

# Arbitrary Power Division Ratio Rat-Race Coupler With Negative Group Delay Characteristics

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**Abstract**—In this letter, we present theoretical and experimental investigations of a rat-race coupler with negative group delay (NGD) characteristics and an arbitrary power division ratio. From the theoretical analysis, the NGD characteristics can be obtained through various transmission paths and have small variations with the power division ratio. The power division ratio is controlled by only the characteristic impedance of the transmission lines. Ideal port isolation and return loss characteristics are obtained at a center frequency for any arbitrary power division ratio. For experimental demonstration, a microstrip line rat-race coupler is implemented with center frequency of 2.14 GHz. The measurement results agree well with simulation results and theoretically predicated values.

**Index Terms**—Arbitrary power division, negative group delay, rat-race coupler, transmission line.

## I. INTRODUCTION

IN microwave circuits and systems, couplers are widely adopted such as in feeding networks of the phased-antenna arrays and, as power splitter/combiner in predistortion power amplifiers [1]. The phased-antenna arrays suffer from a beam-squinting problem, which leads to an unwanted change in direction and shape of the radiation pattern with a frequency. In [2], it is shown that inter-element group delay of a feed network should have an abnormal group delay, called negative group delay (NGD), for beam-squint free operation of the series-fed antenna arrays. Therefore, many researchers have applied NGD circuits to overcome the beam-squinting problem of series-fed antenna arrays [3]–[5]. However, these previous works suffer from high insertion loss as well as separate design procedures of the NGD circuit and feed network.

Modern RF wireless communication systems require highly linear high-power power amplifiers because of the complex modulation techniques that are needed to handle the higher data rate transmissions. A predistortion is one of the cost effective linearization technique and has the advantages of low-power consumption and simple circuit configuration [6], [7]. With this technique, it is crucial to match group delays (GDs), magnitudes, and phases of different paths of the predistortion circuit for a linearity enhancement. For this purpose, a delay element, attenuator and phase shifter are used in such systems. Therefore, if research that can demonstrate a coupler with the

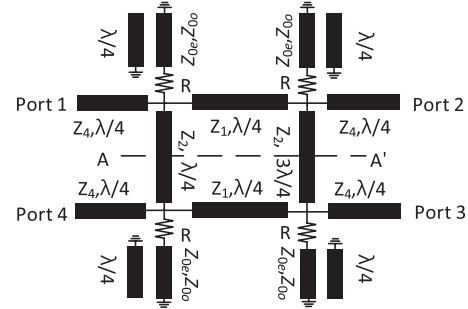


Fig. 1. Proposed structure of arbitrary power division ratio rat-race coupler with negative group delay.

required NGD; would be promising for predistortion amplifier positive group delay compensation that can eliminate the delay element and attenuator. In [8], a power divider with a NGD is presented, but it is limited to an equal power division ratio ( $K = 1$ ). In this letter, the design and implementation of an NGD coupler with an arbitrary  $K$  is presented.

## II. MATHEMATICAL ANALYSIS

Fig. 1 shows the structure of the proposed rat-race coupler with NGD characteristics. It consists of shunt resistor  $R$  connected to  $\lambda/4$  short-circuited coupled lines for generating NGD [9]. The transmission lines  $Z_4$  at each port are used for matching purposes. Since the proposed structure is symmetrical about the AA'-axis, even- and odd-mode analysis can be applied to find the magnitude of scattered waves as well as GDs [1]. For zero reflection from all input ports ( $S_{ii} = 0$ ) at the center frequency ( $f_0$ ), the following relations can be found as:

$$Z_4 = \left\{ \frac{Z_1^2 Z_0^2}{1 + Z_1^2/R^2 + Z_1^2/Z_2^2} \right\}^{\frac{1}{4}} \quad (1)$$

where  $Z_1$  and  $Z_0$  are the characteristic impedances of the series line and reference port impedance, respectively.

Furthermore, the  $S$ -parameters of different transmission paths at  $f_0$  are found with

$$S_{21}|_{f=f_0} = S_{43}|_{f=f_0} = j \frac{R^2}{a_1} \quad (2a)$$

$$S_{41}|_{f=f_0} = -S_{23}|_{f=f_0} = j \frac{R^2 Z_1}{a_1 Z_2} \quad (2b)$$

$$S_{31}|_{f=f_0} = S_{42}|_{f=f_0} = 0 \quad (2c)$$

where

$$a_1 = 2RZ_1 + R^2 \left( \frac{Z_4^2}{Z_0 Z_1} + \frac{Z_1 Z_0}{Z_4^2} \right) + Z_1 Z_4^2 / Z_0 (1 + R^2 / Z_2^2). \quad (3)$$

As seen from (2c), perfect port isolation is naturally satisfied, independent of design variables. Subsequently, the power division ratio  $K$  of the coupler evaluated at  $f_0$  is expressed as

$$\left| \frac{S_{41}}{S_{21}} \right|_{f=f_0} = \left| \frac{S_{23}}{S_{43}} \right|_{f=f_0} = \frac{Z_1}{Z_2} = K \quad (4)$$

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As seen from (4),  $K$  at  $f_0$  is simply determined by  $Z_1$  and  $Z_2$ , independently of  $Z_{0e}$ ,  $Z_{0o}$ , and  $R$ . Furthermore, GDs of different transmission paths evaluated at the  $f_0$  are formulated as

$$\tau_{21}|_{f=f_0} = \tau_{43}|_{f=f_0} = -\frac{1}{4f_0} \times \left( \frac{2Z_c}{RC_{\text{eff}}} - \frac{b_1 + b_2/Z_0 + b_3Z_0}{a_1} \right) \quad (5a)$$

$$\begin{aligned} \tau_{41}|_{f=f_0} = \tau_{23}|_{f=f_0} = \tau_{21}|_{f=f_0} - \frac{Z_4^2}{4f_0Z_0Z_2} \\ = \tau_{43}|_{f=f_0} - \frac{Z_4^2}{4f_0Z_0Z_2} \end{aligned} \quad (5b)$$

where

$$b_1 = 2(1 + Z_4/Z_1 + Z_1/Z_4)R^2 + 2Z_4(Z_1 + KR^2/Z_2) + 4KR^2 + 2Z_1Z_c/C_{\text{eff}} \quad (6a)$$

$$b_2 = 2R \{ KZ_4^2(2 + Z_c/Z_2C_{\text{eff}}) + Z_4(Z_4 + Z_1) + Z_4^2Z_c/Z_1C_{\text{eff}} \} \quad (6b)$$

$$b_3 = 2RZ_1/Z_4(1 + Z_c/Z_4C_{\text{eff}}) \quad (6c)$$

and  $C_{\text{eff}}$  and  $Z_c$  are respectively, the coupling coefficient and equivalent characteristic impedance of the short-circuited coupled lines. The latter is expressed by

$$Z_c = \frac{2Z_{0e}}{Z_{0e}/Z_{0o} - 1} = Z_{0e} \frac{1 - C_{\text{eff}}}{C_{\text{eff}}} = Z_{0o} \frac{1 + C_{\text{eff}}}{C_{\text{eff}}} \quad (7)$$

where  $Z_{0e}$  and  $Z_{0o}$  are the even- and odd-mode impedances of the short-circuited coupled lines [1]. As seen from (7), very high characteristic impedance  $Z_c$  can be obtained if the ratio of  $Z_{0e}$  to  $Z_{0o}$  is close to unity or  $C_{\text{eff}}$  becomes very small.

For a design graph, the calculated  $K$  and GDs/magnitude of  $S_{21}$  and  $S_{31}$  according  $Z_c$ ,  $C_{\text{eff}}$ , and  $R$  are shown in Fig. 2. The 20 to 140  $\Omega$  transmission line can be realized with microstrip practically if a substrate is thicker and dielectric constant of substrate is lower [1]. Therefore, the practical realizable  $K$  lies on range of 0.14 to 7 as shown in Fig. 2(a). Similarly, the GDs are moved toward higher negative values as  $Z_c$  increases and  $R$  decreases. However, the insertion losses are increased as  $R$  decreases. Similarly, the values of  $Z_c$  and  $C_{\text{eff}}$  will effect on the NGD bandwidth (bandwidth with the condition  $\text{NGD} < 0$ ) when the GDs at  $f_0$  are fixed.

To understand the effect of  $Z_c$  and  $C_{\text{eff}}$  on the NGD bandwidth, synthesized results are shown in Fig. 3 by fixing maximum achievable GDs at around  $-1$  ns. As seen from figures, bandwidths of NGDs are decreased as  $Z_c$  increases. However, magnitudes of transmission paths have improved. So there is a trade-off between the NGD bandwidth and magnitudes of  $S$ -parameters.

The circuit parameters of the proposed structure can be found by parametric analysis. Therefore, the design method of the proposed rat-race coupler with NGD characteristics is summarized as follows.

- Specify  $f_0$ , the maximum achievable GD,  $S_{21}$ , and  $S_{41}$ .
- Determine  $R$  for the specified  $S_{21}$  and  $S_{31}$  using (1), (2a) and (2b) by assuming values of  $Z_1$  and  $Z_2$ .
- Calculate the maximum achievable GDs using (5a) and (5b) by providing values of  $Z_c$  and  $C_{\text{eff}}$ .
- Compare the calculated GDs with the required values. If the calculated value is not almost similar to the required value, then repeat step (b) and (c).

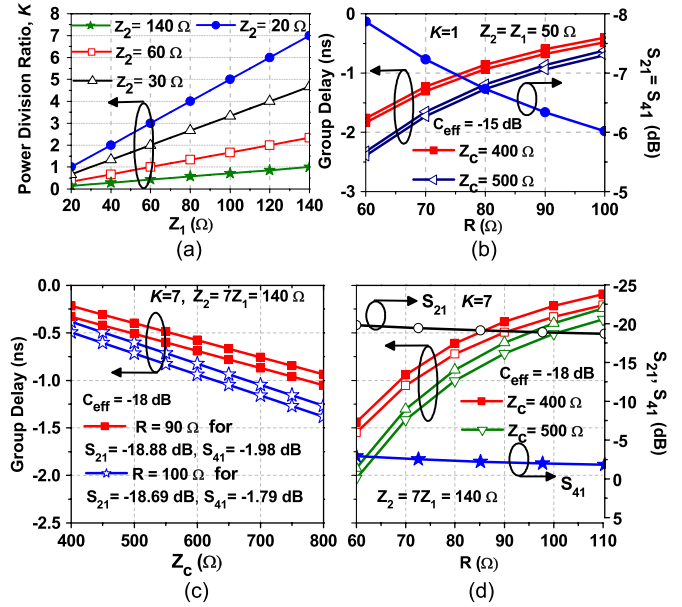


Fig. 2. Calculated power division ratio and magnitudes/group delays at center frequency  $f_0 = 2.14$  GHz according to  $Z_c$ ,  $R$ , and  $K$ : (a) power division ratio  $K$  according to  $Z_1$  and  $Z_2$ , (b) GDs and magnitudes of  $S_{21}$  and  $S_{31}$  according  $R$  for fixed  $Z_c$ ,  $C_{\text{eff}}$ , and  $K = 1$ , (c) GD according to  $Z_c$  with fixed  $R$ ,  $S_{21}$ ,  $S_{31}$ ,  $C_{\text{eff}}$ , and  $K = 7$ , and (d) GDs and magnitudes of  $S_{21}$  and  $S_{31}$  according to  $R$  for fixed  $Z_c$ ,  $C_{\text{eff}}$ , and  $K = 7$ .

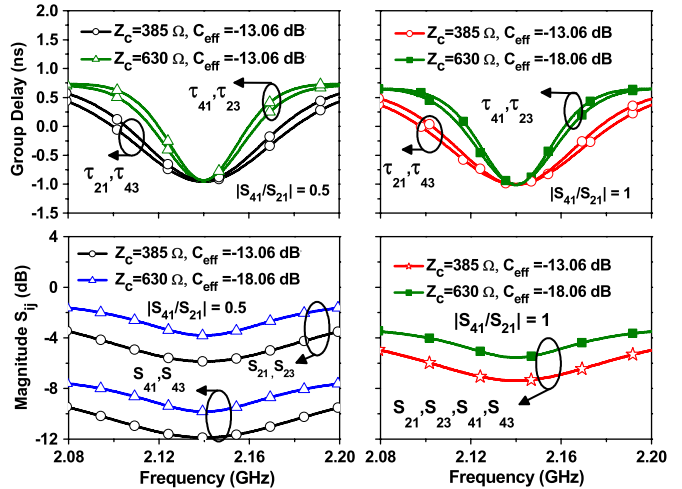


Fig. 3. Synthesized magnitude/group delays of the proposed rat-race coupler for different values of  $Z_c$  and  $K$ .

- Finally, calculate  $Z_{0e}$  and  $Z_{0o}$  of the coupled lines using (7), obtain the width and length of each transmission line according to the substrate information and optimize the physical parameters using a 3-D electromagnetic (EM) simulator.

### III. SIMULATION AND MEASUREMENT RESULTS

For experimental demonstration, the rat-race coupler with  $S_{21} = S_{41} = -6.02$  dB ( $K = 1$ ) was designed and fabricated for an  $f_0$  of 2.14 GHz. The goal was to achieve GDs around  $-0.9$  ns at  $f_0$ . Therefore, the calculated circuit parameters of the rat-race coupler are given as  $Z_1 = 50$   $\Omega$ ,  $Z_2 = 50$   $\Omega$ ,  $Z_c = 600$   $\Omega$ ,  $C_{\text{eff}} = -15$  dB,  $Z_{0e} = 129.77$   $\Omega$ ,  $Z_{0o} = 90.59$   $\Omega$ ,  $Z_4 = 40.82$   $\Omega$ , and  $R = 100$   $\Omega$ . The circuit was fabricated on RT/Duroid 5880 substrate with a dielectric constant ( $\epsilon_r$ ) of 2.2

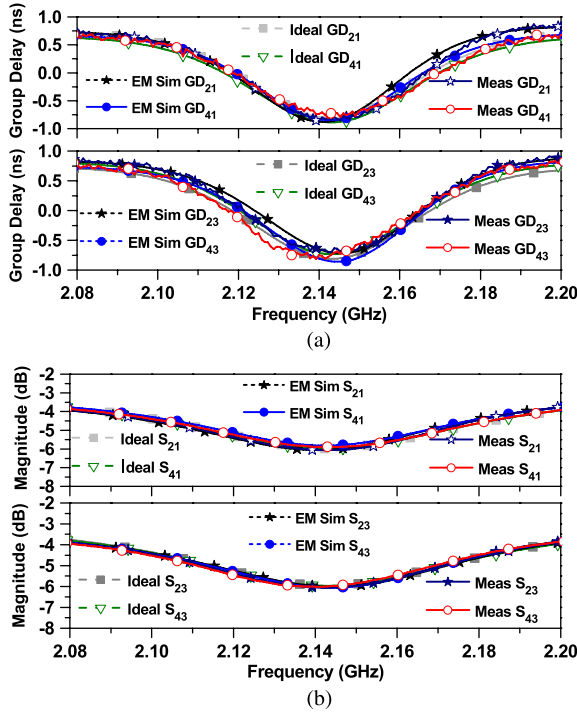


Fig. 4. Simulated and measured results of the proposed rat-race coupler: (a) group delays and (b) magnitude characteristics.

and thickness ( $h$ ) of 0.787 mm. The simulation was performed using ANSYS HFSS 2015.

Fig. 4 shows the simulation and measurement results of magnitude and GD characteristics between different transmission paths. From the measurements, GDs are determined as  $\tau_{21} = -0.86$  ns,  $\tau_{41} = -0.80$  ns,  $\tau_{43} = -0.77$  ns, and  $\tau_{23} = -0.71$  ns at  $f_0 = 2.143$  GHz, respectively. Similarly, achieved magnitudes of  $S$ -parameters are  $S_{21} = -6.05$  dB,  $S_{41} = -5.98$  dB,  $S_{43} = -5.98$  dB, and  $S_{23} = -6.04$  dB. The bandwidth (BW) of the transmission coefficient, which is defined as the 3-dB variation from  $S_{21}$  at  $f_0$ , is 130 MHz. Therefore, NGD-BW products for different transmission paths are determined as 0.12, 0.11, 0.10, and 0.09, respectively. The BW of the NGD rat-race coupler can be enlarged by parallel connection of short-circuited coupled lines with slightly different center frequencies [9].

Fig. 5 show the simulated and measured return loss, isolation, and phase characteristics. From the measurements, return losses are achieved higher than 18.11 dB at  $f_0 = 2.143$  GHz and higher than 15 dB over the bandwidth of 120 MHz. Similarly, isolation characteristics are determined as 42.98 dB at the  $f_0$  and higher than 40 dB in the frequency range of 2.08 – 2.20 GHz. In-phase and out of phase characteristics are determined as  $-1.2 \pm 2^\circ$  and  $182 \pm 2.5^\circ$  over the bandwidth of 120 MHz.

#### IV. CONCLUSION

In this letter, we demonstrated the design and implementation of a rat-race coupler with an arbitrary power division ratio and negative group delay characteristics. Both theoretical analysis and experimental results were provided to validate the proposed structure. The theoretical analysis demonstrated that group delays between transmissions paths are controlled by the resistor value. Moreover, the power division ratio is fully determined by

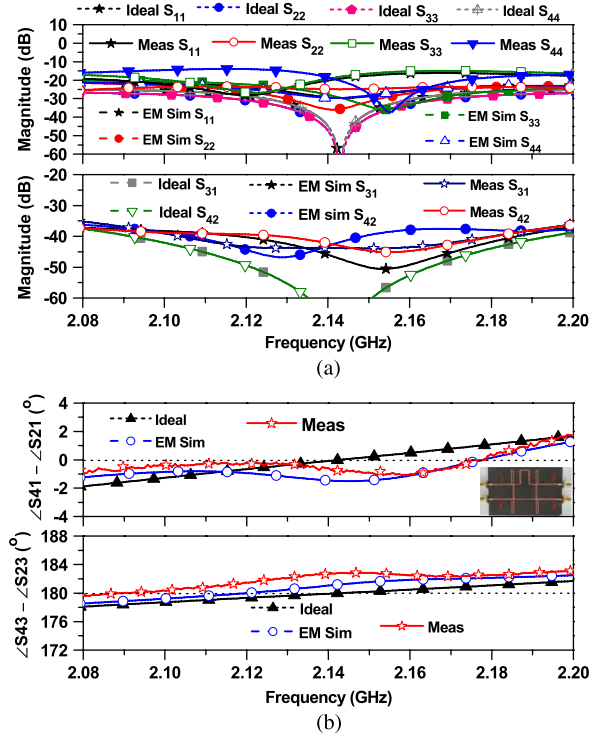


Fig. 5. Simulated and measured return loss, isolation, and phase characteristics of the proposed rat-race coupler: (a) return loss/isolation and (b) phase characteristics.

the ratio of the characteristic impedances of the transmission lines. For experimental verification, the rat-race coupler was designed and fabricated at a center frequency of 2.14 GHz. The measurement results agree well with the simulations. The proposed circuit can be employed in feed networks of series-fed antenna arrays for minimizing beam-squint problem and in predistortion amplifier for group delay matching between different transmission paths.

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