] matrix of (7.9):

$$|S_{12}|^2 + |S_{13}|^2 = 1, \qquad (7.14a)$$

$$|S_{12}|^2 + |S_{24}|^2 = 1, \qquad (7.14b)$$

$$|S_{13}|^2 + |S_{34}|^2 = 1, \qquad (7.14c)$$

$$|S_{24}|^2 + |S_{34}|^2 = 1, \qquad (7.14d)$$

➔ $|S_{13}| = |S_{24}|$ due to Eq.(7.14a) and Eq.(7.14b)

$|S_{12}| = |S_{34}|$ due to Eq.(7.14b) and Eq.(7.14d)

➔ (Coupling and through of) Directional coupler

√ Further simplification can be made by choosing the phase references on three of the four ports.

$S_{12} = S_{34} = \alpha$, $S_{13} = \beta e^{j\theta}$, and $S_{24} = \beta e^{j\phi}$,

where α, β : real,

θ, ϕ : phase constants to be determined

√ Dot product of row 2 (of 1st matrix) and column 3 (of 2nd matrix):

$$S_{12}^* S_{13} + S_{24}^* S_{34} = 0, \qquad (7.15)$$

➔ $$\alpha\beta e^{j\theta} + \alpha\beta e^{-j\phi} = 0$$

$$\theta + \phi = \pi \pm 2n\pi \qquad (7.16)$$

√ If we ignore integer multiples of 2π, there are two particular choices that commonly occur in practice:

1. Symmetrical coupler: $\theta = \phi = \pi/2$

➔The phases of the terms having amplitude β are chosen equal.

$$[S]=\begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix} \qquad (7.17)$$

2. Anti-symmetrical coupler: $\theta = 0$, $\phi = \pi$

➔The phases of the terms having amplitude β are chosen to be 180° apart.

$$[S]=\begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix} \qquad (7.18)$$

√ Basic operation of a directional coupler:

Incident port: port 1

Coupling port: port 3 (Coupling coefficient $|S_{13}|^2 = \beta^2$)

Through port: port 2 (Delivering coefficient $|S_{12}|^2 = \alpha^2 = 1 - \beta^2$)

Isolation port: port 4 (No power is delivered)

$$\text{Coupling} = C = 10\log\frac{P_1}{P_3} = -20\log\beta \text{ [dB]} \qquad (7.20a)$$

$$\text{Directivity} = D = 10\log\frac{P_3}{P_4} = -20\log\frac{\beta}{|S_{14}|} \text{ [dB]} \qquad (7.20b)$$

$$\text{Isolation} = I = 10\log\frac{P_1}{P_4} = -20\log|S_{14}| \text{ [dB]} \qquad (7.20c)$$

$$= 10\log\frac{P_1}{P_3}\frac{P_3}{P_4} = 10\log\frac{P_1}{P_3} + 10\log\frac{P_3}{P_4}$$

$$= C + D \text{ [dB]} \qquad (7.21)$$

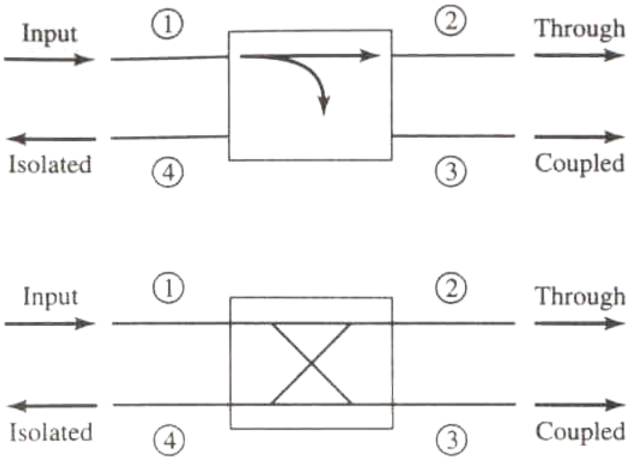


FIGURE 7.4 Two commonly used symbols for directional couplers, and power flow conventions.

√ Hybrid couplers: special cases of directional couplers

Coupling factor: 3 dB $\Leftrightarrow$ $\alpha = \beta = 1/\sqrt{2}$.

1. Type 1: quadrature hybrid
   90° phase shift between ports 2 and 3 ($\theta = \phi = \pi/2$)

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix} \tag{7.22}$$

2. Type 2: magic-T hybrid or rat-race hybrid
   180° phase difference between ports 2 and 3 when fed at port 4

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \tag{7.23}$$

7.2 THE T-JUNCTION POWER DIVIDER

- A simple three-port network that can be used for power division or power combining

- Assume ideal transmission line and lossless junctions

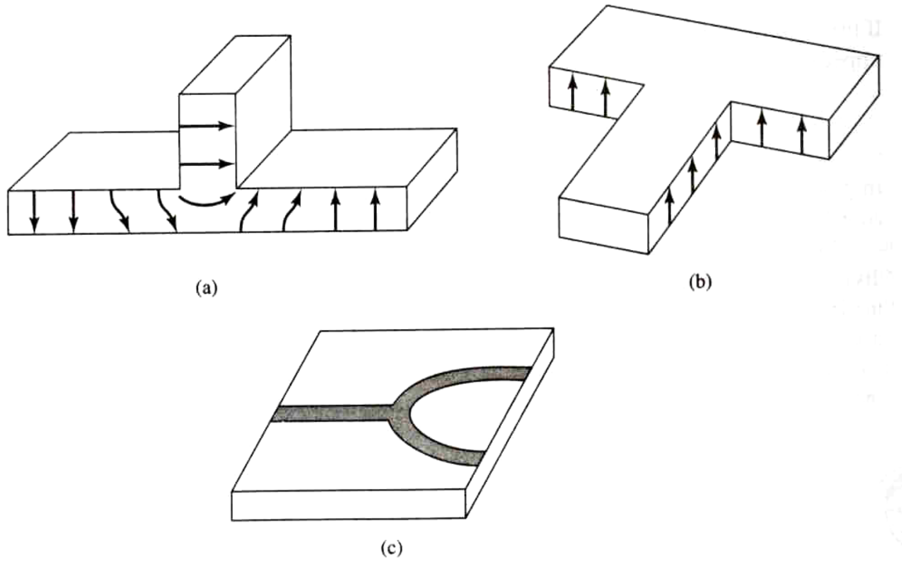


FIGURE 7.5 Various T-junction power dividers. (a) E plane waveguide T. (b) H plane waveguide T. (c) Microstrip T-junction.

- **Lossless Divider**

√ The lossless T-junctions can all be modeled as a junction of three transmission lines.

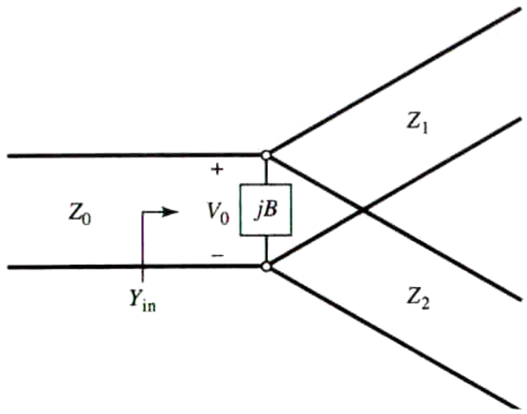


FIGURE 7.6 Transmission line model of a lossless T-junction.

√ Fringing fields and higher order modes associated with the discontinuity at such a junction ➔ stored energy ➔ accounted for by a lumped susceptance, *B*.

√ In order for the divider to be matched to the input line of characteristic impedance Z_0,

$$Y_{\text{in}} = jB + \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0} \tag{7.24}$$

√ If we also assume $B = 0$,

$$\frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0} \tag{7.25}$$

√ In practice, jB can be not negligible. ➔ Need reactive tuning element !

EXAMPLE 7.1 T-junction Power Divider

√ $Z_0 = 50\ \Omega$

√ For 2 : 1 power dividing ratio, Z_1 and Z_2 = ?

√ Reflection coefficients seen looking into the output ports?

Sol.) Input power to the matched divider:

$$P_{\text{in}} = \frac{1}{2}\frac{V_0^2}{Z_0},$$

While the output powers are

$$P_1 = \frac{1}{2}\frac{V_0^2}{Z_1} = \frac{1}{3}P_{\text{in}} = \frac{1}{3}\frac{V_0^2}{2Z_0}$$

$$P_2 = \frac{1}{2}\frac{V_0^2}{Z_2} = \frac{2}{3}P_{\text{in}} = \frac{2}{3}\frac{V_0^2}{2Z_0} = \frac{V_0^2}{3Z_0}$$

These results yield the characteristic impedances as

$$Z_1 = 3Z_0 = 150\ \Omega$$

$$Z_2 = (3Z_0)/2 = 75\ \Omega$$

Then the input impedance to the junction is

$$Z_{\text{in}} = 75 \| 150 = 50\ \Omega, \quad \leftarrow \text{Matched!!}$$

Looking into the 150 Ω output line, we see an impedance of (50 // 75) = 30 Ω, whereas (50 // 150) = 37.5 Ω seen at 75 Ω output line.

$$\Gamma_1 = \frac{30 - 150}{30 + 150} = -0.666$$

$$\Gamma_2 = \frac{37.5 - 75}{37.5 + 75} = -0.333$$

➔ Output matchings are not good!!

- **Resistive Divider**

√ Contains lossy components

√ Matched at all ports

√ Two output ports may not be isolated.

√ An equal-split (-3 dB) divider and unequal power division ratios are also possible.

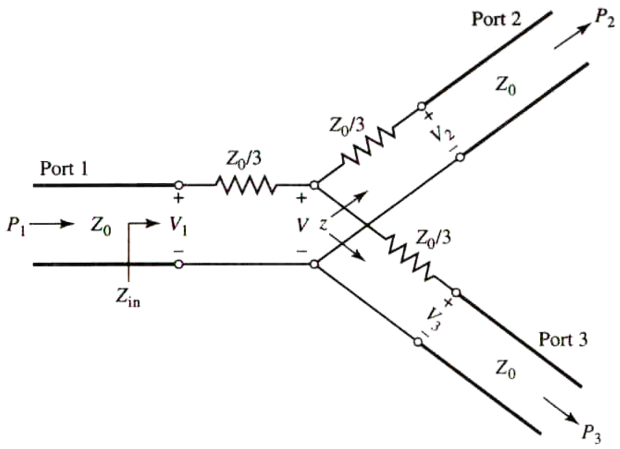


FIGURE 7.7 An equal-split three-port resistive power divider.

√ Assuming that all ports are terminated with Z_0, the impedance Z, seen looking into the $Z_0/3$ resistor followed by the output line, is

$$Z = \frac{Z_0}{3} + Z_0 = \frac{4Z_0}{3} \qquad (7.26)$$

√ Input impedance of the divider:

$$Z_{in} = \frac{Z_0}{3} + \left(\frac{4Z_0}{3} // \frac{4Z_0}{3}\right) = Z_0 \qquad (7.27)$$

➔ Input matched

➔ Since the network is symmetric from all three ports, the output ports are also matched ($S_{11} = S_{22} = S_{33} = 0$).

√ If the voltage at port 1 is V_1, the voltage V at the center of the junction is

$$V = V_1 \frac{2Z_0/3}{Z_0/3 + 2Z_0/3} = \frac{2}{3}V_1 \qquad (7.28)$$

The output voltages are

$$V_2 = V_3 = V \frac{Z_0}{Z_0 + Z_0/3} = \frac{3}{4}V = \frac{1}{2}V_1 \qquad (7.29)$$

√ Scattering matrix: symmetric

$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad (7.30)$$

√ Power delivered to the input of the divider:

$$P_{\text{in}} = \frac{1}{2}\frac{V_1^2}{Z_0} \qquad (7.31)$$

Output powers are

$$P_2 = P_3 = \frac{1}{2}\frac{V_2^2}{Z_0} = \frac{1}{2}\frac{(V_1/2)^2}{Z_0} = \frac{1}{8}\frac{V_1^2}{Z_0} = \frac{1}{4}P_{\text{in}} \qquad (7.32)$$

➔The half of the supplied power is dissipated in the resistors ($P_{\text{in}}/2$).

7.3 THE WILKINSON POWER DIVIDER

- The Wilkinson power divider can be made to give arbitrary power division, but we will first consider the equal-split (3 dB) case.

- This "even-odd" mode analysis technique will also be useful for other networks that we will analyze in later sections.

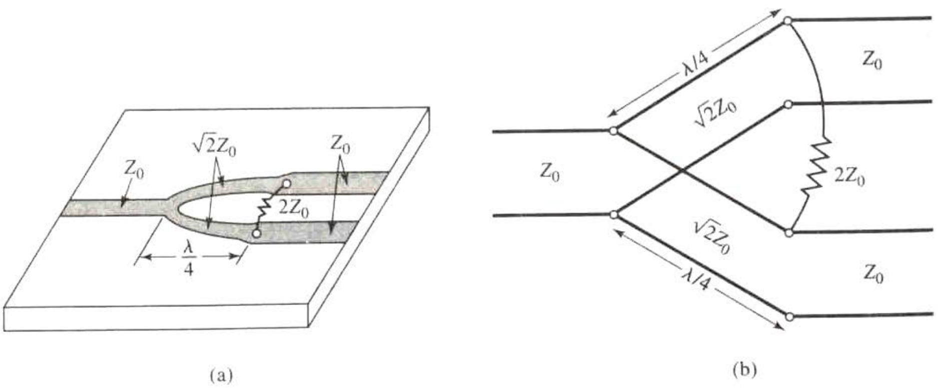


FIGURE 7.8 The Wilkinson power divider. (a) An equal-split Wilkinson power divider in microstrip form. (b) Equivalent transmission line circuit.

•Even-Odd Mode Analysis

√ Normalize all impedances to the characteristic impedance Z_0.

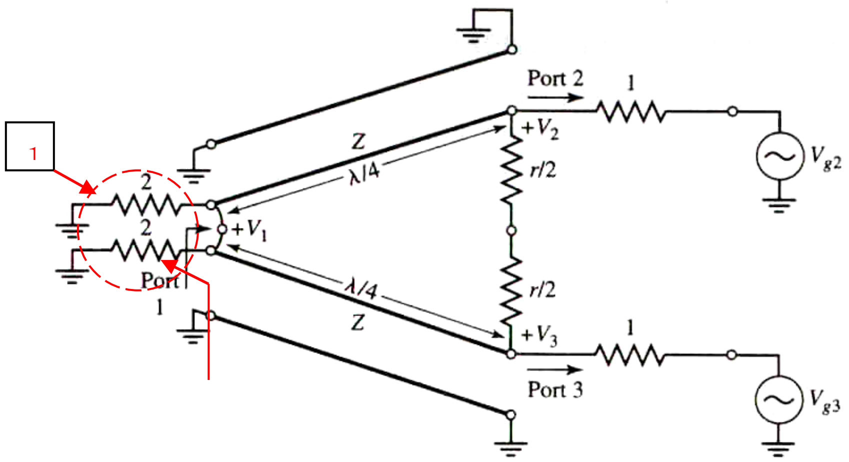


FIGURE 7.9 The Wilkinson power divider circuit in normalized and symmetric form.

√ Symmetric across the mid-plane

√ **Even mode**.

♦ $V_{g2} = V_{g3} = 2\mathrm{V}$

♦ $V_2^e = V_3^e$

➔ No current flow through the $r/2$ resistors
➔ Bisect the network of Figure 7.9 with open circuits

♦ Impedance looking into port 2

$$Z_{\text{in}}^e = \frac{Z^2}{2} \qquad (7.33)$$ ←Quarter-wave transformer

If $Z = \sqrt{2}$, port 2 will be matched for even mode excitation.

➔ $V_2^e = \mathrm{V}$ (since $Z_{\text{in}}^e = 1$ and $V_{g2} = 2\mathrm{V}$).

♦ Let $x = 0$ at port 1 and $x = -\lambda/4$ at port 2, the voltage on the transmission line section can be written as

$$V(x) = V^+(e^{-j\beta x} + \Gamma e^{j\beta x})$$

Then,

$$V_2^e = V(-\lambda/4) = V^+(e^{j\frac{2\pi}{\lambda}\frac{\lambda}{4}} + \Gamma e^{-j\frac{2\pi}{\lambda}\frac{\lambda}{4}})$$

$$= jV^+(1-\Gamma) = \mathrm{V} \qquad (7.34)$$

$$\Rightarrow V^+ = j\frac{\mathrm{V}}{\Gamma - 1}$$

$$V_1^e = V(0) = V^+(1+\Gamma) = j\mathrm{V}\frac{\Gamma+1}{\Gamma-1}$$

♦ Reflection coefficient (Γ) seen at port 1, looking toward termination resistor of normalized value 2:

$$\Gamma = \frac{2-\sqrt{2}}{2+\sqrt{2}},$$

$$V_1^e = jV\frac{\dfrac{2-\sqrt{2}}{2+\sqrt{2}}+1}{\dfrac{2-\sqrt{2}}{2+\sqrt{2}}-1} = jV\frac{4}{-2\sqrt{2}} = -j\sqrt{2}V \qquad (7.35)$$

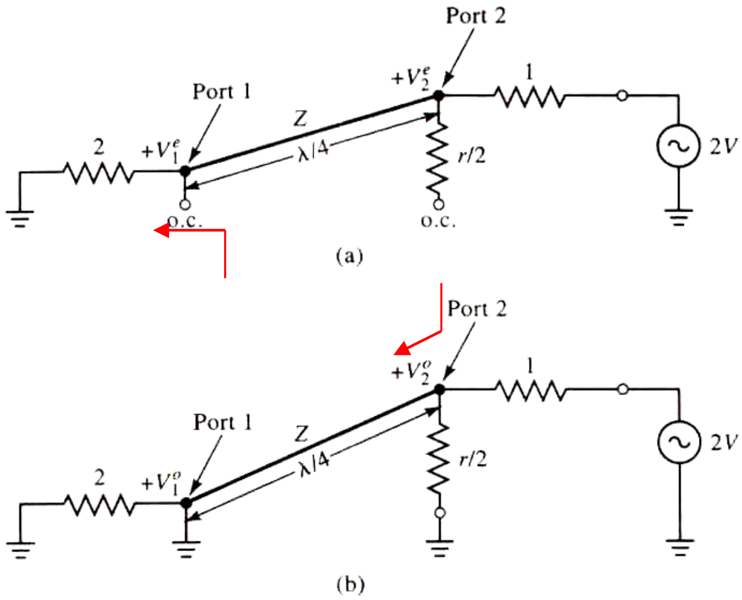


FIGURE 7.10 Bisection of the circuit of Figure 7.9. (a) For even-mode excitation. (b) For odd-mode excitation.

√ **Odd mode.**

- ♦ V_{g2} = -V_{g3} = 2V
- ♦ $V_2^o = -V_3^o$
- ♦ Impedance seen from port 2 to port 1 is like an open circuit. If port2 will be matched for odd mode excitation,

  we select r = 2. ➔ $V_2^o = V$ and $V_1^o = 0$;

♦ Input impedance at port 1 of the Wilkinson divider when ports 2 and 3 are terminated in matched loads.

$$Z_{\text{in}} = \left\{\frac{(\sqrt{2})^2}{1}\right\} // \left\{\frac{(\sqrt{2})^2}{1}\right\} = \frac{1}{2}(\sqrt{2})^2 = 1 \qquad (7.36)$$

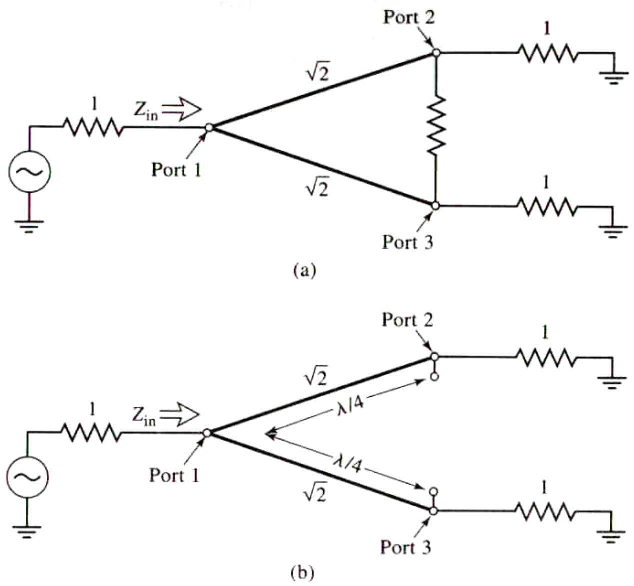


FIGURE 7.11 Analysis of the Wilkinson divider to find S_{11}. (a) The terminated Wilkinson divider. (b) Bisection of the circuit in (a).

♦ *S*-parameters characteristics for the Wilkinson divider:

$S_{11} = 0$ ($Z_{\text{in}} = 1$ at port 1)

$S_{22} = S_{33} = 0$ (Ports 2 and 3 are matched for even and odd modes)

$$S_{12} = S_{21} = \frac{V_1^e + V_1^o}{V_2^e + V_2^o} = \frac{-j\mathrm{V}\sqrt{2} + 0}{\mathrm{V}+\mathrm{V}} = -j/\sqrt{2}$$

(Symmetry due to reciprocity)

$S_{13} = S_{31} = -j/\sqrt{2}$ (Symmetry of ports 2 and 3)

$S_{23} = S_{32} = 0$ (Due to short or open at bisection)

EXAMPLE 7.2 Design and Performance of a Wilkinson Divider

√ Design an equal-split Wilkinson power divider for a 50 Ω system impedance at frequency f_0

Sol.) Characteristic impedance of $\lambda/4$ transmission line:

$$Z = \sqrt{2}Z_0 = 70.7\ \Omega,$$

Value of shunt resistor:

$$R = 2Z_0 = 100\ \Omega$$

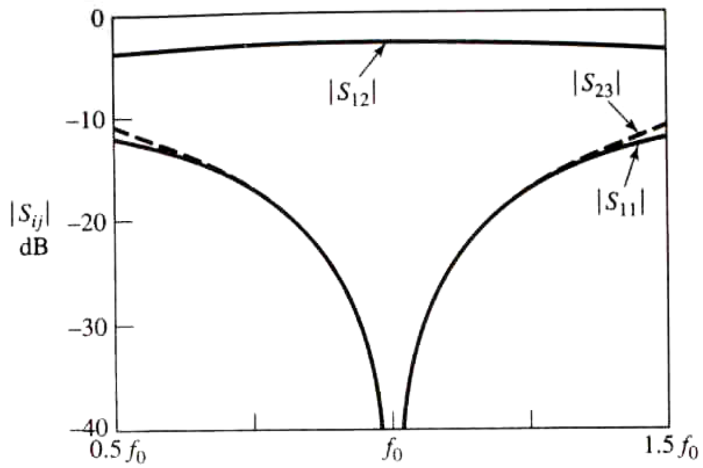


FIGURE 7.12 Frequency response of an equal-split Wilkinson power divider. Port 1 is the input port; ports 2 and 3 are the output ports.

• Unequal Power Division and *N*-Way Wilkinson Dividers

√ Unequal power splits; a microstrip version is shown in Figure 7.13.

√ If the power ratio between ports 2 and 3 is $K^2 = P_3 / P_2$,

$$Z_{03} = Z_0 \sqrt{\frac{1+K^2}{K^3}} \qquad (7.37a)$$

$$Z_{02} = K^2 Z_{03} = Z_0 \sqrt{K(1+K^2)} \qquad (7.37b)$$

$$R = Z_0 \left(K + \frac{1}{K} \right) \qquad (7.37c)$$

√ The output lines are matched to the impedances $R_2 = Z_0K$ and $R_3 = Z_0 / K$.

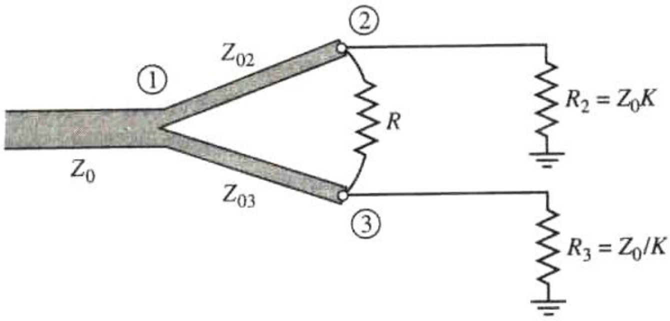


FIGURE 7.13 A Wilkinson power divider in microstrip form having unequal power division.

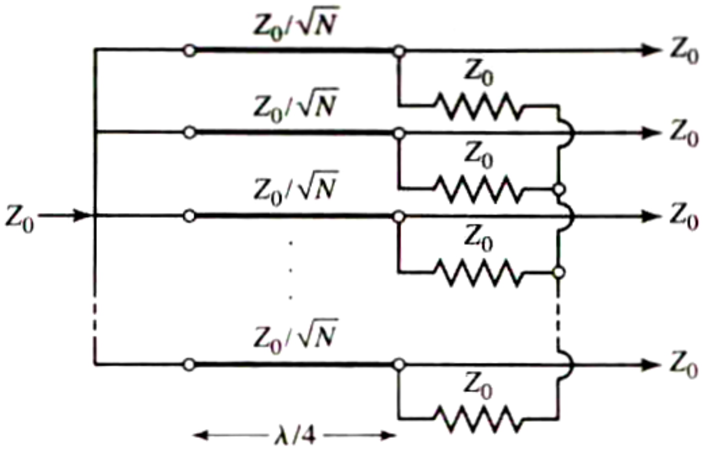
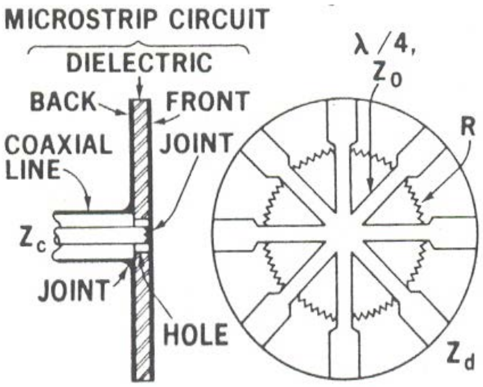
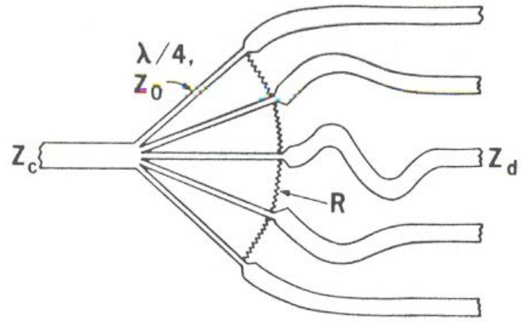


FIGURE 7.14 An *N*-way, equal-split Wilkinson power divider.





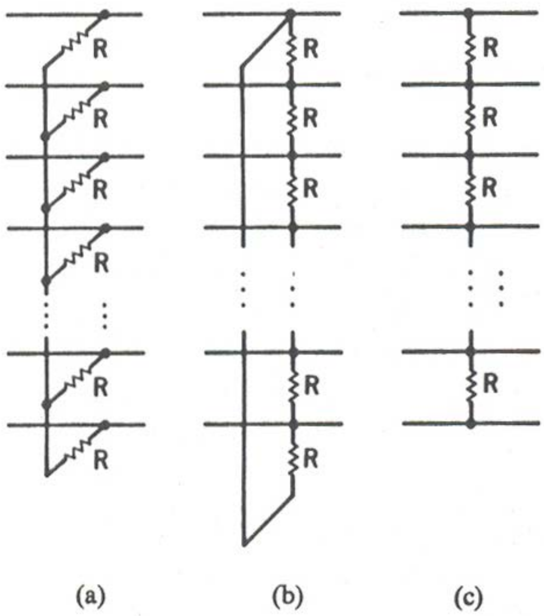


Fig. 5. The arrangements of the isolation resistors for the Wilkinson (a), radial (b), and fork (c) n-way hybrids.

7.5 QUADRATURE (90°) HYBRID (or Branch line hybrid)

- Quadrature hybrids are 3 dB directional couplers with a 90° phase difference in the outputs of the through and coupled arms.

- Made with microstrip or stripline forms

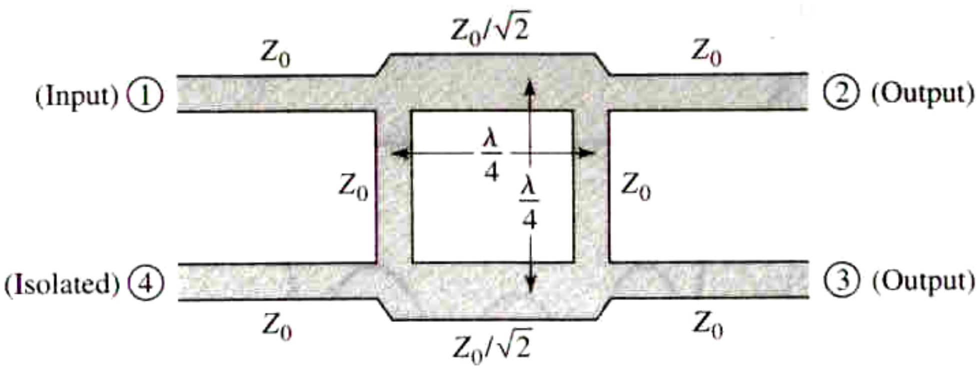


FIGURE 7.21 Geometry of a branch-line coupler.

- All ports are matched.

- Input port #: 1
  Output ports #: 2 (through), 3 (coupling) with a 90° phase shift
  Isolation port #: 4

- $[S]$ matrix:

$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix} \qquad (7.61)$$

cf.) Eq. (7.22)

- High degree of symmetry

• Even-Odd Mode Analysis

√ The branch-line coupler in normalized form that is normalized to Z_0

√ Unit amplitude of input wave: $A_1 = 1$

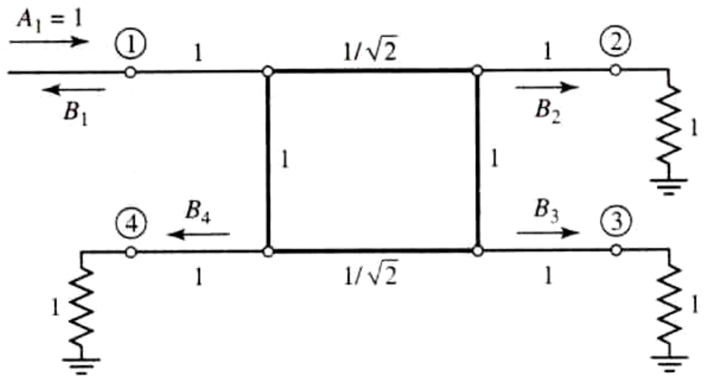


FIGURE 7.22 Circuit of the branch-line hybrid coupler in normalized form.

√ Decomposed into the <u>superposition</u> of an even-mode excitation and an odd-mode excitation.

➔ The actual response (the scattered waves) can be obtained from the sum of the responses to the even and odd-mode excitations.

➔ (4-port network) → (2-port network)

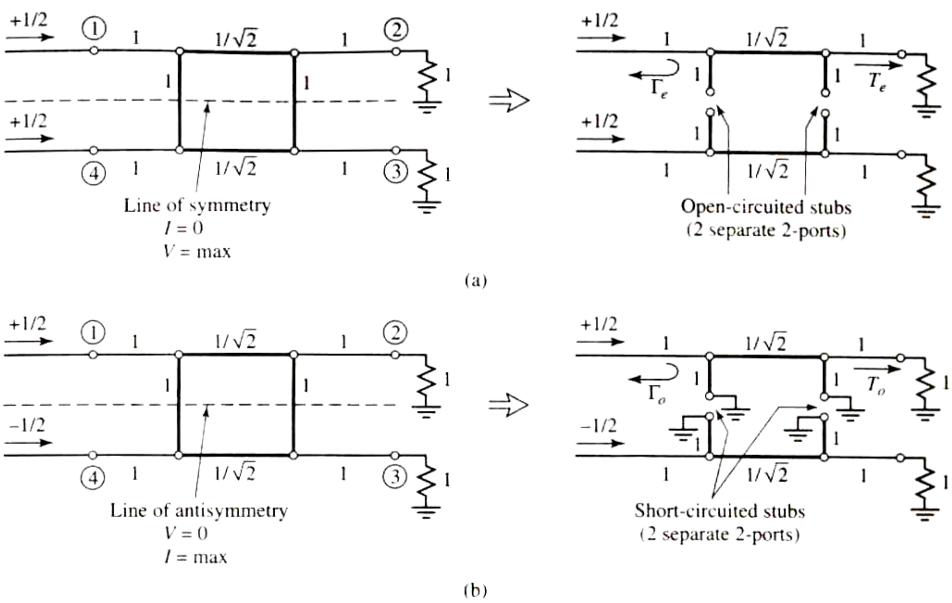


FIGURE 7.23 Decomposition of the branch-line coupler into even- and odd-mode excitations. (a) Even mode (e). (b) Odd mode (o).

√ Since the amplitudes of the incident waves for these two-ports are ±1/2, the amplitudes of the emerging wave at each port of the branch-line hybrid can be expressed as;

$$B_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o \qquad (7.62a)$$

$$B_2 = \frac{1}{2}T_e + \frac{1}{2}T_o \qquad (7.62b)$$

$$B_3 = \frac{1}{2}T_e - \frac{1}{2}T_o \qquad (7.62c)$$

$$B_4 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o \qquad (7.62d)$$

where $\Gamma_{e,o}$ and $T_{e,o}$: even- and odd-mode reflections and transmission coefficients

√ Even-mode *ABCD* matrices of each cascade component:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix} \begin{bmatrix} 0 & j/\sqrt{2} \\ j\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & j \\ j & -1 \end{bmatrix} \qquad (7.63)$$

Shunt Y=*j*	λ/4 Transmission line	Shunt Y=*j*

where admittance of shunt open-circuited $\lambda/8$ normalized stubs:

$Y = j\tan\beta l = j\tan(2\pi/\lambda \cdot \lambda/8) = j$

√ Using conversion from *ABCD*- to *S*-parameter with $Z_0 = 1$ and Table 4.2, the reflection and transmission coefficients are below;

$$\Gamma_e = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} = \frac{(-1 + j - j + 1)/\sqrt{2}}{(-1 + j + j - 1)/\sqrt{2}} = 0 \qquad (7.64a)$$

$$T_e = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{(-1 + j + j - 1)/\sqrt{2}} = \frac{-1}{\sqrt{2}}(1 + j) \qquad (7.64b)$$

√ Odd-mode *ABCD* matrix:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix} \begin{bmatrix} 0 & j/\sqrt{2} \\ j\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix} \quad (7.65)$$

➔ Reflection and transmission coefficients:

$$\Gamma_o = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} = \frac{(1 + j - j - 1)/\sqrt{2}}{(1 + j + j + 1)/\sqrt{2}} = 0 \quad (7.66a)$$

$$T_o = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{(1 + j + j + 1)/\sqrt{2}} = \frac{1}{\sqrt{2}}(1 - j) \quad (7.66b)$$

√ Using (7.64) and (7.66) in (7.62),

$$B_1 = 0 \quad \text{(port 1 is matched)} \quad (7.67a)$$

$$B_2 = -\frac{j}{\sqrt{2}} \quad \text{(half - power, -}90^{\circ}\text{ phase shift from port 1 to 2)} \quad (7.67b)$$

$$B_3 = -\frac{1}{\sqrt{2}} \quad \text{(half - power, -}180^{\circ}\text{ phase shift from port 1 to 3)} \quad (7.67c)$$

$$B_4 = 0 \quad \text{(no power to port 4)} \quad (7.67d)$$

➔Agree with the first row and column of the [*S*] matrix given in (7.61)

√ Due to the quarter-wave length requirement, the bandwidth of a branchline hybrid is limited to 10 - 20%.

➔Multisection branch-line hybrid for more broad bandwidth

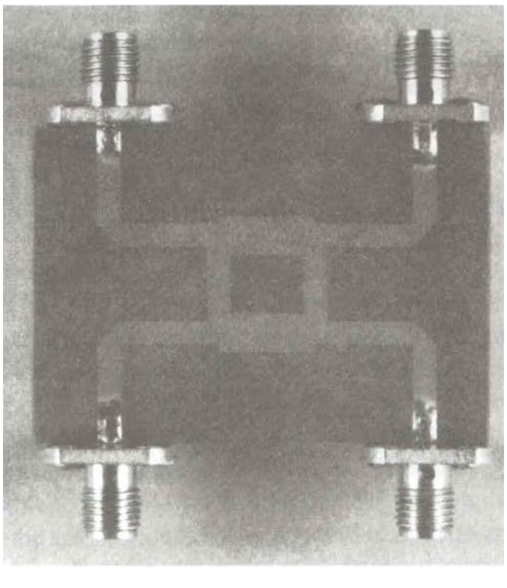


FIGURE 7.24 Photograph of a microstrip quadrature gybrid prototype. Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.

EXAMPLE 7.5 Design and Performance of a Quadrature Hybrid

√ Design a 50Ω branch-line quadrature hybrid junction.
√ Plot the S parameter magnitudes from $0.5f_0$ to $1.5f_0$, where f_0 is the design frequency.

Sol.) Branch-line impedance:

$$\frac{Z_0}{\sqrt{2}} = \frac{50}{\sqrt{2}} = 35.4\ \Omega.$$

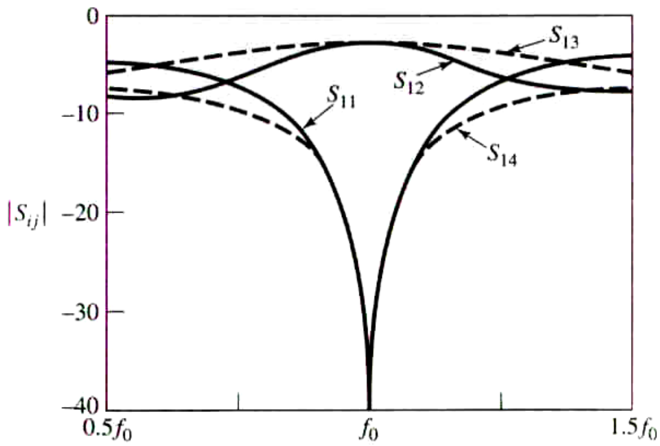


FIGURE 7.25 S parameter magnitudes versus frequency for the branch-line coupler of Example 7.5.

7.6 COUPLED LINE DIRECTIONAL COUPLERS

- Coupled transmission lines: Power can be coupled between the lines due to the interaction of the electromagnetic fields of each line.

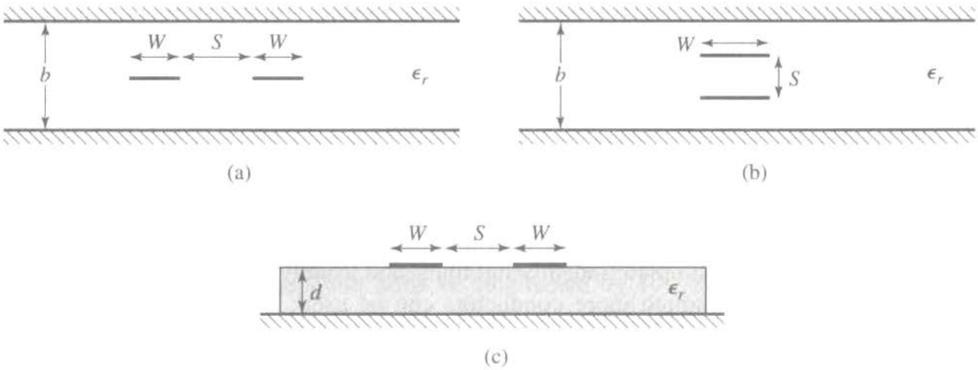


FIGURE 7.26 Various coupled transmission line geometries. (a) Coupled stripline (planar. or edge-coupled). (b) Coupled stripline (stacked, or broadside-coupled). (c) Coupled microstrip

- TEM mode operation: Stripline
  Quasi-TEM mode operation: Microstrip structures.

- **Coupled Line Theory**

  √ Three-wire line

  √ TEM propagation: The electrical characteristics of the coupled lines can be completely determined from the effective capacitances between the lines and the velocity of propagation on the line.

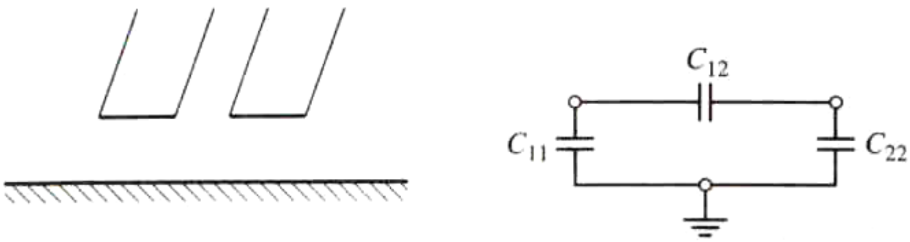


FIGURE 7.27 A three-wire coupled transmission line and its equivalent capacitance network.

√ C_{12}: capacitance the two strip conductors in the absence of the ground conductor

√ C_{11} and C_{22}: capacitances between one strip conductor and ground, in the absence of the other strip conductor.

➔ If the strip conductors are identical in size and location relative to the ground conductor, then $C_{11} = C_{22}$.

√ Even mode excitation: The currents in the strip conductors are equal in amplitude and in the same direction

➔ Even symmetry about the center line, and no current flows between the two strip conductors ($C_{12} = 0$)

$$C_e = C_{11} = C_{22} \qquad (7.68)$$

➔ Characteristic impedance for the even mode:

$$Z_{0e} = \sqrt{\frac{L}{C_e}} = \frac{\sqrt{LC_e}}{C_e} = \frac{1}{vC_e} \qquad (7.69)$$

where v: velocity of propagation on the line.

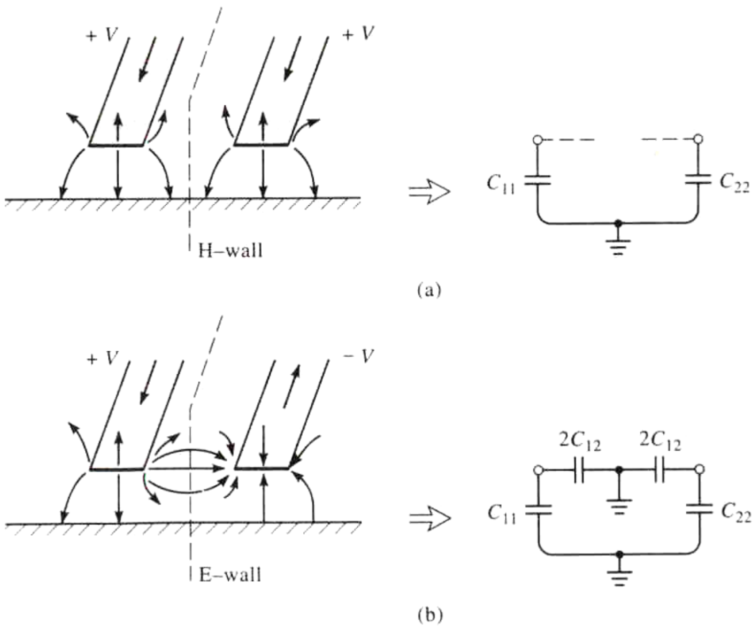


FIGURE 7.28 Even- and odd-mode excitations for a coupled line, and the resulting equivalent capacitance networks. (a) Even-mode excitation. (h) Odd-mode excitation

√ Odd mode excitation: The currents in the strip conductors are equal in amplitude but in opposite directions.

➔ Odd symmetry about the center line, and a voltage null exists between the two strip conductors.

$$C_o = C_{11} + 2C_{12} = C_{22} + 2C_{12} \qquad (7.70)$$

➔ Characteristic impedance for the odd mode

$$Z_{0o} = \frac{1}{vC_o} \qquad (7.71)$$

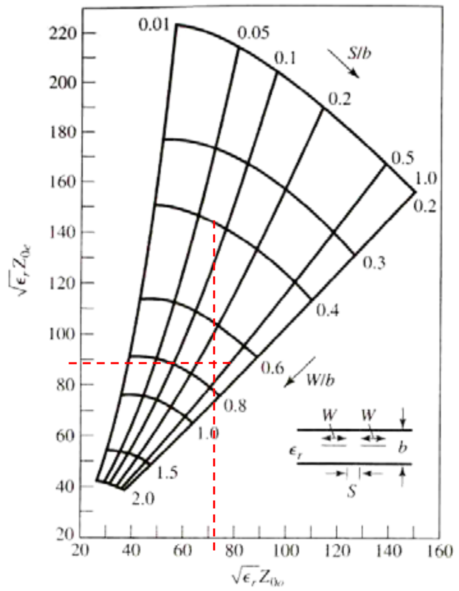


FIGURE 7.29 Normalized even- and odd-mode characteristic impedance design data for edge coupled striplines.

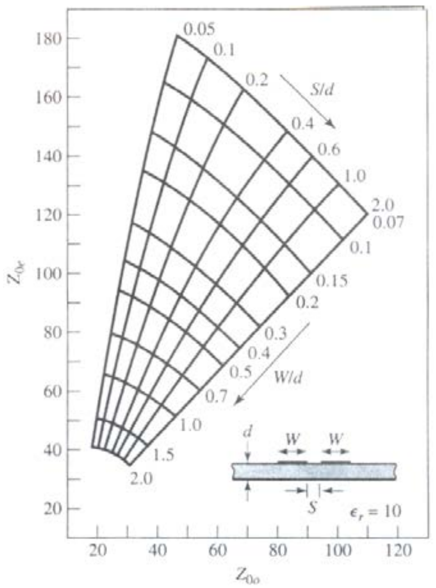


FIGURE 7.30 Even- and odd-mode characteristic impedance design data for for coupled microstrip lines.

EXAMPLE 7.6 Impedance of a Simple Coupled Line

√ For figure 7.26b ($W >> S$, $W << b$), determine the even- and odd-mode characteristic impedances.

Sol.) √ Since the line is symmetric, $C_{11} = C_{22}$.

√ Capacitance of a parallel plate capacitor with plate area, A, and plate separation, d:

$$C = \frac{\varepsilon A}{d},$$

where ε: permittivity of the material between the plates.

√ Capacitance per unit length:

$$\overline{C}_{11} = \frac{\varepsilon_r \varepsilon_o W}{(b-S)/2} + \frac{\varepsilon_r \varepsilon_0 W}{b - \frac{(b-S)}{2}}$$

$$= \frac{\varepsilon_r \varepsilon_o W}{(b-S)/2} + \frac{\varepsilon_r \varepsilon_0 W}{(b+S)/2} = \frac{4b\varepsilon_r \varepsilon_0 W}{b^2 - S^2} \text{[Fd/m]}$$

√ Capacitance between the strips per unit length:

$$\overline{C}_{12} = \frac{\varepsilon_r \varepsilon_o W}{S} \text{[Fd/m]}$$

√ Even- and odd-mode capacitances:

$$\overline{C}_e = \overline{C}_{11} = \frac{4b\varepsilon_r \varepsilon_0 W}{b^2 - S^2} \text{ [Fd/m]}$$

$$\overline{C}_o = \overline{C}_{11} + 2\overline{C}_{12} = 2\varepsilon_r \varepsilon_0 W \left(\frac{2b}{b^2 - S^2} + \frac{1}{S} \right) \quad \text{[Fd/m]}$$

√ Phase velocity on the line:

$$v = 1/\sqrt{\varepsilon_r \varepsilon_0 \mu_0}$$

√ Characteristic impedances:

$$Z_{0e} = \frac{1}{v\overline{C}_e} = \sqrt{\varepsilon_r \varepsilon_0 \mu_0} \frac{b^2 - S^2}{4b\varepsilon_r \varepsilon_0 W} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{b^2 - S^2}{4bW\sqrt{\varepsilon_r}} = Z_0 \frac{b^2 - S^2}{4bW\sqrt{\varepsilon_r}},$$

$$Z_{0o} = \frac{1}{v\overline{C}_o} = \sqrt{\varepsilon_r \varepsilon_0 \mu_0} \frac{1}{2\varepsilon_r \varepsilon_0 W[2b/(b^2 - S^2) + 1/S]}$$

$$= \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{2W\sqrt{\varepsilon_r}[2b/(b^2 - S^2) + 1/S]} = Z_0 \frac{1}{2W\sqrt{\varepsilon_r}[2b/(b^2 - S^2) + 1/S]}$$

• Design of Coupled Line Couplers

√ Four-port network is terminated in the impedance Z_0.

√ Driven with a voltage generator of 2V at port 1

√ A ground conductor is understood to be common to both strip conductors.

√ By superposition, the excitation at port 1 can be treated as the sum of the even- and odd-mode excitations.

√ From symmetry,

$$I_1^e = I_3^e,\ I_4^e = I_2^e,\ V_1^e = V_3^e,\ \text{and}\ V_4^e = V_2^e \quad \text{for even-mode}$$
$$I_1^o = -I_3^o,\ I_4^o = -I_2^o,\ V_1^o = -V_3^o,\ \text{and}\ V_4^o = -V_2^o \quad \text{for odd-mode}$$

√ Input impedance at port 1:

$$Z_{\text{in}} = \frac{V_1}{I_1} = \frac{V_1^e + V_1^o}{I_1^e + I_1^o} \qquad (7.72)$$

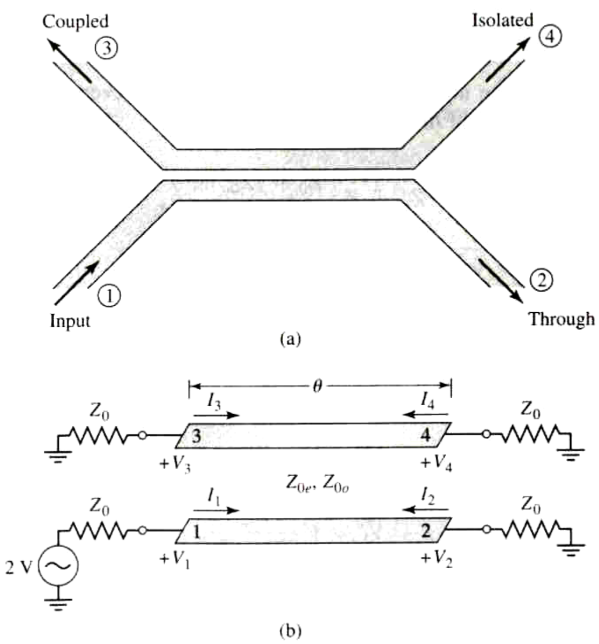


FIGURE 7.31 A single-section coupled line coupler. (a) Geometry and port designations. (b) The schematic circuit.

√ Z^e_{in}: input impedance at port 1 for the even mode,
Z^o_{in}: input impedance at port 1 for the odd mode,

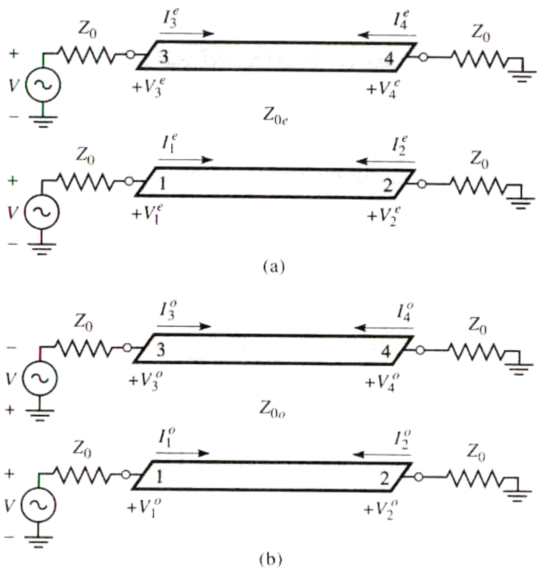


FIGURE 7.32 Decomposition of the coupled line coupler circuit of Figure 7.31 into even- and odd-mode excitations. (a) Even mode. (b) Odd mode.

$$Z_{in}^{e} = Z_{0e} \frac{Z_0 + jZ_{0e} \tan\theta}{Z_{0e} + jZ_0 \tan\theta} \qquad (7.73a)$$

$$Z_{in}^{o} = Z_{0o} \frac{Z_0 + jZ_{0o} \tan\theta}{Z_{0o} + jZ_0 \tan\theta} \qquad (7.73b)$$

√ By voltage division

$$V_1^o = \mathrm{V} \frac{Z_{in}^o}{Z_{in}^o + Z_0} \qquad (7.74a)$$

$$V_1^e = \mathrm{V} \frac{Z_{in}^e}{Z_{in}^e + Z_0} \qquad (7.74b)$$

$$I_1^o = \frac{\mathrm{V}}{Z_{in}^o + Z_0} \qquad (7.74c)$$

$$I_1^e = \frac{\mathrm{V}}{Z_{in}^e + Z_0} \qquad (7.74d)$$

√ Input impedance using Eq. (7.72):

$$Z_{\text{in}} = \frac{V_1^e + V_1^o}{I_1^e + I_1^o} = \frac{\mathrm{V}\dfrac{Z_{\text{in}}^e}{Z_{\text{in}}^e + Z_0} + \mathrm{V}\dfrac{Z_{\text{in}}^o}{Z_{\text{in}}^o + Z_0}}{\dfrac{\mathrm{V}}{Z_{\text{in}}^e + Z_0} + \dfrac{\mathrm{V}}{Z_{\text{in}}^o + Z_0}}$$

$$= \frac{Z_{\text{in}}^o (Z_{\text{in}}^e + Z_0) + Z_{\text{in}}^e (Z_{\text{in}}^o + Z_0)}{Z_{\text{in}}^e + Z_{\text{in}}^o + 2Z_0} = \frac{2Z_{\text{in}}^o Z_{\text{in}}^e + Z_0 (Z_{\text{in}}^e + Z_{\text{in}}^o) + 2Z_0^2 - 2Z_0^2}{Z_{\text{in}}^e + Z_{\text{in}}^o + 2Z_0}$$

$$= Z_0 + \frac{2(Z_{\text{in}}^o Z_{\text{in}}^e - Z_0^2)}{Z_{\text{in}}^e + Z_{\text{in}}^o + 2Z_0} \qquad (7.76)$$

If we let

$$Z_0 = \sqrt{Z_{0e} Z_{0o}} \qquad (7.77),$$

then (7.73*a*, *b*) reduce to

$$Z_{\text{in}}^e = Z_{0e} \frac{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}}\tan\theta}{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}}\tan\theta}$$

$$Z_{\text{in}}^o = Z_{0o} \frac{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}}\tan\theta}{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}}\tan\theta}$$

$$\Rightarrow Z_{\text{in}}^e Z_{\text{in}}^o = Z_{0e} Z_{0o} = Z_0^2$$

$$Z_{\text{in}} = Z_0 \qquad (7.78) \quad \Leftarrow \text{Eq.(7.76)}$$

√ As long as (7.77) is satisfied, port 1 (and, by symmetry, all other ports) will be matched.

$$V_1 = \frac{Z_{\text{in}}}{Z_0 + Z_{\text{in}}} 2\mathrm{V} = \frac{Z_0}{Z_0 + Z_0} 2\mathrm{V} = \mathrm{V}$$

The voltage at port 3:

$$V_3 = V_3^e + V_3^o = V_1^e - V_1^o = \mathrm{V}\left[\frac{Z_{\text{in}}^e}{Z_{\text{in}}^e + Z_0} - \frac{Z_{\text{in}}^o}{Z_{\text{in}}^o + Z_0}\right] \qquad (7.79)$$

√ From (7.73) and (7.77),

$$\frac{Z_{\text{in}}^e}{Z_{\text{in}}^e + Z_0} = \frac{Z_{0e}\dfrac{Z_0 + jZ_{0e}\tan\theta}{Z_{0e} + jZ_0\tan\theta}}{Z_{0e}\dfrac{Z_0 + jZ_{0e}\tan\theta}{Z_{0e} + jZ_0\tan\theta} + Z_0} = \frac{Z_{0e}(Z_0 + jZ_{0e}\tan\theta)}{2Z_{0e}Z_0 + jZ_{0e}^2\tan\theta + jZ_0^2\tan\theta}$$

$$= \frac{Z_{0e}(Z_0 + jZ_{0e}\tan\theta)}{2Z_{0e}Z_0 + jZ_{0e}(Z_{0e} + Z_{0o})\tan\theta} = \frac{Z_0 + jZ_{0e}\tan\theta}{2Z_0 + j(Z_{0e} + Z_{0o})\tan\theta}$$

$$\frac{Z_{\text{in}}^o}{Z_{\text{in}}^o + Z_0} = \frac{Z_{0o}\dfrac{Z_0 + jZ_{0o}\tan\theta}{Z_{0o} + jZ_0\tan\theta}}{Z_{0o}\dfrac{Z_0 + jZ_{0o}\tan\theta}{Z_{0o} + jZ_0\tan\theta} + Z_0} = \frac{Z_0 + jZ_{0o}\tan\theta}{2Z_0 + j(Z_{0e} + Z_{0o})\tan\theta}$$

So that (7.79) reduces to

$$V_3 = V\frac{j(Z_{0e} - Z_{0o})\tan\theta}{2Z_0 + j(Z_{0e} + Z_{0o})\tan\theta} \qquad (7.80)$$

√ C : coupling ratio at the midband voltage (= V_3 / V)

$$C \equiv \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}} \qquad (7.81)$$

Then

$$\sqrt{1 - C^2} = \sqrt{\frac{(Z_{0e} + Z_{0o})^2 - (Z_{0e} - Z_{0o})^2}{(Z_{0e} + Z_{0o})^2}} = \sqrt{\frac{4Z_{0e}Z_{0o}}{(Z_{0e} + Z_{0o})^2}} = \frac{2Z_0}{Z_{0e} + Z_{0o}}$$

$$V_3 = \mathrm{V}\frac{j(Z_{0e} - Z_{0o})\tan\theta}{2Z_0 + j(Z_{0e} + Z_{0o})\tan\theta} = \mathrm{V}\frac{j\dfrac{(Z_{0e} - Z_{0o})}{(Z_{0e} + Z_{0o})}\tan\theta}{2\dfrac{Z_o}{(Z_{0e} + Z_{0o})} + j\tan\theta}$$

$$= \mathrm{V}\frac{jC\tan\theta}{\sqrt{1 - C^2} + j\tan\theta} \qquad (7.82)$$

With another method, $V_1 = V_1^e + V_1^o$, $\quad I_1 = I_1^e + I_1^o$

$V_2 = V_2^e + V_2^o$, $\quad I_2 = I_2^e + I_2^o$

$V_3 = V_1^e - V_1^o$, $\quad I_3 = I_1^e - I_1^o$

$V_4 = V_2^e - V_2^o$, $\quad I_4 = I_2^e - I_2^o$

√ Even excitation + Odd excitation:

$$\begin{bmatrix} V_1^e \\ I_1^e \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_{0e}\sin\theta \\ jY_{0e}\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_2^e \\ I_2^e \end{bmatrix}$$

$$\begin{bmatrix} V_1^o \\ I_1^o \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_{0o}\sin\theta \\ jY_{0o}\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_2^o \\ I_2^o \end{bmatrix}$$

Terminal conditions: $V_2^e = Z_0 I_2^e$, $\quad V_2^o = Z_0 I_2^o$

$V_1^e + Z_0 I_1^e = \mathrm{V}$, $\quad V_1^o + Z_0 I_1^o = \mathrm{V}$

$$\left[\begin{array}{l} V_1^e = \cos\theta \cdot V_2^e + jZ_{0e}\sin\theta \cdot I_2^e \\ I_1^e = jY_{0e}\sin\theta \cdot V_2^e + \cos\theta \cdot I_2^e \end{array}\right.$$

$$\left[\begin{array}{l} V_1^o = \cos\theta \cdot V_2^o + jZ_{0o}\sin\theta \cdot I_2^o \\ I_1^o = jY_{0o}\sin\theta \cdot V_2^o + \cos\theta \cdot I_2^o \end{array}\right.$$

$$\left[\begin{array}{l} V_1^e = \cos\theta \cdot Z_0 I_2^e + jZ_{0e}\sin\theta \cdot I_2^e = (Z_0\cos\theta + jZ_{0e}\sin\theta) I_2^e \\ I_1^e = jY_{0e}\sin\theta \cdot Z_0 I_2^e + \cos\theta \cdot I_2^e = (jZ_0 Y_{0e}\sin\theta + \cos\theta) I_2^e \end{array}\right. \qquad (*)$$

$$\left[\begin{array}{l} V_1^o = \cos\theta \cdot Z_0 I_2^o + jZ_{0o}\sin\theta \cdot I_2^o = (Z_0\cos\theta + jZ_{0o}\sin\theta) I_2^o \\ I_1^o = jY_{0o}\sin\theta \cdot Z_0 I_2^o + \cos\theta \cdot I_2^o = (jZ_0 Y_{0o}\sin\theta + \cos\theta) I_2^o \end{array}\right.$$

$$\rightarrow V_1^e = \frac{Z_0\cos\theta + jZ_{0e}\sin\theta}{\cos\theta + jZ_0 Y_{0e}\sin\theta} I_1^e \quad \text{or} \quad I_1^e = \frac{\cos\theta + jZ_0 Y_{0e}\sin\theta}{Z_0\cos\theta + jZ_{0e}\sin\theta} V_1^e$$

$$V_1^o = \frac{Z_0\cos\theta + jZ_{0o}\sin\theta}{\cos\theta + jZ_0 Y_{0o}\sin\theta} I_1^o \quad \text{or} \quad I_1^o = \frac{\cos\theta + jZ_0 Y_{0o}\sin\theta}{Z_0\cos\theta + jZ_{0o}\sin\theta} V_1^o$$

$$V_1^e + Z_0 I_1^e = V_1^e \left[1 + \frac{Z_0 \cos\theta + jZ_0^2 Y_{0e} \sin\theta}{Z_0 \cos\theta + jZ_{0e} \sin\theta} \right] = \mathrm{V}$$

$$V_1^e = \frac{Z_0 \cos\theta + jZ_{0e} \sin\theta}{2Z_0 \cos\theta + j\sin\theta(Z_0^2 Y_{0e} + Z_{0e})} \mathrm{V} = \frac{Z_0 Z_{0e} \cos\theta + jZ_{0e}^2 \sin\theta}{2Z_0 Z_{0e} \cos\theta + j(Z_0^2 + Z_{0e}^2)\sin\theta} \mathrm{V}$$

$$= \frac{Z_0 Z_{0e} \cos\theta + jZ_{0e}^2 \sin\theta}{D} \mathrm{V}$$

where $D = 2Z_0 Z_{0e} \cos\theta + j(Z_0^2 + Z_{0e}^2)\sin\theta$

$$I_1^e = \frac{\cos\theta + jZ_0 Y_{0e} \sin\theta}{Z_0 \cos\theta + jZ_{0e} \sin\theta} V_{1e} = \frac{\cos\theta + jZ_0 Y_{0e} \sin\theta}{Z_0 \cos\theta + jZ_{0e} \sin\theta} \frac{Z_{0e}(Z_0 \cos\theta + jZ_{0e} \sin\theta)}{2Z_0 Z_{0e} \cos\theta + j(Z_0^2 + Z_{0e}^2)\sin\theta} \mathrm{V}$$

$$= \frac{Z_{0e} \cos\theta + jZ_0 \sin\theta}{2Z_0 Z_{0e} \cos\theta + j(Z_0^2 + Z_{0e}^2)\sin\theta} \mathrm{V} = \frac{Z_{0e} \cos\theta + jZ_0 \sin\theta}{D} \mathrm{V}$$

$$I_2^e = \frac{I_1^e}{jZ_0 Y_{0e} \sin\theta + \cos\theta} = \frac{Z_{0e} \cos\theta + jZ_0 \sin\theta}{(\cos\theta + jZ_0 Y_{0e} \sin\theta) \cdot D} \mathrm{V} = \frac{Z_{0e}}{D} \mathrm{V}$$

$$V_2^e = Z_0 I_2^e = \frac{Z_{0e} Z_0}{D} \mathrm{V}$$

In the same manner for the odd mode,

$$V_1^o = \frac{Z_0 \cos\theta + jZ_{0o} \sin\theta}{2Z_0 \cos\theta + j\sin\theta(Z_0^2 Y_{0o} + Z_{0o})} \mathrm{V} = \frac{Z_0 Z_{0o} \cos\theta + jZ_{0o}^2 \sin\theta}{2Z_0 Z_{0o} \cos\theta + j(Z_0^2 + Z_{0o}^2)\sin\theta} \mathrm{V}$$

$$= \frac{Z_0 Z_{0o} \cos\theta + jZ_{0o}^2 \sin\theta}{D'} \mathrm{V}$$

where $D' = 2Z_0 Z_{0o} \cos\theta + j(Z_0^2 + Z_{0o}^2)\sin\theta$

$$I_1^o = \frac{Z_{0o} \cos\theta + jZ_0 \sin\theta}{D'} \mathrm{V}, \qquad I_2^o = \frac{Z_{0o}}{D'} \mathrm{V}$$

$$V_2^o = \frac{Z_0 Z_{0o}}{D'} \mathrm{V}$$

$$V_1^e = \frac{Z_0 Z_{0e}\cos\theta + jZ_{0e}^2\sin\theta}{2Z_0Z_{0e}\cos\theta + j(Z_0^2 + Z_{0e}^2)\sin\theta}\text{V} = \frac{Z_0 Z_{0e}\cos\theta + jZ_{0e}^2\sin\theta}{2Z_0Z_{0e}\cos\theta + j(Z_{0e}Z_{0o} + Z_{0e}^2)\sin\theta}\text{V}$$

$$= \frac{Z_0\cos\theta + jZ_{0e}\sin\theta}{2Z_0\cos\theta + j(Z_{0e}+Z_{0o})\sin\theta}\text{V} = \frac{Z_0\cos\theta + jZ_{0e}\sin\theta}{D''}\text{V}$$

where $D'' = 2Z_0\cos\theta + j(Z_{0e}+Z_{0o})\sin\theta$

$$V_2^e = \frac{Z_0}{D''}\text{V}$$

$$V_1^o = \frac{Z_0\cos\theta + jZ_{0o}\sin\theta}{D''}\text{V}$$

$$V_2^o = \frac{Z_0}{D''}\text{V}$$

√ As results,

$$V_4 = V_4^e + V_4^o = V_2^e - V_2^o = 0 \qquad (7.83)$$

$$V_2 = V_2^e + V_2^o = \frac{2Z_0}{2Z_0\cos\theta + j(Z_{0e}+Z_{0o})\sin\theta}\text{V}$$

$$= \frac{\dfrac{2Z_0}{Z_{0e}+Z_{0o}}}{\dfrac{2Z_0}{Z_{0e}+Z_{0o}}\cos\theta + j\sin\theta}\text{V} = \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta}\text{V} \qquad (7.84)$$

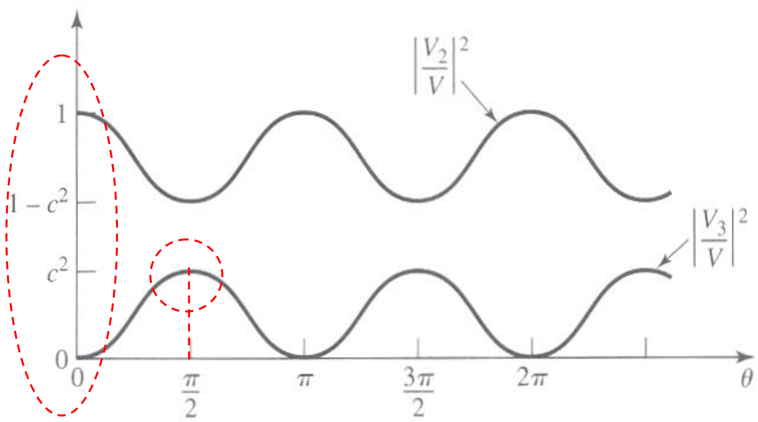


FIGURE7.33 coupled and through port voltages (squared) versus frequency for the coupled line coupler of Figure 7.31.

√ At very low frequencies ($\theta << \pi$), virtually all power is transmitted through port 2, with none being coupled to port 3.

For $\theta = \pi/2$, the coupling to port 3 is at its first maximum
➔ This is where the coupler is generally operated, for small size and minimum line loss.

√ The response is periodic, with maxima in V_3 for $\theta = \pi/2, 3\pi/2, \cdots$

√ $\theta = \pi/2 \Leftrightarrow$ The coupler is $\lambda/4$ long.

$$\frac{V_3}{V} = C \qquad (7.85) \leftarrow (7.82)$$

$$\frac{V_2}{V} = -j\sqrt{1-C^2} \qquad (7.86) \leftarrow (7.84)$$

√ Input Power: $P_{\text{in}} = (1/2)|V|^2 / Z_0$

Output power: $P_2 = (1/2)|V_2|^2 / Z_0 = (1/2)(1-C^2)|V|^2 / Z_0$

$$P_3 = (1/2)|C|^2 |V|^2 / Z_0$$

$$P_4 = 0$$

➔ $P_{\text{in}} = P_2 + P_3 + P_4$: Power conservation

√ 90° phase shift between the two output port voltages
➔ Quadrature hybrid.

√ By Eq.(7.77) and (7.81):

$$Z_{0e} = Z_0\sqrt{\frac{1+C}{1-C}} \qquad (7.87a)$$

$$Z_{0o} = Z_0\sqrt{\frac{1-C}{1+C}} \qquad (7.87b)$$

√ For a coupled microstrip (quasi-TEM operation), or other non-TEM line, this condition will generally not be satisfied, and the coupler will have **poor directivity because of unequal even- and odd-mode phase velocities.**

➔Techniques for compensating coupled microstrip lines to achieve equal even- and odd-mode phase velocities include the use of dielectric overlays and anistropic substrates.

EXAMPLE 7.7 Single-Section Coupler Design and Performance

√ Design a 20 dB single-section coupled line coupler in stripline with a 0.158 cm ground plane spacing (b = 0.158).

√ $\varepsilon_r = 2.56$, $Z_0 = 50\ \Omega$, $f_0 = 3$ GHz.

√ Plot the coupling and directivity from 1 to 5 GHz.

Sol.) √ Voltage coupling factor: $C = 10^{-20/20} = 0.1$

√ Even- and odd-mode characteristic impedances are

$$Z_{0e} = Z_0\sqrt{\frac{1+C}{1-C}} = 50\sqrt{\frac{1.1}{0.9}} = 55.28\ \Omega,$$

$$Z_{0o} = Z_0\sqrt{\frac{1-C}{1+C}} = 50\sqrt{\frac{0.9}{1.1}} = 45.23\ \Omega$$

√ To use Figure 7.29,

$$\sqrt{\varepsilon_r}Z_{0e} = 88.4,$$

$$\sqrt{\varepsilon_r}Z_{0o} = 72.4,$$

So $W/b = 0.72$ ➔ $W = 0.72b = 0.114$ cm

$S/b = 0.34$ ➔ $S = 0.34b = 0.054$ cm

➔These lines are quite close together, which may make fabrication difficult.

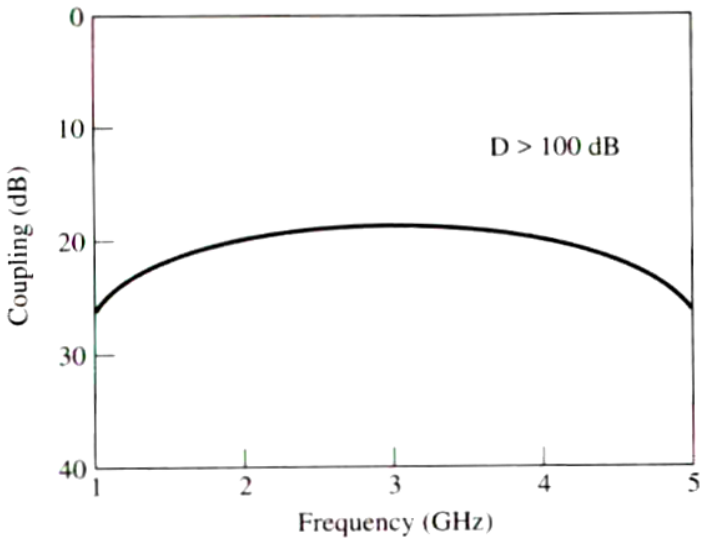


FIGURE 7.34 Coupling versus frequency for the single-section coupler of example 7.7

•Design of Multi-section Coupled Line Couplers

√ Broad bandwidth
√ Generally made with an odd number (N) of sections.
√ Weak coupling ($C \leq 10$ dB)
√ Each section is $\lambda/4$ long ($\theta = \pi/2$) at the center frequency.
√ For a single coupled line section, with $C << 1$,

$$\frac{V_3}{V_1} = \frac{jC\tan\theta}{\sqrt{1-C^2} + j\tan\theta} \approx \frac{jC\tan\theta}{1 + j\tan\theta} = \frac{jC\sin\theta}{\cos\theta + j\sin\theta} = jC\sin\theta e^{-j\theta} \quad (7.88a)$$

$$\frac{V_2}{V_1} = \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta} \approx \frac{1}{\cos\theta + j\sin\theta} = e^{-j\theta} \quad (7.88b)$$

➔For $\theta = \pi/2$, $V_3/V_1 = C$ and $V_2/V_1 = -j$.
➔No power is lost on the through path from one section to the next.
➔It is a good assumption for small C, even though power conservation violation

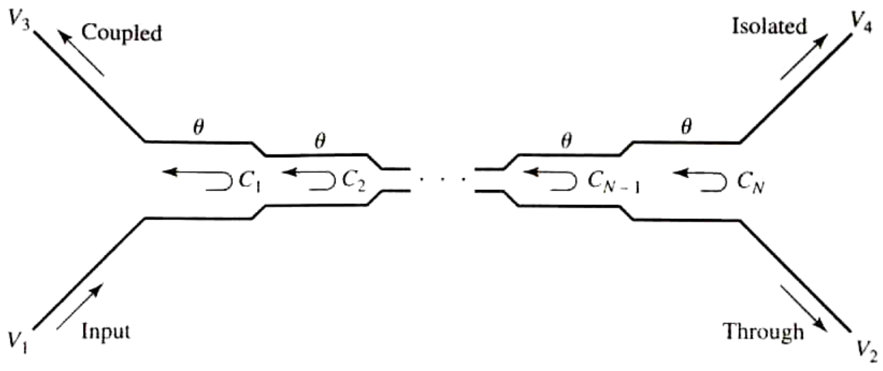


FIGURE 7.35 An N –section coupled line coupler.

√ Total voltage at the coupled port (port 3) of the cascaded coupler:

$$V_3 = (jC_1 \sin\theta e^{-j\theta})V_1 + (jC_2 \sin\theta e^{-j\theta})V_1 e^{-2j\theta} + \cdots + (jC_N \sin\theta e^{-j\theta})V_1 e^{-2j(N-1)\theta} \qquad (7.89)$$

where C_n: voltage coupling coefficient of the n^{th} section

√ If we assume that the coupler is symmetric,

$C_1 = C_N$, $C_2 = C_{N-1}$, ⋯.

Then (7.89) can be simplified to

$$\begin{aligned} V_3 &= jV_1 \sin\theta e^{-j\theta}[C_1(1+e^{-2j(N-1)\theta}) + C_2(e^{-2j\theta} + e^{-2j(N-2)\theta}) \\ &\qquad + \cdots + C_M e^{-j(N-1)\theta}] \\ &= 2jV_1 \sin\theta e^{-jN\theta}[C_1(e^{j(N-1)\theta} + e^{-j(N-1)\theta})/2 + C_2(e^{j(N-3)\theta} + e^{-j(N-3)\theta})/2 \\ &\qquad + \cdots + C_M/2] \\ &= 2jV_1 \sin\theta e^{-jN\theta}[C_1 \cos(N-1)\theta + C_2 \cos(N-3)\theta + \cdots + C_M/2] \qquad (7.90) \end{aligned}$$

where $M = (N+1)/2$.

√ C_0: Voltage coupling factor at the center frequency

$$C_0 = \left|\frac{V_3}{V_1}\right|_{\theta=\pi/2} \qquad (7.91)$$

➔ Obtainable wideband coupling characteristic by choosing the coupling coefficients, C_n.

√ Multisection couplers of this form can achieve decade bandwidths, but coupling levels must be low.

√ Because of the longer electrical length, it is more critical to have equal even- and odd-mode phase velocities than it is for the single-section coupler. ➔ Stripline is the preferred medium for good coupler directivity

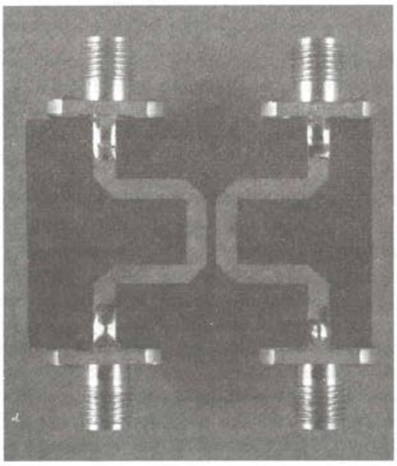


FIGURE 7.36 Photograph of a single-section microstrip coupled line coupler. Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.

EXAMPLE 7.8 Multisection Coupler Design and Performance

- √ Three-section 20 dB coupler with a maximally flat response
- √ System impedance of 50 Ω
- √ Center frequency: 3 GHz.
- √ Plot the coupling and directivity from 1 to 5 GHz.

Sol.) √ For a mamximally flat response for a three-section (N = 3) coupler,

$$\left.\frac{d^n}{d\theta^n}C(\theta)\right|_{\theta=\pi/2} = 0 \quad \text{for} \quad n = 1, 2$$

√ From (7.90),

$$C = \left|\frac{V_3}{V_1}\right| = 2\sin\theta[C_1\cos 2\theta + \frac{1}{2}C_2] = 2C_1\sin\theta\cos 2\theta + C_2\sin\theta$$

$$= C_1(\sin 3\theta - \sin\theta) + C_2\sin\theta$$

$$= C_1\sin 3\theta + (C_2 - C_1)\sin\theta$$

So $$\frac{dC}{d\theta} = \left.[3C_1\cos 3\theta + (C_2 - C_1)\cos\theta]\right|_{\pi/2} = 0,$$

$$\frac{d^2C}{d\theta^2} = \left.[-9C_1\sin 3\theta - (C_2 - C_1)\sin\theta]\right|_{\pi/2} = 10C_1 - C_2 = 0$$

√ At midband, $\theta = \pi/2$ and C_0 = 20 dB.

$$\Rightarrow \begin{aligned} C &= 10^{-20/20} = 0.1 = C_2 - 2C_1 \\ 10C_1 - C_2 &= 0 \end{aligned}$$

➔ $C_1 = C_3 = 0.0125$, $C_2 = 0.125$

√ Even- and odd-mode characteristic impedances for each section:

$$Z_{0e}^1 = Z_{0e}^3 = Z_0\sqrt{\frac{1+C_1}{1-C_1}} = 50\sqrt{\frac{1.0125}{0.9875}} = 50.63\ \Omega,$$

$$Z_{0o}^{1} = Z_{0o}^{3} = Z_0\sqrt{\frac{1-C_1}{1+C_1}} = 50\sqrt{\frac{0.9875}{1.0125}} = 49.38\ \Omega,$$

$$Z_{0e}^{2} = 50\sqrt{\frac{1.125}{0.875}} = 56.69\ \Omega,$$

$$Z_{0o}^{2} = 50\sqrt{\frac{0.875}{1.125}} = 44.10\ \Omega$$

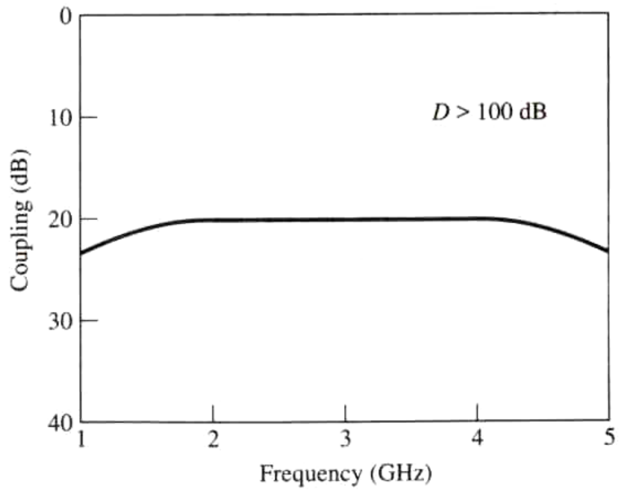


FIGURE 3.37 Coupling versus frequency for the three-section binomial coupler of Example 7.8.

➔ compare Fig. 3.37 with Fig. 7.34

7.7 THE LANGE COUPLER

- Four coupled lines are used with interconnections to provide tight coupling.

- Easily achieve 3 dB coupling ratios, with an octave or more bandwidth.

- 90° phase difference between the output lines (ports 2 and 3)
  ➔ Quadrature coupler

- The practical disadvantages:
  √ Very narrow lines and closing line gap
  √ Difficult to fabricate the necessary bonding wires across the lines.

- The unfolded Lange coupler operates essentially the same as the original Lange coupler, but is ea it.

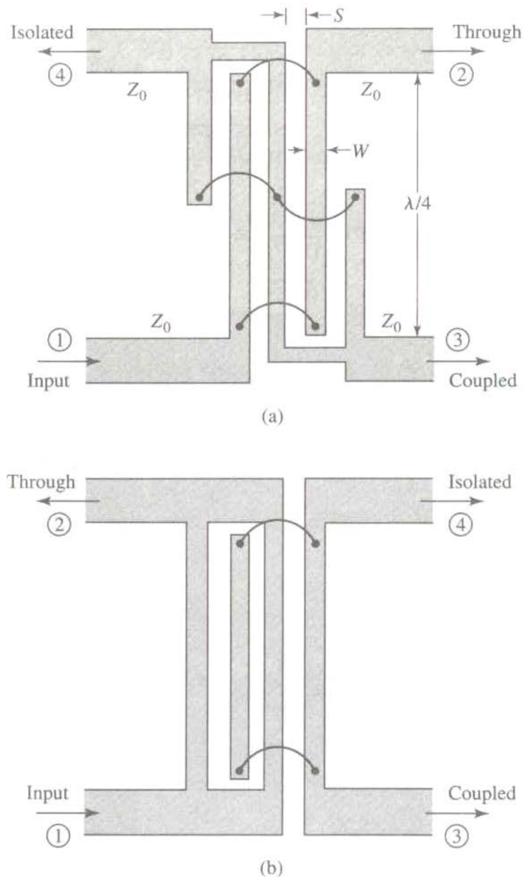


FIGURE 7.38 The Lange coupler.
(a) Layout in microstrip form.
(b) The unfolded Lange coupler.

- The unfolded Lange coupler
  - √ Four-wire coupled line structure
  - √ The same width and spacing
  - √ The reasonable assumption: Each line couples only to its nearest neighbor, and ignore more distant couplings.
  - √ Z_{e4}: The even-mode characteristic impedances of the four-wire circuit

    Z_{o4}: The odd-mode characteristic impedances of the four-wire circuit

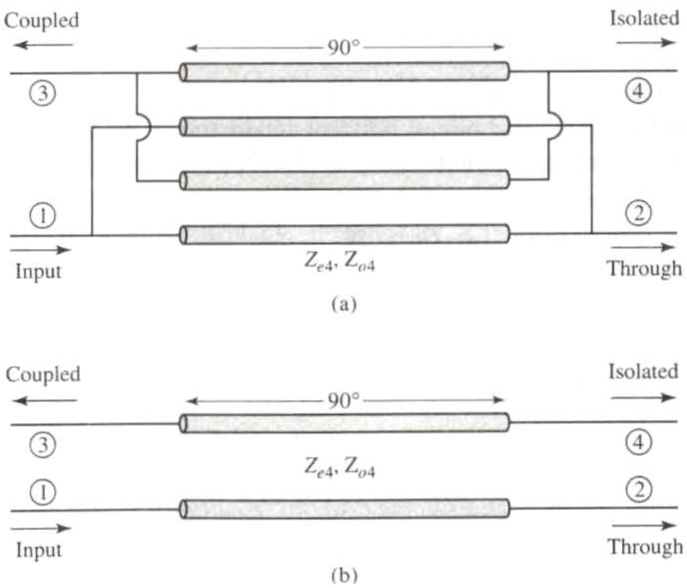


FIGURE 7.39 Equivalent circuits for the unfolded Lange coupler. (a) Four-wire coupled line model. (b) Approximate two-wire coupled line model.

- √ The effective capacitances between the conductors of the four wire coupled line

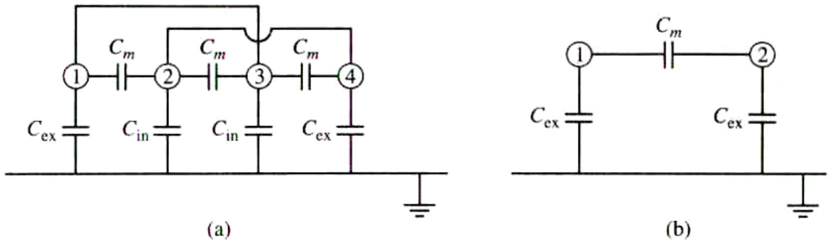


FIGURE 7.40 Effective capacitance networks for the unfolded Lange coupler equivalent circuits of Figure 7.39. (a) Effective capacitance for the four-wire model. (b) Effective capacitance for the two-wire model.

√ The capacitances of the four lines to ground are different depending on whether the line is on the outside (1 and 4), or on the inside (2 and 3)

$$C_{\text{in}} = C_{\text{ex}} - \frac{C_{\text{ex}} C_m}{C_{\text{ex}} + C_m} \qquad (7.92)$$

√ For an even-mode excitation, all four conductors are at the same potential, C_m has no effect.
The total capacitance of any line to ground:

$$C_{e4} = C_{\text{ex}} + C_{\text{in}} \qquad (7.93a)$$

√ For an odd-mode excitation, electric walls effectively exist through the middle of each C_m.
The capacitance of any line to ground:

$$C_{o4} = C_{\text{ex}} + C_{\text{in}} + 6C_m \qquad (7.93b)$$

√ The even- and odd-mode characteristic impedances:

$$Z_{e4} = \frac{1}{vC_{e4}} \qquad (7.94a)$$

$$Z_{o4} = \frac{1}{vC_{o4}} \qquad (7.94b)$$

where v is the velocity of propagation on the line.

√ Consider any isolated pair of adjacent conductors in the four-line model: the effective capacitances are as shown in Figure 7.40b. The even- and odd-mode capacitances are

$$C_e = C_{\text{ex}} \qquad (7.95a)$$

$$C_o = C_{\text{ex}} + 2C_m \qquad (7.95b)$$

➔ $C_{\text{ex}}=(C_e + C_o - 2C_m)/2$
$C_m=(C_o - C_e)/2$

√ Substituting into (7.93) with the aid of (7.92) gives the even-odd mode capacitances of the four-wire line in terms of a two-wire coupled line:

$$C_{e4} = \frac{C_e(3C_e + C_o)}{C_e + C_o} \qquad (7.96a)$$

$$C_{o4} = \frac{C_o(3C_o + C_e)}{C_e + C_o} \qquad (7.96b)$$

√ Since Z_0=1/vC (or C=1/vZ_0), the even/odd mode characteristic impedances of the Lange coupler are

$$Z_{e4} = \frac{Z_{0o} + Z_{0e}}{3Z_{0o} + Z_{0e}} Z_{0e} \qquad (7.97a)$$

$$Z_{o4} = \frac{Z_{0o} + Z_{0e}}{3Z_{0e} + Z_{0o}} Z_{0o} \qquad (7.97b)$$

where Z_{0e}, Z_{0o}: The even- and odd-mode characteristic impedances of the two-conductor pair.

√ Since $Z_0 = \sqrt{Z_{0e}Z_{0o}}$, characteristic impedance:

$$Z_0 = \sqrt{Z_{e4}Z_{o4}} = \sqrt{\frac{Z_{0e}Z_{0o}(Z_{0o} + Z_{0e})^2}{(3Z_{0o} + Z_{0e})(3Z_{0e} + Z_{0o})}} \qquad (7.98)$$

The voltage coupling coefficient:

$$C = \frac{Z_{e4} - Z_{o4}}{Z_{e4} + Z_{o4}} = \frac{3(Z_{0e}^2 - Z_{0o}^2)}{3(Z_{0e}^2 + Z_{0o}^2) + 2Z_{0e}Z_{0o}} \quad (7.99) \;\leftarrow C = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}$$

√ For design purposes, it is useful to invert these results to give the necessary even- and odd-mode impedances for a desirable characteristic impedance and coupling coefficient:

$$Z_{0e} = \frac{4C - 3 + \sqrt{9 - 8C^2}}{2C\sqrt{(1-C)/(1+C)}} Z_0 \qquad (7.100a)$$

$$Z_{0o} = \frac{4C + 3 - \sqrt{9 - 8C^2}}{2C\sqrt{(1+C)/(1-C)}} Z_0 \qquad (7.100b)$$

7.8 THE 180° HYBRID

- Four-port network with 180° and 0° phase shift between the two output ports according to operation

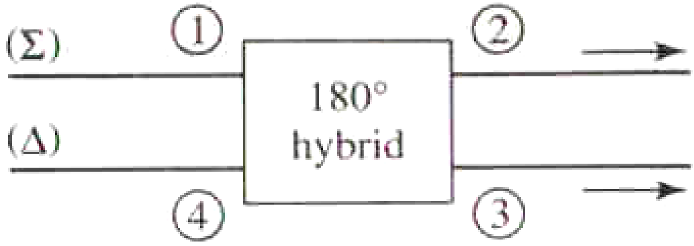


FIGURE 7.41 Symbol for a 180^o hybrid junction

- Input port 1 ➔ 1) Evenly split into two in-phase components at ports 2 and 3
  2) Isolation port 4

  Input port 4 ➔ 1) Evenly split into two out-of-phase components at ports 2 and 3
  2) Isolation port 1

  Input port 2, 3 (as a combiner) ➔ Summation at port 1
  Difference at port 4

- Scattering parameter:

$$[S]=\frac{-j}{\sqrt{2}}\begin{bmatrix}0 & 1 & 1 & 0\\ 1 & 0 & 0 & -1\\ 1 & 0 & 0 & 1\\ 0 & -1 & 1 & 0\end{bmatrix} \quad (7.101)$$

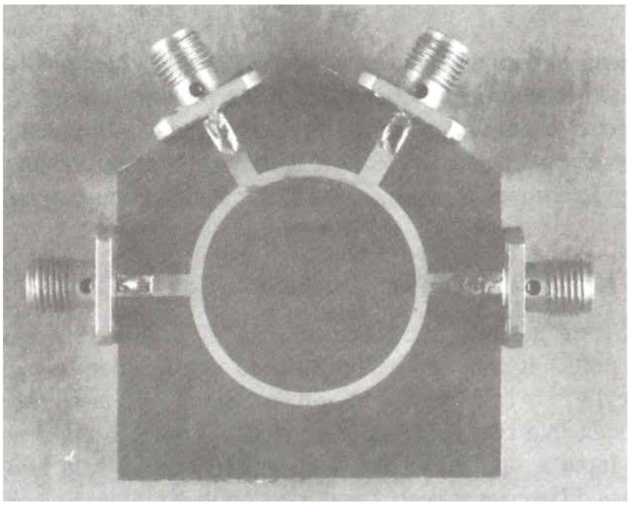


FIGURE 7.42 Photograph of a microstrip ring hybrid. Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory. Lexington. Mass.

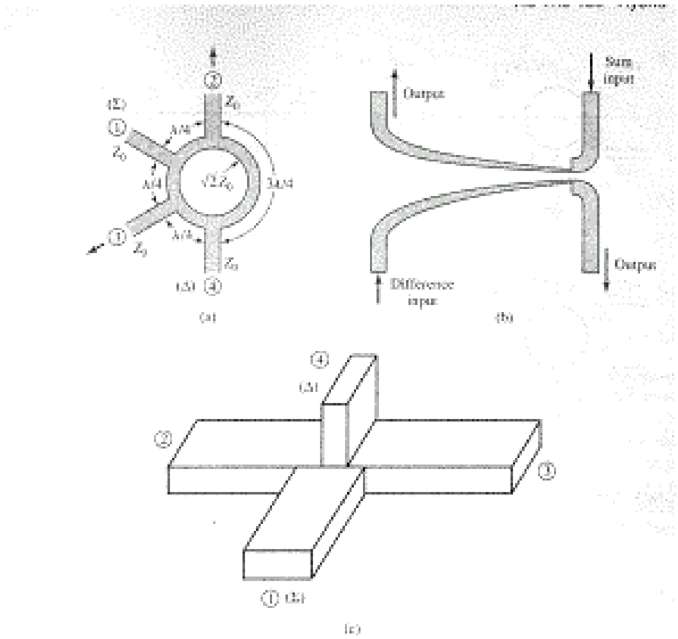


FIGURE 7.43 Hybrid junctions. (a) A ring hybrid, or *rat-race*, in microstrip or stripline form. (b) A tapered coupled line hybrid. (c) A waveguide hybrid junction, or *magic-T*.

●Even-Odd Mode Analysis of the Ring Hybrid

√ Unit amplitude wave incident at port 1 (the sum port) of the ring hybrid of Figure 7.43a.

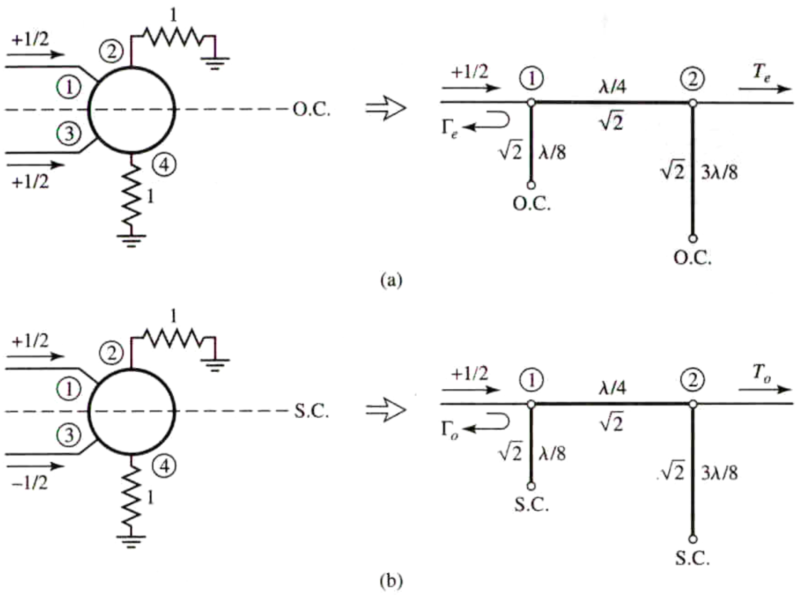


FIGURE 7.44 Even- and odd-mode decomposition of the ring hybrid when port I is excited with a unit amplitude incident wave. (a) Even mode. (b) Odd mode

√ Amplitudes of the scattered waves from the ring hybrid:

$$B_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o \qquad (7.102a)$$

$$B_2 = \frac{1}{2}T_e + \frac{1}{2}T_o \qquad (7.102b)$$

$$B_3 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o \qquad (7.102c)$$

$$B_4 = \frac{1}{2}T_e - \frac{1}{2}T_o \qquad (7.102d)$$

√ *ABCD* matrix for the even- and odd- mode two-port circuits

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & j\sqrt{2} \\ j\sqrt{2} & -1 \end{bmatrix} \quad (7.103a)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{bmatrix} = \begin{bmatrix} -1 & j\sqrt{2} \\ j\sqrt{2} & 1 \end{bmatrix} \quad (7.103b)$$

√ With the aid Table 4.2

$$\Gamma_e = \frac{-j}{\sqrt{2}} \qquad (7.104a) \leftarrow (7.64a)$$

$$T_e = \frac{-j}{\sqrt{2}} \qquad (7.104b) \leftarrow (7.64b)$$

$$\Gamma_o = \frac{j}{\sqrt{2}} \qquad (7.104c) \leftarrow (7.66b)$$

$$T_o = \frac{-j}{\sqrt{2}} \qquad (7.104d) \leftarrow (7.66b)$$

√ Using these results in (7.102),

$$B_1 = 0 \qquad (7.105a)$$

$$B_2 = \frac{-j}{\sqrt{2}} \qquad (7.105b)$$

$$B_3 = \frac{-j}{\sqrt{2}} \qquad (7.105c)$$

➔Input is matched, port 4 is isolated.

$$B_4 = 0 \qquad (7.105d)$$

√ Consider a unit amplitude wave incident at port 4 (the difference port) of the ring hybrid.

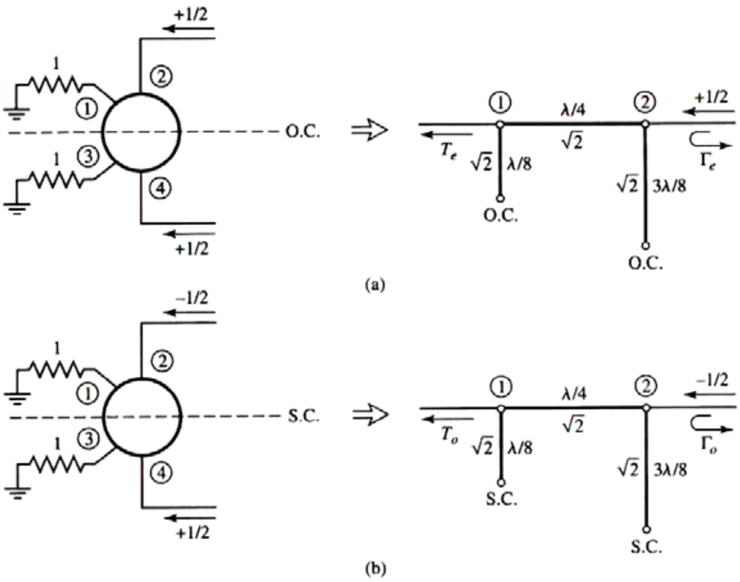


FIGURE 7.45 Even- and odd-mode decomposition of the ring hybrid when port 4 is excited with a unit amplitude incident wave. (a) Even mode. (b) Odd mode.

√ Amplitudes of the scattered waves:

$$B_1 = \frac{1}{2}T_e - \frac{1}{2}T_o \qquad (7.106a)$$

$$B_2 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o \qquad (7.106b)$$

$$B_3 = \frac{1}{2}T_e + \frac{1}{2}T_o \qquad (7.106c)$$

$$B_4 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o \qquad (7.106d)$$

√ *ABCD* matrix for the even- and odd- mode two-port circuits

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{bmatrix}\begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{bmatrix} = \begin{bmatrix} -1 & j\sqrt{2} \\ j\sqrt{2} & 1 \end{bmatrix} \qquad (7.107a)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{bmatrix}\begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & j\sqrt{2} \\ j\sqrt{2} & -1 \end{bmatrix} \qquad (7.107b)$$

√ With the aid Table 4.2

$$\Gamma_e = \frac{j}{\sqrt{2}} \qquad (7.108a)$$

$$T_e = \frac{-j}{\sqrt{2}} \qquad (7.108b)$$

$$\Gamma_o = \frac{-j}{\sqrt{2}} \qquad (7.108c)$$

$$\Gamma_o = \frac{-j}{\sqrt{2}} \qquad (7.108d)$$

√ Using these results in (7.106),

$$B_1 = 0 \qquad (7.109a)$$

$$B_2 = \frac{j}{\sqrt{2}} \qquad (7.109b)$$

$$B_3 = \frac{-j}{\sqrt{2}} \qquad (7.109a)$$

$$B_4 = 0 \qquad (7.109a)$$

➔ The input port is matched, port 1 is isolated, and the input power is evenly divided into ports 2 and 3 with a 180^o phase difference.

√ Bandwidth of the ring hybrid: 20 - 30%. (limited by the frequency dependence of the ring lengths)

EXAMPLE 7.9 Design and check performance of a ring hybrid

√ Design a 180° ring hybrid for a 50 Ω system impedance

√ Plot the magnitude of the S-parameters (S_{ij}) from $0.5f_0$ to $1.5f_0$, where f_0 is the design frequency

Sol.) √ Characteristic impedance of the ring transmission line:

$$\sqrt{2}Z_o = 70.7\Omega$$

√ Feedline impedances: 50Ω

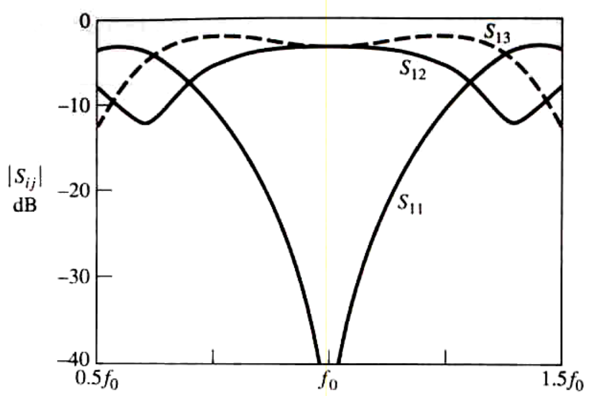


FIGURE 7.46 S parameter magnitudes versus frequency for the ring hybrid of Example 7.9.

Even-Odd Mode Analysis of the Tapered Coupled Line Hybrid

• The tapered coupled line 180° hybrid
➔ Klopfenstein taper

• Any power division ratio with a bandwidth of a decade or more.

• The schematic circuit
√ Two coupled lines with tapering characteristic impedances over the length $0 < z < L$.
√ At z=0, the lines are very weakly coupled so that $Z_{0e}(0)=Z_{0o}(0) = Z_0$
√ At $z=L$, the coupling coefficient is $Z_{0e}(L)=Z_0/k$, $Z_{0o}(L)= kZ_0$, where $0\le k \le 1$: Coupling factor
√ At $L < z < 2L$, the lines are uncoupled, and both have a characteristic impedance Z_0;

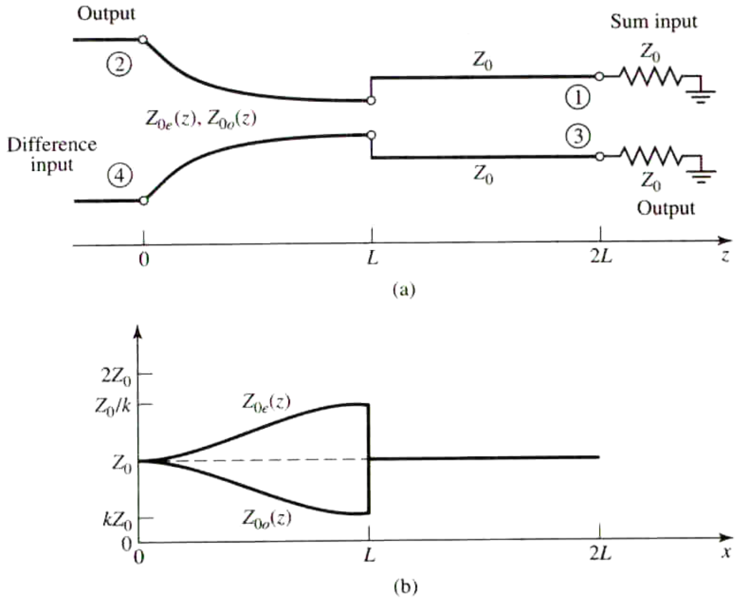


FIGURE 7.47 (a) Schematic diagram of the tapered coupled line hybrid. (b) The variation of characteristic impedances

√ The even mode of the coupled line thus matches a load impedance of Z_0/k (at $z=L$) to Z_0, while the odd mode matches a load of kZ_0 to Z_0.

➔ $Z_{0e}(z)Z_{0o}(z) = Z_0^2$ for all z

√ Consider an incident voltage wave of amplitude V applied to port 4 (the difference input).

➔ The superposition of an even-mode excitation and an odd-mode excitation:

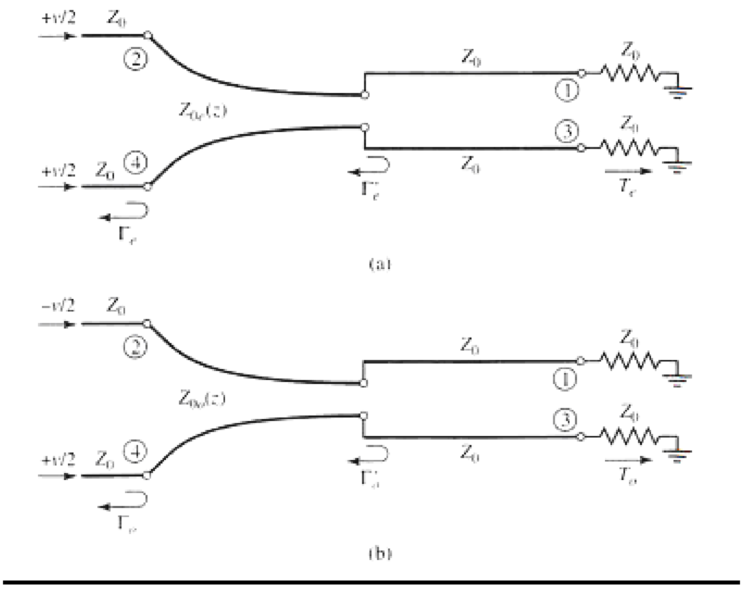


FIGURE 7.48 Excitation of the tapered coupled line hybrid. (a) Even-mode excitation. (b) Odd-mode excitation.

√ At the junctions of the coupled and uncoupled lines (z=L), the reflection coefficients seen by the even or odd modes of the tapered lines are

$$\Gamma'_e = \frac{Z_o - Z_o/k}{Z_o + Z_o/k} = \frac{k-1}{k+1} \qquad (7.110a)$$

$$\Gamma'_o = \frac{Z_o - kZ_o}{Z_o + kZ_o} = \frac{1-k}{1+k} \qquad (7.110b)$$

At z=0,

$$\Gamma_e = \Gamma'_e e^{-2j\theta} = \frac{k-1}{k+1} e^{-2j\theta} \qquad (7.111a)$$

$$\Gamma_o = \Gamma'_o e^{-2j\theta} = \frac{1-k}{1+k} e^{-2j\theta} \qquad (7.111b)$$

√ The scattering parameters of ports 2 and 4 due to superposition:

$$S_{44} = \frac{1}{2}(\Gamma_e + \Gamma_o) = 0 \qquad (7.112a)$$

$$S_{22} = \frac{1}{2}(\Gamma_e - \Gamma_o) = \frac{k-1}{k+1} e^{-2j\theta} \qquad (7.112b)$$

➔By symmetry, S_{22}=0, S_{42}=S_{24}.

√ *ABCD* parameters for the equivalent circuits of the tapered matching sections is equivalent to *ABCD* matrix of the transformer:

$$\begin{bmatrix} \sqrt{k} & 0 \\ 0 & 1/\sqrt{k} \end{bmatrix} \quad : \text{even mode}$$

$$\begin{bmatrix} 1/\sqrt{k} & 0 \\ 0 & \sqrt{k} \end{bmatrix} \quad : \text{odd mode}$$

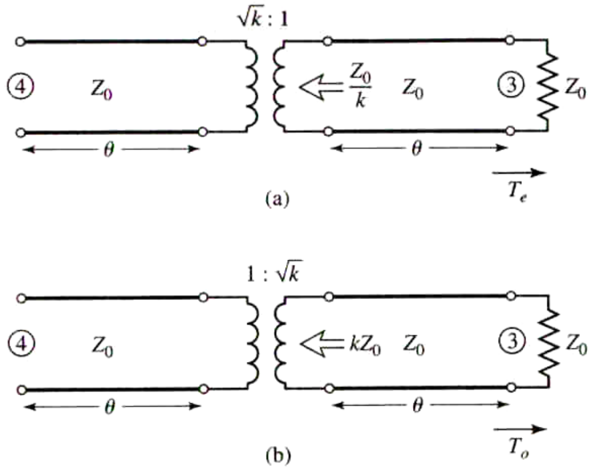


FIGURE 7.49 Equivalent circuits for the tapered coupled line hybrid, for transmission from port 4 to port 3. (a) Even-mode case. (b) Odd-mode case.

√ The even- and odd-mode transmission coefficients:

$$T_e = T_o = \frac{2\sqrt{k}}{k+1} e^{-j\theta} \qquad (7.113)$$

$$\Leftarrow T = 2/(A + B/Z_o + CZ_o + D) = 2\sqrt{k}/(k+1)$$

$e^{-2j\theta}$: The phase delay of the two transmission line sections.

√ *S*- parameters:

$$S_{34} = \frac{1}{2}(T_e + T_o) = \frac{2\sqrt{k}}{k+1} e^{-2j\theta} \qquad (7.114a)$$

$$S_{34} = \frac{1}{2}(T_e - T_o) = 0 \qquad (7.114b)$$

√ The voltage coupling factor from port 4 to port 2:

$$\beta = | S_{34} | = \frac{2\sqrt{k}}{k+1}, \qquad 0 < \beta < 1 \qquad (7.115a)$$

The voltage coupling factor from port 4 to port2:

$$\alpha = | S_{24} | = -\frac{k-1}{k+1} \qquad 0 < \alpha < 1 \qquad (7.115b)$$

√ Power conservation is verified.

$$| S_{24} |^2 + | S_{34} |^2 = \alpha^2 + \beta^2 = 1$$

- If we now apply even- and odd-mode excitations at ports 1 and 3, so that superposition yields an incident voltage wave at port 1, we can derive the remaining scattering parameters.

√ With a phase reference at the input ports, the even- and odd-mode reflection coefficients at port1 will be

$$\Gamma_e = \frac{1-k}{1+k} e^{-2j\theta} \qquad (7.116a)$$

$$\Gamma_o = \frac{k-1}{k+1} e^{-2j\theta} \qquad (7.116b)$$

√ S- parameters:

$$S_{11} = \frac{1}{2}(\Gamma_e + \Gamma_o) = 0 \qquad (7.117a)$$

$$S_{31} = \frac{1}{2}(\Gamma_e - \Gamma_o) = \frac{1-k}{1+k} e^{-2j\theta} = \alpha e^{-2j\theta} \qquad (7.117b)$$

➔ S_{33}=0, S_{13}=S_{31}, S_{14}=S_{32}, S_{12}=S_{34} (Symmetric property)

√ S- parameters of the tapered coupled line 180° hybrid:

$$[S]=\begin{bmatrix} 0 & \beta & \alpha & 0 \\ \beta & 0 & 0 & -\alpha \\ \alpha & 0 & 0 & \beta \\ 0 & -\alpha & \beta & 0 \end{bmatrix} e^{-2j\theta}$$

Waveguide Magic-T

• Scattering matrix is similar in form to (7.101).

$$[S]=\frac{-j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \qquad (7.101)$$

• TE_{10} mode incident

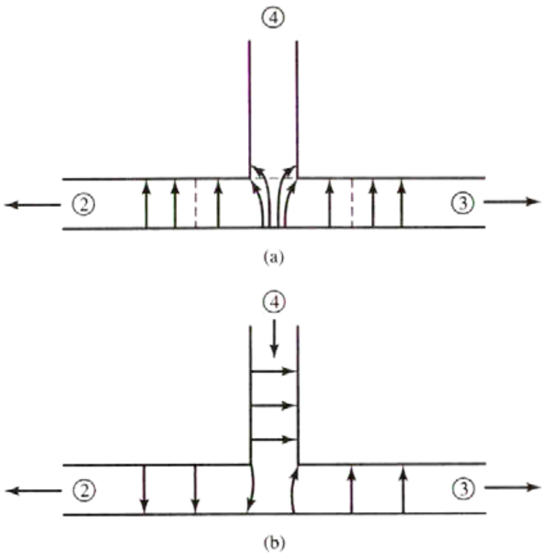


FIGURE 7.51 The Moreno crossed-guide coupler.