Hai *et al. EURASIP Journal on Wireless Communications and Networking* (2017) 2017:103
DOI 10.1186/s13638-017-0891-6

EURASIP Journal on Wireless Communications and Networking

RESEARCH Open Access

Low complexity precoding codebook design based on DFRST for limited feedback precoded OSTBC systems

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Abstract

Precoding techniques can be used in multiple input multiple output (MIMO) system to assign resources such as power and information optimally. This paper is concerned with limited feedback in precoded orthogonal space time block code (OSTBC) systems, which is a precoding technique for incomplete channel information response. The conventional approach to design the precoding codebook is based on the discrete Fourier transform (DFT) matrix, which has high computational complexity. In this paper, a new approach to design the precoding codebook for the limited feedback precoded OSTBC systems is proposed, which is based on the discrete fractional sine transform (DFRST) matrix. The proposed DFRST-based precoding codebook design approach has lower computational complexity comparing to that of the DFT-based design with very similar bit error rate (BER) performance.

Keywords: MIMO, OSTBC, Precoding, Codebook, DFRST

1 Introduction

Multiple input multiple output (MIMO) systems are widely used in many practical wireless communications system [1–4]. MIMO systems can increase channel capacity gain (multiplexing gain) by using multiple transmit antennas and receive antennas. Some techniques have been developed to improve the performance at transmitter and the receiver by using appropriate signal processing techniques, e.g., the V-BLAST [5] scheme.

MIMO systems with precoding for orthogonal STBC (OSTBC) by using limited feedback was proposed in [6] and then investigated in [7–10]. This precoding technique for the MIMO system can improve the system performance significantly by using the channel state information (CSI) [11–14] at the transmitter. This system model with limited feedback studied by Love and Heath in [7] has been adopted in many communication systems, such as the Long Term Evolution (LTE) [1–4] and spatial modulation [15, 16]. The CSI at the transmitter should be updated with the channel varying in a time varying channel. However, the feedback channel capacity is always limited in the practical system; therefore, it is impossible to use full CSI to select precoder. Hence, the index of the precoder can be obtained based on the CSI. Then, the corresponding index can be fed back to the transmitter instead of full CSI.

The conventional precoding codebook design approach proposed by Love and Heath is based on discrete Fourier transform (DFT) matrix, which is called unitary precoding. The idea of unitary precoding is as follows: a precoding codebook $\mathcal{P} = \left\{\mathbf{P}_1, \mathbf{P}_2, \ldots, \mathbf{P}_Q\right\}$ is first designed based on the DFT matrix. The optimal precoder $\mathbf{P}_i$ can be chosen from the codebook $\mathcal{P}$. Then, the corresponding precoding index i can be determined based on the optimal precoder $\mathbf{P}_i$ and feed the index i back to the transmitter. Obviously, the quality of precoder improved with the increase of the codebook size Q. And $L = \lceil \log_2 Q \rceil$ feedback bits determine the codebook size Q. The conventional approach to design the codebook is based on the DFT matrix [17]. However, this design approach is of high computational complexity.

In this paper, we propose a new approach to design the precoding codebook. Differing from the conventional approach, this new design approach is based on the discrete fractional sine transform (DFRST) matrix [17]. We

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show that the DFRST-based codebook design approach has lower computational complexity than that of the conventional approach without bit error rate (BER) performance loss.

Notations: Bold lowercase and capital letters are used for vectors and matrices, respectively. $(\cdot)^H$ denotes Hermitian transposition. A complex Gaussian distribution with mean u and variance σ^2 is denoted by $\mathcal{CN}(u,\sigma^2)$. $\lfloor\cdot\rfloor$ is used for flooring operation. A hat over the variable name is used to signify the detected signals; $E\{\cdot\}$ denotes the expectation. $\mathcal{U}(M_t,M)$ denotes the set of $M_t \times M$ matrices with orthonormal columns. $\|\cdot\|_F$ is used to represent the Frobenius norm of a vector, and $\lambda_i\{\mathbf{A}\}$ denotes the i-th largest singular value of the matrix $\mathbf{A}$.

2 Preliminary

In this section, we introduce some background concepts that will be used throughout this paper.

Consider a limited feedback precoded OSTBC system with M_t transmit antennas and M_r receive antennas, shown in Fig. 1, over a Rayleigh fading channel [18, 19]. Assume that $\mathcal{P} = \left\{\mathbf{P}_1, \mathbf{P}_2, \ldots, \mathbf{P}_Q\right\}$ is constructed before the transmission at both the transmitter and the receiver, and there is an L-bit error-free feedback channel from the receiver to the transmitter. First, an optimal precoding matrix $\mathbf{P}_i$ in the codebook $\mathcal{P}$ can be selected based according to the following formula [7]

$$\mathbf{P}_i = \arg\min_{\mathbf{P}_i\in\mathcal{P}} \|\mathbf{HP}_i\|_F^2. \tag{1}$$

Then, the index i of $\mathbf{P}_i$ is conveyed to the transmitter from the receiver by using the L bits feedback channel. Then, the M data symbols are mapped into M_t transmit antennas via the precoding matrix $\mathbf{P}_i$ at the transmitter. Let $\mathbf{X} =[x_1,\cdots,x_T]$ be an $M \times T$ space-time codeword whose t-th column $\mathbf{X}_t$ specifies the OSTBC encoder output at the t-th time slot. The received signal can be formulated as

$$\mathbf{Y} = \sqrt{\frac{\rho}{M}}\mathbf{HP}_i\mathbf{X} + \mathbf{Z}, \tag{2}$$

where ρ is the signal-to-noise ratio (SNR), $\mathbf{H}$ is a $M_r \times M_t$ channel matrix with independent entries distributed according to $\mathcal{CN}(0,1)$, and $\mathbf{Z}$ is an $M_r \times T$ noise matrix with independent entries distributed according to $\mathcal{CN}(0,1)$. Finally, the receiver performs the maximum likelihood (ML) decoding on the received signals by

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}} \left\|\mathbf{Y} - \sqrt{\frac{\rho}{M}}\mathbf{HP}_i\mathbf{X}\right\|_F^2. \tag{3}$$

The discrete fractional sine transform matrix was proposed in [17], which is given by

$$\mathbf{S}_N=\sqrt{\frac{2}{N+1}}\begin{pmatrix} \sin\frac{\pi}{N+1} & \sin\frac{2\pi}{N+1} & \cdots & \sin\frac{(N-1)\pi}{N+1} & \sin\frac{N\pi}{N+1} \\ \sin\frac{2\pi}{N+1} & \sin\frac{4\pi}{N+1} & \cdots & \sin\frac{2(N-1)\pi}{N+1} & \sin\frac{2N\pi}{N+1} \\ \vdots & & \cdots & & \vdots \\ \sin\frac{(N-1)\pi}{N+1} & \sin\frac{2(N-1)\pi}{N+1} & \cdots & \sin\frac{(N-1)^2\pi}{N+1} & \sin\frac{N(N-1)\pi}{N+1} \\ \sin\frac{N\pi}{N+1} & \sin\frac{2N\pi}{N+1} & \cdots & \sin\frac{N(N-1)\pi}{N+1} & \sin\frac{N^2\pi}{N+1} \end{pmatrix}, \tag{4}$$

where N is the size of DFRST kernel. This DFRST matrix will be used in the proposed design approach which has lower computational complexity compared to the conventional approach.

3 Codebook design approach based on DFRST

In this section, we propose a new approach to design the precoding codebook $\mathcal{P}$ based on the DFRST matrix.

3.1 Problem formulation

Consider the total effective power $\|\mathbf{HP}_i\|_F^2$ in (2), where $\mathbf{P}_i \in \mathcal{P}$. The loss in received channel power is expressed as $\min_{\mathbf{P}\in\mathcal{P}}\left(\|\mathbf{HP}_{opt}\|_F^2 - \|\mathbf{HP}_i\|_F^2\right)$, where $\mathbf{P}_{opt}$ is the optimal precoding matrix. As a measure of distortion, the average of the loss in received channel power is given as $E\left\{\min_{\mathbf{P}\in\mathcal{P}}\left(\|\mathbf{HP}_{opt}\|_F^2 - \|\mathbf{HP}_i\|_F^2\right)\right\}$.

Since $\|\mathbf{HP}_{opt}\|_F^2 \geq \|\mathbf{HP}_i\|_F^2$ for all $\mathbf{P} \in \mathcal{U}(M_t,M)$, the loss in received channel power is always non-negative and upper-bounded by [7]

$$\begin{aligned} &\min_{\mathbf{P}\in\mathcal{P}}\left(\|\mathbf{HP}_{opt}\|_F^2 - \left\|\mathbf{HP}_i\right\|_F^2\right) \\ &\leq \lambda_1^2\{\mathbf{H}\}E\left\{\min_{\mathbf{P}\in\mathcal{P}}\frac{1}{2}\left\|\mathbf{P}_{opt}\mathbf{P}_{opt}^* - \mathbf{P}_i\mathbf{P}_i^*\right\|_F^2\right\}, \end{aligned} \tag{5}$$

where $\lambda_1^2\{\mathbf{H}\}$ denotes the the largest singular value of $\mathbf{H}$.

Since $\lambda_1^2\{\mathbf{H}\}$ is determined by the channel gain, the loss in received channel power can be minimized by doing

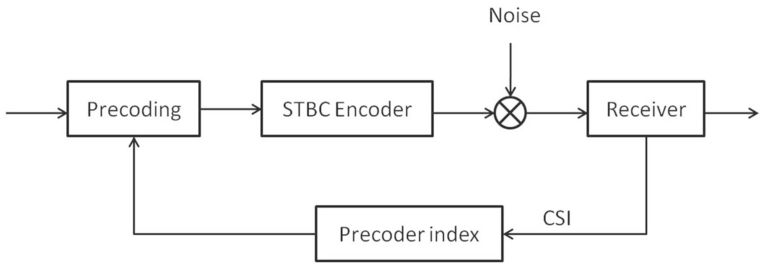

Fig. 1 Block diagram of system model

the minimization $E\left\{\min_{\mathbf{P}\in\mathcal{P}} \frac{1}{2}\left\|\mathbf{P}_{opt}\mathbf{P}_{opt}^{*}-\mathbf{P}_{i}\mathbf{P}_{i}^{*}\right\|_{F}^{2}\right\}$. This minimization problem equivalents to maximize the minimum subspace distance between any pair $\{\mathbf{P}_k, \mathbf{P}_t\}$ of codebook column spaces [8, 20, 21]. In another word, the codebook $\mathcal{P} = \{\mathbf{P}_1, \mathbf{P}_2, \ldots, \mathbf{P}_Q\}$ is designed to maximize

$$\gamma = \min_{1\leq k\neq t\leq Q} \frac{1}{\sqrt{2}}\left\|\mathbf{P}_k\mathbf{P}_k^{*} - \mathbf{P}_t\mathbf{P}_t^{*}\right\|_F. \tag{6}$$

3.2 Proposed codebook design approach

In this subsection, we propose a new codebook design approach as follows.

Firstly, construct the precoding matrix $\mathbf{P}_R$. The $M_t \times M$ matrix $\mathbf{P}_R$ is formed by any M columns of the $M_t \times M_t$ DFRST matrix as follows,

$$P_R=\sqrt{\frac{2}{M_t+1}}\begin{pmatrix} \sin\frac{\pi}{M_t+1} & \cdots & \sin\frac{t\pi}{M_t+1} & \cdots & \sin\frac{M\pi}{M_t+1} \\ \sin\frac{2\pi}{M_t+1} & \cdots & \sin\frac{2t\pi}{M_t+1} & \cdots & \sin\frac{2M\pi}{M_t+1} \\ \vdots & & & \cdots & \vdots \\ \sin\frac{(M_t-1)\pi}{M_t+1} & \cdots & \sin\frac{t(M_t-1)\pi}{M_t+1} & \cdots & \sin\frac{(M_t-1)M\pi}{M_t+1} \\ \sin\frac{M_t\pi}{M_t+1} & \cdots & \sin\frac{tM_t\pi}{M_t+1} & \cdots & \sin\frac{M_tM\pi}{M_t+1} \end{pmatrix}, \tag{7}$$

where $1 \leq t \leq M$.

Secondly, determine the integers $\mathbf{u}$ $=[u_1, u_2, \ldots, u_{M_t-1}]$ Following the literature [7, 17], let

$$\Theta = \begin{pmatrix} 1 & & & & & \\ & e^{-j2u_1} & & & & \\ & & \ddots & & & \\ & & & e^{-j2(i-1)u_{i-1}} & & \\ & & & & \ddots & \\ & & & & & e^{-j2(M_t-1)u_{M_t-1}} \end{pmatrix}. \tag{8}$$

and

$$\mathbf{P}_i = \Theta^{i-1}\mathbf{P}_R, \tag{9}$$

where $i = 1, 2, \ldots, Q$. Then, we can choose $u_1, u_2, \ldots, u_{M_t-1}$ from the set $\mathcal{Z} = \{\mathbf{u} \in \mathbb{Z}^{M_t-1} | \forall k, 0 \leq u_k \leq Q-1\}$ by

$$\mathbf{u} = \arg\max_{\mathcal{Z}} \min_{2\leq i\leq Q} \frac{1}{\sqrt{2}}\|\mathbf{P}_R\mathbf{P}_R^{*} - \mathbf{P}_i\mathbf{P}_i^{*}\|_F. \tag{10}$$

where $\mathcal{Z} = \{\mathbf{u} \in \mathbb{Z}^{M_t-1} | \forall k, 0 \leq u_k \leq Q-1\}$ denotes the set that contains all the possible combinations of $\mathbf{u} = \left[u_1, u_2, \ldots, u_{M_t-1}\right]$, for any $k \in [1, M_t-1]$, $u_k \in [0, Q-1]$.

The precoding codebook $\mathcal{P}$ can be stored at the transmitter/receiver using $\lceil (M_t-1)\log_2 Q\rceil$ bits as only $u_1, u_2, \ldots, u_{M_t-1}$ must be stored. Then, the chosen precoding matrix $\mathbf{P}_i$ at the transmitter, corresponding to the feedback index i, can be easily calculated by computing $\Theta^{i-1}\mathbf{P}_R, i = 2, 3, \ldots, Q$. The proposed codebook design approach can be described by Algorithm 1. Note that the proposed approach is to design the codebook for the precoding system, and this procedure can be completed offline. Therefore, the proposed approach can be applied into the different scenarios, including different transmit antennas, receive antennas, and modulation scheme. Hence, the proposed approach can be applied for vast number of data transmission.

Algorithm 1 Proposed codebook design

```
Initialization: Set DFRST matrix $\mathbf{S}_N$,and the size of precoding matrix $M_t, M$.
Construct the precoding codeword matrix $\mathbf{P}_R$:
for $k = 1$ to $M_t$ do
    for $l = 1$ to $M$ do
        $\mathbf{P}_R(k,l) = \sqrt{\frac{2}{M_t+1}}\sin\frac{kl\pi}{M_t+1}$
    end for
end for
Determine the integers $u_1, \ldots, u_{M_t-1}$:
$\gamma_{\max} = 0$
for $u_1 = 0$ to $Q-1$ do
    for $u_2 = 0$ to $Q-1$ do
        ⋮
            for $u_{M_t-1} = 0$ to $Q-1$ do
                $\Theta = \text{zeros}(M_t, M_t)$
                $\Theta(1,1) = e^{-j0}$
                for $i = 2$ to $Q-1$ do
                    $\Theta(i,i) = e^{-j2(i-1)u_{i-1}}$
                end for
                $\gamma_{\min} = \frac{1}{\sqrt{2}}\|\mathbf{P}_R\mathbf{P}_R^{*} - \Theta\mathbf{P}_R(\Theta\mathbf{P}_R)^{*}\|_F$
                for $i = 2$ to $Q-1$ do
                    $\gamma = \frac{1}{\sqrt{2}}\|\mathbf{P}_R\mathbf{P}_R^{*} - \Theta^{i}\mathbf{P}_R(\Theta^{i}\mathbf{P}_R)^{*}\|_F$
                    if $\gamma < \gamma_{\min}$ then $\gamma_{\min} = \gamma$
                    end if
                end for
                if $\gamma_{\min} > \gamma_{\max}$ then
                    $\mathbf{u} = [u_1, \ldots, u_{M_t-1}]$
                    $\gamma_{\max} = \gamma_{\min}$
                end if
            end for
        ⋮
    end for
end for
Construct the codebook:
$\mathcal{P} = \{\mathbf{P}_1, \mathbf{P}_2, \ldots, \mathbf{P}_i, \ldots, \mathbf{P}_Q\}$,where $\mathbf{P}_i = \Theta^{i-1}\mathbf{P}_R, i = 1, 2, \ldots, Q$.
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4 Performance and complexity analysis

In this section, we apply our DFRST-based precoding codebook to the limited feedback precoded OSTBC systems. The simulation results show that the performance of the OSTBC system with the DFRST precoders is very similar to that with the conventional one. However, from the complexity analysis, we can see that the proposed precoding codebook design approach has lower computational complexity. The OSTBC system with the DFRST precoders have been simulated in the following examples. For comparison, the OSTBC with the DFT precoders has also been simulated [7]. The channel models in all the schemes are Rayleigh fading channels.

Example 1 *The first example compares the OSTBC with the DFRST precoders and DFT precoders on a* 2 × 1 *MIMO system. Both of them use Alamouti codes* [22] *and QPSK. The BER performance for the* 2 × 1 *Alamouti code is also shown for comparison. The simulation results are shown in Fig.* 2*. In all simulations,* 6 *bits feedback and the ML detection are used. It can be seen that the BER performance of DFRST-based precoding codebooks is very similar to that of the DFT-based precoding codebooks.*

Example 2 *Figure* 3 *considers a* 2 × 2 *OSTBC system using the precoded Alamouti code and QPSK modulation. Three cases are considered: the OSTBC with the DFRST precoders, the OSTBC with the DFT precoders, and the OSTBC system without precoding. The feedback rate is* 6 *bits and the simulation results are shown in Fig.* 2*. In all simulations, the ML detection is used. It can be seen that the BER performance of DFRST precoding codebooks is very similar to that of the DFT codebooks.*

Let us analyze the computational complexity of the proposed precoding codebook design approach. In order to construct the rotation matrix Θ, we must determine the integers $\mathbf{u} = [u_1, \ldots, u_{M_t-1}]$ from (10). Based on the equation , we compute (10) for every possible values of $u_1, \ldots, u_{M_t-1}$ and i. For each $u_i \in \mathbf{u}, i = 1, 2, \ldots, M_t - 1$, there are Q different possibilities that must be considered, which means that there are Q^{M_t-1} different possibilities for Θ. Moreover, there are $Q - 1$ different possibilities for i that must be considered. It is possible to do an exhaustive search over all possibilities of $\mathbf{u}$ in set $\mathcal{Z}$ if the transmit antennas number or low feedback bits is small. Therefore, exhaustive search can be employed for designing codebook $\mathcal{P}$. This method tests all the possible values of $\mathbf{u}$ in the set $\mathcal{Z}$ to optimize the cost function (10). Therefore, the computational complexity of DFRST codebook design approach is $Q^{Mt-1}(Q-1)$. However, the conventional precoding codebook design approach must determine the integers $u_1, \ldots, u_{M_t}$, which means that there are Q^{M_t} different possibilities for the rotation matrix Θ in conventional codebook design approach. Therefore, the computational complexity of conventional codebook design approach is $Q^{Mt}(Q-1)$. To measure the computational complexity of proposed approach, we define the computational complexity reduction ration (CCRR), which is given by the formula:

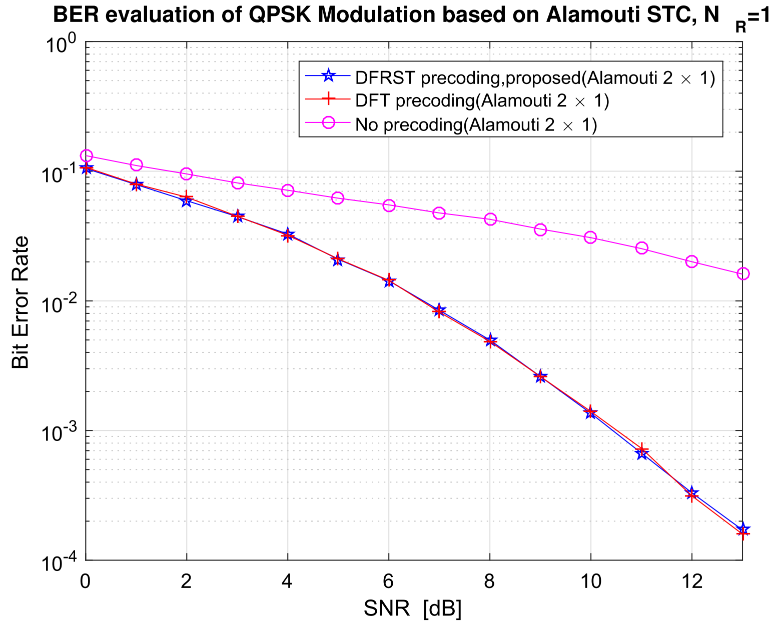

Fig. 2 BER evaluation of QPSK modulation based on Alamouti code (2 × 1) with 6 bits feedback

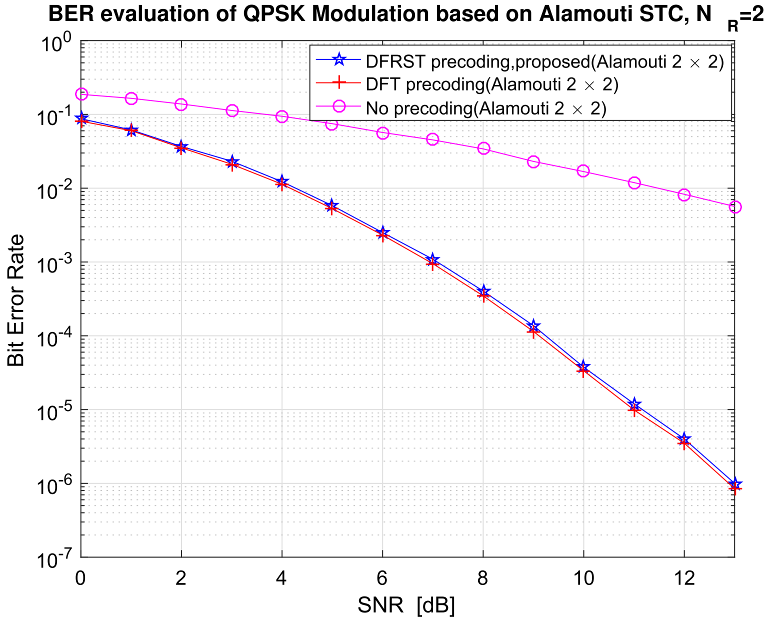

Fig. 3 BER evaluation of QPSK Modulation based on Alamouti code (2 × 2) with 6 bits feedback

$$\beta = \frac{C_c - C_P}{C_C} = 1 - \frac{C_P}{C_C}, \tag{11}$$

where C_P denotes number of computations in proposed approach and C_C denotes number of computations in conventional approach.

Table 1 shows the comparison of the computational payload for the precoding codebook design approach. We compare the computational complexity in 4 × 4, 8 × 8, and 16 × 16 MIMO systems with 4, 6 and 8 bits feedback, respectively. Comparing with the DFT-based precoding codebook design approach, our DFRST-based approach reduce 93.75% of the computational complexity when 4 transmit antennas with 4 bits feedback are used. It can be seen that the CCRR increases as the number of antennas also increased as the number of feedback bits increases.

Table 1 Comparison of the computational payload for codebook design approach

MIMO	Feedback	DFT	DFRST	CCRR
4 × 4	2 bits	768	192	75%
	4 bits	983,040	61,440	93.75%
	6 bits	1.057×10^{9}	1.652×10^{7}	98.44%
6 × 6	2 bits	12,288	3072	75%
	4 bits	2.517×10^{9}	1.573×10^{7}	93.75%
	6 bits	4.329×10^{12}	6.765×10^{10}	98.44%
8 × 8	2 bits	196,608	49,152	75%
	4 bits	6.443×10^{10}	4.027×10^{9}	93.75%
	6 bits	1.773×10^{16}	2.771×10^{14}	98.44%

Note that DFRST is proposed based on the DST in [17]. In this paper, we used the kernel matrix (8) of DFRST to design the codebooks. Based on the special structure of DFRST, the kernel matrix of DFRST is different from the kernel matrix of DFT. More specifically, DFRST has a smaller size of kernel matrix [17]. Therefore, the proposed approach can reduce the computational complexity comparing to the conventional DFT approach.

5 Conclusions

In this paper, we have proposed a new approach to design the precoding codebook for the precoded OSTBC systems with limited feedback. The proposed approach was based on DFRST matrix. Since DFRST matrix has a simpler structure than that of the DFT matrix, this can reduce the computational complexity of codebook design approach from $Q^{Mt}(Q-1)$ to $Q^{Mt-1}(Q-1)$ than that of the DFT matrix with very similar BER performance.

Acknowledgements
This work was supported by the Brain Korea 21 PLUS Project, NRF, Korea; MEST (2015R1A2A1A05000977), NRF, Korea; and National Natural Science Foundation of China (61671143).

Competing interests
The authors declare that they have no competing interests.

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Received: 16 February 2017 Accepted: 22 May 2017

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