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Complex Hadamard Matrix-Aided Generalized Space Shift Keying Modulation

HAN HAI[1], XUE-QIN JIANG[2], (Member, IEEE), WEI DUAN[3], JUN LI[4], (Member, IEEE), MOON HO LEE[1], (Senior Member, IEEE), AND YONGCHAE JEONG[1], (Senior Member, IEEE)

[1]Division of Electronics and Information Engineering, Chonbuk National University, Jeonju 54896, South Korea
[2]School of Information Science and Technology, Donghua University, Shanghai, 201620, China
[3]School of Electronics and Information, Nantong University, Nantong 226019, China
[4]School of Mechanical and Electrical Engineering, Guangzhou University, Guangzhou 510006, China

Corresponding author: Moon Ho Lee (moonho@jbnu.ac.kr)

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ABSTRACT In this paper, we present a complex Hadamard matrix-aided generalized space shift keying (HSSK) modulation scheme, which introduces complex-Hadamard-based signal vectors at the transmitter to provide higher spectrum efficiency than generalized space shift keying (GSSK). It also has lower maximum-likelihood detection complexity than the multiple active spatial modulation (MA-SM) and generalized spatial modulation (GSM) systems due to the corresponding fast complex Hadamard transform at the receiver. Based on the Lee distance of our signal vectors, we provide an optimized mapping between our data bits and signal vectors. We also analyze the upper bound on the average bit error rate of our HSSK system, which agrees well with the simulation results. Finally, the performance of HSSK is compared with MA-SM, GSM, and GSSK systems through Monte Carlo simulations. It is shown that for the same transmission rate the HSSK system performs better than the GSM, MA-SM, and GSSK systems.

INDEX TERMS Generalized space shift keying (GSSK), generalized spatial modulation (GSM), multiple active spatial modulation (MA-SM), complex Hadamard matrix, maximum-likelihood (ML) detection.

I. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) wireless systems are the most promising techniques in wireless communications, since they offer high bandwidth efficiency and good system performance. Due to this, they have attracted huge research interest in recent years leading to exploit the diversity and multiplexing gains, such as Vertical-Bell Lab Layered Space-Time (V-BLAST) and Space Time Block Code (STBC) [1], [2]. Spatial Modulation (SM) is an emerging transmission technique for MIMO wireless systems that exploits spatial domain to convey information [3]. Since SM activates only one antenna each time, it provides benefits such as no inter-channel interference, no inter-antenna synchronization, and reduction of Radio Frequency (RF) chains compared with conventional MIMO systems. Recent works have advanced research in SM, e.g., Differential Spatial Modulation (DSM) [4], [5], Space-Time Block Coded Spatial Modulation (STBC-SM) [6], Orthogonal Frequency Division Multiplexing with Index Modulation (OFDM-IM) [7]–[9]. If readers want to know more on these topics, they can refer to [10].

In [11], a special case of SM is studied with fixed gain of active antennas, which is called Space Shift Keying (SSK). In SSK, only the index of active antennas can be used to transmit information to avoid conventional modulations. SSK can further reduce the detection complexity, while the performance is almost the same as SM. Therefore, it provides trade-off in receiver complexity. The SSK can be easily integrated within communication systems due to the simplicity of its framework. Consequently, a lot of studies have focused on the the performance of SSK modulation [12], [13].

In order to further exploit the spatial domain to transmit more data bits, SSK is generalized to a fixed number of active antennas in Generalized SSK (GSSK) [14], [15]. An alternative approach to exploit high spectral efficiency is the Generalized Spatial Modulation (GSM) [16]. However, all the selected active antennas transmit the same symbol at each time slot in the GSM system. In order to fully exploit the spectral efficiency to transmit data bits, a Multiple Active Spatial Modulation (MA-SM) is proposed to allow multiple transmitter antennas in GSM system to transmit different symbols at the same time [17]. GSM and MA-SM

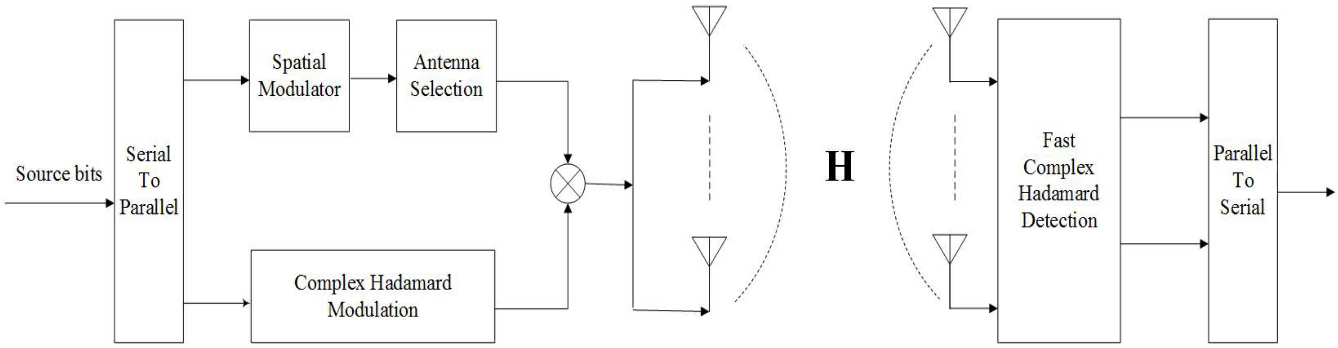

FIGURE 1. System model of proposed HSSK system.

achieve higher spectrum efficiency than the GSSK scheme by combining the amplitude/phase modulation techniques with antenna index modulation [10], [17]–[19]. The detection of GSSK, GSM and MA-SM signals is conventionally performed using the Maximum-Likelihood (ML) criterion. Although the GSM and MA-SM are able to provide higher spectrum efficiency than GSSK, their ML detection algorithm, which jointly searches all active antenna combinations and the modulated symbols, increases exponentially in complexity with the number of transmit antennas and modulation levels [16].

Hadamard matrices have many applications in data compression [20], coding theory [21] and MIMO systems [22], [23]. The popularity of Hadamard matrices is due to their simplicity and efficiency in execution of the corresponding fast Hadamard transforms. A complex Hadamard matrix of order n is an $n \times n$ matrix $\mathbf{U}_n$ whose entries are $\{\pm 1, \pm j\}$, where $j = \sqrt{-1}$, and satisfies the orthogonal requirement [24]–[27]. In this paper, we present a complex Hadamard matrix-aided generalized Space Shift Keying (HSSK) modulation scheme, which introduces complex Hadamard matrix based mapping operations at the transmitter to provide higher spectrum efficiency than GSSK. It also has lower ML detection complexity than MA-SM and GSM with better Bit Error Rate (BER) performance.

The main contribution of this paper is twofold: (a) A novel HSSK scheme is proposed. By applying the fast complex Hadamard transform, the HSSK has low detection complexity like GSSK and high data rate close to that of GSM and MA-SM systems; (b) More importantly, compared to GSM, MA-SM and GSSK, the HSSK has better BER performance.

The remainder of this paper is organized as follows. Section II presents the system model of the HSSK system. Section III introduces the HSSK modulation and detection algorithms. In Section IV, we analyze the complexity and BER upper bound of the HSSK system with the fast Hadamard transform based ML detection. In addition, an optimized mapping between source bit and symbol vector is also proposed. Simulation results are given in Section V. Finally, Section VI concludes the paper.

Notation: Bold lower case letters represent vectors, while bold upper case letters denote matrices. $(\cdot)^T$, $(\cdot)^H$ and $\|\cdot\|_F$ denote transpose, Hermitian, and Frobenius norm operations, respectively. $\mathbb{C}^{m\times n}$ stands for the complex space of $m \times n$ dimensions. $\mathbb{E}[\cdot]$ evaluates the expectation with respect to all random variables within the bracket. $\lfloor\cdot\rfloor$ indicates flooring operator and $\binom{\cdot}{\cdot}$ represents the binomial coefficient. $\mathbf{I}_n$ denotes the identity matrix of order n and Re denotes the imaginary part of a complex number. The determinant of a matrix $\mathbf{A}$ is indicated by $\det(\mathbf{A})$. $\otimes$ denotes kronecker product and $vec(\mathbf{A})$ stacks all columns of $\mathbf{A}$ into a vector.

II. SYSTEM MODEL

As shown in Fig. 1, an HSSK system consists of a MIMO wireless system with N_t transmitter antennas and N_r receiver antennas. In the HSSK system, only n antennas are active in each channel use. Therefore, there are $L = \binom{N_t}{n}$ combinations when choosing n ($n < N_t$) active transmitter antennas out of N_t. For each channel use, $\lfloor \log_2 L \rfloor$ bits b_A are used to select the active antenna combination A, and $(\log_2 n + 2)$ bits b_s are mapped to a symbol vector, respectively. Following [28], the transmission rate is defined as the number of bits per channel use (bpcu). Therefore, the transmission rate of the HSSK system is

$$\gamma = \lfloor \log_2 L \rfloor + \log_2 n + 2. \tag{1}$$

Let $\mathbf{A} = (a_1, a_2, \cdots, a_n)$ denote an active antenna combination, where a_k is the index of the k-th active antenna, and let $s = [s_1, s_2, \ldots, s_n]^T$ denote the symbol vector. In this paper, the signals s are chosen from the columns of the complex Hadamard matrix. Then, the transmission signal vector can be represented by $\mathbf{x} = [0, \ldots, 0, s_{a_1}, 0, \ldots, s_{a_2}, 0, \ldots, s_{a_n}, 0, \ldots, 0]^T$.

Let $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ be the MIMO channel matrix from the transmitter to the receiver, whose entries follow $\mathcal{CN}(0, 1)$. Let $\mathbf{h}_{a_k}$ be the a_k-th column of channel matrix $\mathbf{H}$ and $\mathbf{H}_A = (\mathbf{h}_{a_1}, \mathbf{h}_{a_2}, \cdots, \mathbf{h}_{a_n})$ be the submatrix consisting of n columns of $\mathbf{H}$, corresponding to the combination $\mathbf{A}$. Then, the received signal can be formulated as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \boldsymbol{\eta} = \mathbf{H}_A\mathbf{s} + \boldsymbol{\eta} \tag{2}$$

where $\boldsymbol{\eta} \in \mathbb{C}^{N_r \times 1}$ denotes the noise vector whose elements follow $\mathcal{CN}(0, \sigma^2)$. Assuming perfect channel state information (CSI) at the receiver, the ML detection can be formulated as

$$\left(\hat{\mathbf{A}}, \hat{\mathbf{s}}\right) = \arg \min_{\mathbf{A} \in \mathbb{A},\ \mathbf{s} \in S_n} \|y - \mathbf{H}_A \mathbf{s}\|_F^2, \tag{3}$$

where $\mathbb{A} = \{0, 1, \ldots, 2^{\lfloor \log_2 L \rfloor} - 1\}$ and $\mathbf{S}_n$ is the set of columns of the complex Hadamard matrix.

III. HSSK SYSTEM

A. HSSK MODULATION

The elements of a complex Hadamard matrix $\mathbf{U}_n$ of order n can be represented by [24]:

$$\mathbf{U}_n(u, v) = j^{uv} s(u, v), \tag{4}$$

where $n = 2^m$, $u, v = 0, 1, \ldots, 2^m - 1$ and

$$s(u, v) = \begin{cases} 1, & \text{for } m = 2; \\ (-1)^{\sum_{r=3}^{m} \lfloor u/2^{r-1} \rfloor \lfloor v/2^{r-1} \rfloor}, & \text{for } m = 3, 4, 5, \ldots. \end{cases} \tag{5}$$

is the sign function. Then, columns of

$$\mathbf{S}_n = [\mathbf{U}_n, \ -\mathbf{U}_n, \ j\mathbf{U}_n, \ -j\mathbf{U}_n] \tag{6}$$

will be used as the symbol of our proposed HSSK system.

Example 1: Let $n = 4$, the corresponding complex Hadamard matrix of order 4 is

$$\mathbf{U}_4 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{pmatrix}. \tag{7}$$

The sequential mapping between the incoming data bits and the symbol vectors selected from $\mathbf{S}_4$ are given in Table 1. Assuming $b_s = [\,0\ 1\ 1\ 0\,]$, the corresponding signal vector is $s = [-1, +1, -1, +1\,]^T$. □

TABLE 1. Symbol vectors for $n = 4$ with sequential mapping.

l	Source bits $\mathbf{b}_s$	Symbol vector $\mathbf{s}^T$
0	0000	$[+1, +1, +1, +1]$
1	0001	$[+1, +j, -1, -j]$
2	0010	$[+1, -1, +1, -1]$
3	0011	$[+1, -j, -1, +j]$
4	0100	$[-1, -1, -1, -1]$
5	0101	$[-1, -j, +1, +j]$
6	0110	$[-1, +1, -1, +1]$
7	0111	$[-1, +j, +1, -j]$
8	1000	$[+j, +j, +j, +j]$
9	1001	$[+j, -1, -j, +1]$
10	1010	$[+j, -j, +j, -j]$
11	1011	$[+j, +1, -j, -1]$
12	1100	$[-j, -j, -j, -j]$
13	1101	$[-j, +1, +j, -1]$
14	1110	$[-j, +j, -j, +j]$
15	1111	$[-j, -1, +j, +1]$

In our proposed HSSK system, the transmitter work as follows:

Step 1. Map b_I of $\lfloor \log_2 L \rfloor$ bits to a positive integer l, $0 \le l \le 2^{\lfloor \log_2 L \rfloor} - 1$.

Step 2. Map b_s of $(\log_2 n + 2)$ bits to the decimal presentation k, and select the k-th column vector of $\mathbf{S}_n$ as the symbol vector $s = [s_1, s_2, \ldots, s_n]^T$.

Step 3. Select the l-th active antenna combination $\mathbf{A}$ and construct the transmit signal vector $x = [0, \ldots, 0, s_{a_1}, 0, \ldots, s_{a_2}, 0, \ldots, s_{a_n}, 0, \ldots, 0]^T$.

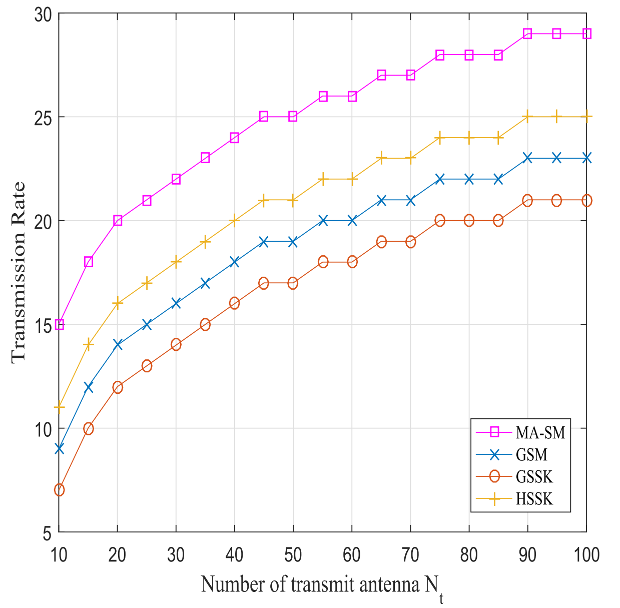

FIGURE 2. Transmission rate comparison of MA-SM, GSSK, HSSK systems for varying N_t, $n = 4$ and $M = 4$.

We compare the transmission rates γ of different systems with increasing N_t and $M = 4$ in Fig. 2. It is clear that HSSK can achieve higher transmission rate than GSSK and GSM.

B. HSSK DETECTION

For the MA-SM, GSM and GSSK systems, the ML detection leads to excessive complexity that grows exponentially with N_t, which limits their applicability. To reduce the detection complexity of our HSSK system, we propose a modified ML detection algorithm. At the receiver, instead of excessively searching through all the column vectors in $\mathbf{S}_n$ directly by the ML detection (2), we can detect s by only searching columns in $\mathbf{U}_n$. The steps of the modified ML detection are as follows.

Step 1. For $0 \le l \le 2^{\lfloor \log_2 L \rfloor} - 1$, select the l-th active antenna combination $\mathbf{A}$ from a mapping table.

Step 2. Compute $\alpha = [\alpha_0, \alpha_1, \ldots, \alpha_{n-1}] = \mathbf{H}_A \mathbf{U}_n$ by the well-known fast complex Hadamard transform [27]

$$\alpha^T = \begin{bmatrix} \mathbf{I}_{n-1} & \mathbf{I}_{n-1} \\ \mathbf{I}_{n-1} & -\mathbf{I}_{n-1} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{n-1} & 0 \\ 0 & \mathbf{G}_{n-1} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{U}_{n-1} & 0 \\ 0 & \mathbf{U}_{n-1} \end{bmatrix} \mathbf{H}_A^T, \tag{8}$$

where $\mathbf{G}_k = \begin{bmatrix} \mathbf{I}_{k-1} & 0 \\ 0 & j\mathbf{I}_{k-1} \end{bmatrix}$ and $\mathbf{I}_k$ denotes the identity matrix of order k.

Step 3. The output of modified ML detection is given by

$$\left(\hat{\mathbf{A}}, \hat{\mathbf{s}}\right) = \arg \min_{t \in T,\ k \in K} \|y - j^t \alpha_k\|_F^2, \tag{9}$$

where $T = \{0, 1, \ldots, 3\}$ and $K = \{0, 1, \ldots, n-1\}$.

Note that, the modified ML detection is equivalent to the ML detection (3), mathematically. However, the modified ML detection has lower complexity since the fast complex Hadamard transform is used.

IV. COMPLEXITY AND THEORETICAL PERFORMANCE

In this section, we analyze the complexity and the theoretical performance of the proposed HSSK modulation.

A. DETECTION COMPLEXITY

In this subsection, the complexity of detection in the MA-SM, GSM, GSSK and our proposed HSSK systems is analyzed and compared. For fair comparison, we denote the complexity as the total number of multiplications normalized by the transmission rate. We assume that the ML detection is applied in the GSSK, GSM and our proposed HSSK system. To estimate the computational complexity of ML detection in these systems, we use the number of multiplications required in the detection process. The number of additions can be shown to have a similar view. However, since the complexity of addition is much lower than that of the multiplication, the number of the additions is omitted in the analysis of detection complexity.

Let M denotes the modulation order used in the MA-SM and GSM systems, C_M denotes the number of multiplications and C_A denotes the number of additions. If we apply the general ML detection to the MA-SM system, the squared Euclidean distance $\|y - Hx\|_F^2$ needs $(n+2)$ multiplications and $N_r(n+1)$ additions, which is calculated $2^{\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor} M^n N_r$ times [16]. Consequently, the ML detection in the MA-SM system applies exhaustive search employing as high as

$$C_M^{MA} = 2^{\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor} M^n N_r (n+2) \tag{10}$$

multiplications and

$$C_A^{MA} = 2^{\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor} M^n N_r^2 (n+1) \tag{11}$$

additions, respectively. Since there is only one symbol transmitted via the activate transmit antennas, the ML detection in GSM system employs

$$C_M^{GSM} = 2^{\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor} M N_r (n+2) \tag{12}$$

multiplications and

$$C_A^{GSM} = 2^{\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor} M N_r^2 (n+1) \tag{13}$$

additions, respectively. In the GSSK system, since there is no symbol vector transmitted via the activate transmitter antennas and therefore the squared Euclidean distance becomes $\| \mathbf{y} - \mathbf{H}_A \|$. Consequently, the ML detection in the GSSK system employs

$$C_M^{GSSK} = 2 \cdot 2^{\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor} N_r \tag{14}$$

multiplications and

$$C_A^{GSSK} = 2 \cdot 2^{\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor} N_r^2 (n+1) \tag{15}$$

additions, respectively.

Now, let us analyze the complexity of the modified ML detection for our HSSK system. First, the $[\alpha_0, \alpha_1, \ldots, \alpha_{n-1}] = \mathbf{H}_A \mathbf{U_n}$ can be computed by the fast complex Hadamard transform (8), which only needs $n \log_2 n$ additions or subtractions and no multiplication [26]. The multiplication $j^t \alpha_k$ needs 4 multiplications. Then, the ML detection (9) needs 6 multiplications and $N_r(n+1)$ additions or subtractions. Thus, in the HSSK system, the ML detection applies

$$C_M^{HSSK} = 6 \cdot 2^{\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor} N_r \tag{16}$$

multiplications and

$$C_A^{HSSK} = n \log_2 n + 2^{\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor} N_r^2 (n+1) \tag{17}$$

additions, respectively.

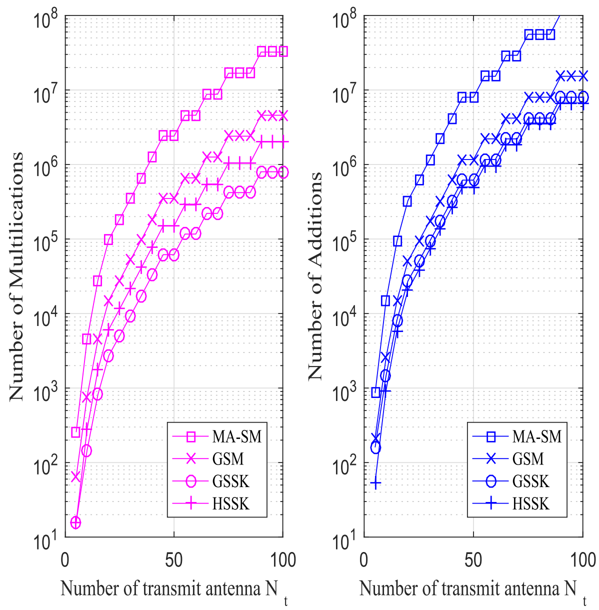

FIGURE 3. Detection complexity comparison for MA-SM, GSSK, HSSK systems for varying N_t , $n = 4$ and $N_r = 8$.

For varying N_t, we show the numbers of multiplications and additions in the ML detection in MA-SM, GSM, GSSK and HSSK in Fig. 3 with the configurations given in Table 2. For fairness, the detection complexities of MA-SM, GSM, GSSK and HSSK systems have to be divided by their transmission rate, respectively. From Fig. 3, we can see that the ML detection complexity of the HSSK system is much lower than that of the MA-SM and GSM systems and close to that of the GSSK system.

TABLE 2. Configurations for MA-SM, GSSK, and HSSK systems.

Modulation Scheme	n	M	N_r	Transmission rate γ	Number of Multiplications C_M	Number of Additions C_A
MA-SM	4	2	8	$\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor + n \log_2 M$	$2^{\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor} M^n N_r(n+2)$	$2^{\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor} M^n N_r^2(n+1)$
GSM	4	2	8	$\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor + \log_2 M$	$2^{\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor} M N_r(n+2)$	$2^{\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor} M N_r^2(n+1)$
GSSK	4	/	8	$\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor$	$2 \cdot 2^{\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor} N_r$	$2 \cdot 2^{\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor} N_r^2(n+1)$
HSSK	4	/	8	$\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor + \log_2 n + 2$	$6 \cdot 2^{\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor} N_r$	$n \log_2 n + 2^{\left\lfloor \log_2 \binom{N_t}{n} \right\rfloor} N_r^2(n+1)$

B. THEORETICAL PERFORMANCE

In this subsection, we first analyze the Lee distance [29] of symbol vectors in the HSSK system and propose an optimized mapping between the data bits and symbol vectors selected from $\mathbf{S}_n$. Then, we also provide an upper bound on the BER of the ML detection for the HSSK system.

Theorem 1: If the complex Hadamard matrix $\mathbf{U}_n$ is used in the HSSK system, then the minimal Lee distance of columns in $\mathbf{S}_n$ is n and the maximum Lee distance is $2n$.

Proof. Since any two different column vectors s_i and s_j of matrix $\mathbf{U}_n$ are conjugate orthogonal, by

$$\begin{aligned} d_L(s_i, s_j) &= \|s_i - s_j\|_F^2 \\ &= \|s_i\|_F + \|s_j\|_F - 2Re(s_i^H s_j) \\ &= 2n - 2Re(s_i^H s_j), \end{aligned} \tag{18}$$

and we have the Lee distance

$$d_L(s_i, s_j) = \begin{cases} 2n, & \text{if } s_i = s_j + \vec{2}; \\ n, & \text{Otherwise}, \end{cases} \tag{19}$$

where $\vec{2}$ denotes all 2 vector of length n. The proof is completed. ■

TABLE 3. Symbol vectors for $n = 4$ with optimized mapping.

l	Source bits $\mathbf{b}_s$	Symbol vector $\mathbf{s}^T$
0	0000	$[+1, +1, +1, +1]$
1	0001	$[+1, +j, -1, -j]$
2	0011	$[+1, -1, +1, -1]$
3	0010	$[+1, -j, -1, +j]$
4	1111	$[-1, -1, -1, -1]$
5	1110	$[-1, -j, +1, +j]$
6	1100	$[-1, +1, -1, +1]$
7	1101	$[-1, +j, +1, -j]$
8	1000	$[+j, +j, +j, +j]$
9	1001	$[+j, -1, -j, +1]$
10	1011	$[+j, -j, +j, -j]$
11	1010	$[+j, +1, -j, -1]$
12	0111	$[-j, -j, -j, -j]$
13	0110	$[-j, +1, +j, -1]$
14	0100	$[-j, +j, -j, +j]$
15	0101	$[-j, -1, +j, +1]$

Based on the Lee distance (19), we can use the gray mapping for s_i, s_j belong to the same submatrix $j^k\mathbf{U}_n$, and inverse mapping for $s_i = s_j + \vec{2}$. An example of the new mapping is given in in Table 3. In this paper, we refer to this new mapping as optimized mapping.

Next, we derive an upper bound on the average BER for our HSSK system based on the Pairwise Error Probabilities (PEP) $\Pr(x \to \hat{x})$, which denote the probability of detecting $\hat{x}$ given that x is transmitted. From (3), the PEP can be formulated as

$$\begin{aligned} &\Pr(x \to \hat{x}|\mathbf{H}) \\ &= \left\{ \Pr\left(\|y - Hx\|_F^2 > \|y - H\hat{x}\|_F^2 \right) \right\} \\ &= \left\{ Q\left(\sqrt{\frac{1}{2\sigma^2} \|H(x - \hat{x})\|_F^2} \right) \right\}. \end{aligned} \tag{20}$$

We can apply Craig's representation of Q-function

$$Q(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{x^2}{2\sin^2\theta} \right) d\theta \tag{21}$$

to obtain a theoretical result. By using (21), (20) can be rewritten as [30]

$$\Pr(x \to \hat{x}|\mathbf{H}) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{\| \mathbf{H}(x - \hat{x}) \|_F^2}{4\sigma^2 \sin^2\theta} \right) d\theta. \tag{22}$$

Following [31], we have

$$\Pr(x \to \hat{x}) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \Psi\left(-\frac{1}{4\sigma^2 \sin^2\theta} \right) d\theta, \tag{23}$$

where $\Psi(\cdot)$ is the Moment-Generating Function (MGF) of the random variable $\|\mathbf{H}(x - \hat{x})\|_F^2$. Clearly, $\|\mathbf{H}(x - \hat{x})\|_F^2$ can be expressed as a complex quadratic form like in [32] as

$$\begin{aligned} &\| \mathbf{H}(x - \hat{x}) \|^2 \\ &= vec(\mathbf{H}_W^H)^H \mathbf{R}_s^{\frac{H}{2}} (\mathbf{I}_N \otimes (x - \hat{x})(x - \hat{x})^H) \mathbf{R}_s^{\frac{1}{2}} vec(\mathbf{H}_W^H), \end{aligned} \tag{24}$$

where $\mathbf{R}_s = \mathbf{R}_{rx} \otimes \mathbf{R}_{tx}$, $\mathbf{R}_s$ denotes the spatial correlation matrix, $\mathbf{R}_{rx}$ denotes transmitter correlation matrix and $\mathbf{R}_{rx}$ denotes receiver correlation matrix, respectively. $\mathbf{H}_W$ is obtained by $\mathbf{H} = \mathbf{Q}_r^H \mathbf{H}_W \mathbf{Q}_t$, where the matrices $\mathbf{Q}_r$ and $\mathbf{Q}_t$ are the square roots of $\mathbf{R}_{rx} = \mathbf{Q}_r^H \mathbf{Q}_r$ and $\mathbf{R}_{tx} = \mathbf{Q}_t^H \mathbf{Q}_t$, respectively. Using the results derived in [31], the MGF of (24) can then be expressed as

$$\Psi(s) = \frac{1}{\det\left(\mathbf{I} - s\mathbf{R}_s^{\frac{H}{2}} \left(\mathbf{I}_{N_t} \otimes (x - \hat{x})(x - \hat{x})^H \right) \mathbf{R}_s^{\frac{1}{2}} \right)}. \tag{25}$$

Following [32], mathematical manipulations on (25) lead to

$$\Psi(s) = \prod_{i=1}^{N_t} \prod_{j=1}^{N_r} \frac{1}{1 - s\lambda_i \tau_j}, \tag{26}$$

where the parameters λ_i and τ_j denote the eigenvalues of the matrices $(x-\hat{x})(x-\hat{x})^H \mathbf{R}_{tx}$ and $\mathbf{R}_{rx}$, respectively. Substituting (26) into (23), we have

$$\Pr(x \to \hat{x}) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{i=1}^{N_t} \prod_{j=1}^{N_r} \frac{1}{1 + \frac{1}{4\sigma^2 \sin^2\theta} \lambda_i \tau_j} d\theta. \quad (27)$$

Then, the BER of the HSSK with the ML detector can be bounded as

$$\begin{aligned} P_{e,bit}(x) &= \mathbb{E}\left[\frac{1}{\gamma} \sum_{\hat{\mathbf{A}} \in \mathbb{A}, \hat{\mathbf{s}} \in \mathbf{S}_n} \Pr(x \to \hat{x}) e(x, \hat{x}) \right] \\ &= \frac{1}{\gamma 2^\gamma} \sum_{\mathbf{A} \in \mathbb{A}, \mathbf{s} \in \mathbf{S}_n} \sum_{\hat{\mathbf{A}} \in \mathbb{I}, \hat{\mathbf{s}} \in \mathbf{S}_n} \Pr(x \to \hat{x}) e(x, \hat{x}), \quad (28) \end{aligned}$$

where $e(x, \hat{x})$ denotes the number of bits in error between x and $\hat{x}$.

V. SIMULATION

In this section, we first present some examples to compare the BER performance of HSSK system with the optimized mapping to that of the sequential mapping. Then, we show the theoretical and simulation results for BER performance of HSSK. The theoretical results are obtained using our theoretical analysis result (28). Finally, we also compare the BER performance of our proposed HSSK system with MA-SM, GSM and GSSK systems.

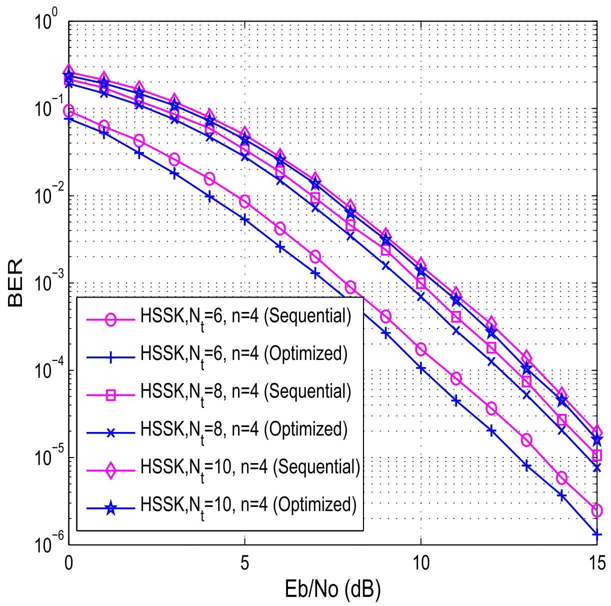

FIGURE 4. BER performance comparison between HSSK system with optimized mapping and sequential mapping ($N_r = 4$).

A. PERFORMANCE COMPARISON BETWEEN HSSK SYSTEM WITH OPTIMIZED MAPPING AND SEQUENTIAL MAPPING

For $n = 4$ and varying N_t, Fig. 4 compares the BER performance between HSSK system with optimized mapping and sequential mapping. Specifically, for $N_t = 6$, HSSK system with optimized mapping has about 0.8 dB performance gain compared to HSSK system with sequential mapping at BER $= 1 \times 10^{-5}$. For $N_t = 8$, HSSK system with optimized mapping has about 0.5 dB performance gain compared to HSSK system with sequential mapping at BER $= 1 \times 10^{-5}$. For $N_t = 10$, HSSK system with optimized mapping has about 0.3 dB performance gain compared to HSSK system with sequential mapping at BER $= 1 \times 10^{-5}$. Therefore, the simulation results show that HSSK system with optimized mapping can achieve better BER performance, which complies with our theoretical analysis.

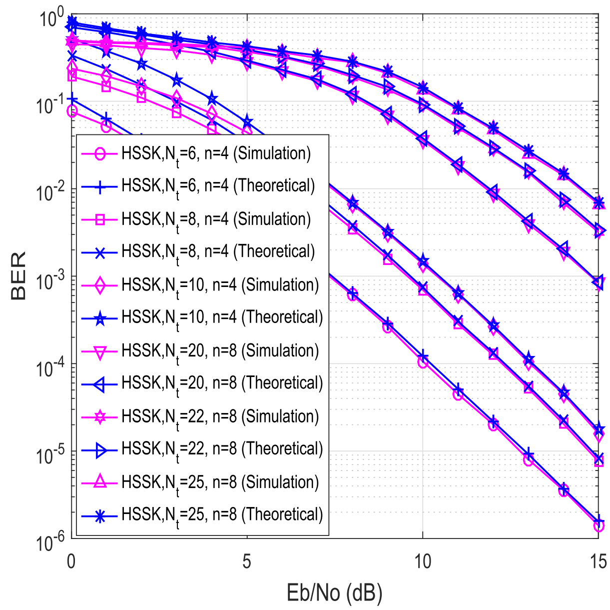

FIGURE 5. BER performance comparison of theoretical and simulation results for HSSK ($N_r = 4$).

B. THEORETICAL AND SIMULATION RESULTS OF THE HSSK SYSTEM

Fig. 5 illustrates the theoretical bounds (28) and simulation results of BER for HSSK with $n = 4$ for $N_t = 6, 8, 10$ and $n = 8$ for $N_t = 20, 22, 25$, respectively. It is observed that, as expected, both analysis and simulation predict that the increase of N_t degrades the BER rapidly. It can also be seen that the deviation between simulated and analytical results is almost negligible, especially in the high SNR region. The deviations between the theoretical results and the simulation results, in low SNR region, are due to the reason that some approximation premise in the theoretical derivation cannot be satisfied.

C. PERFORMANCE COMPARISON OF MA-SM, GSM, GSSK AND HSSK SYSTEMS

Fig. 6 compares the BER performance of the HSSK system to that of the MA-SM system with the same transmission rate γ and number of transmit antennas N_t. In order to achieve

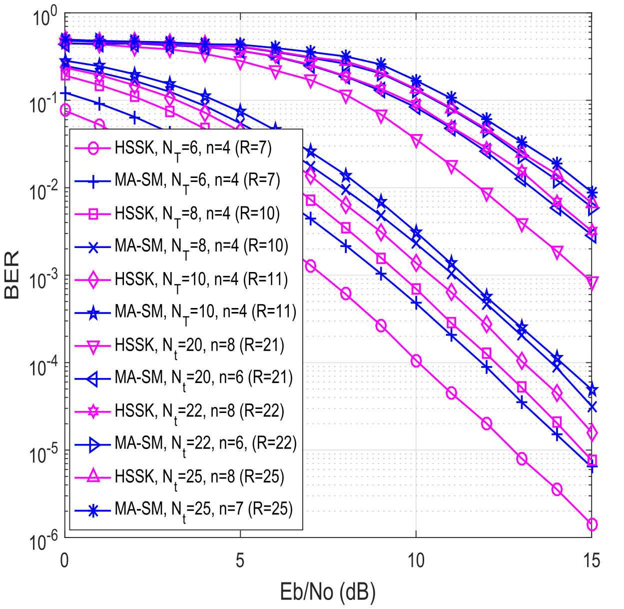

FIGURE 6. **BER performance comparison of the MA-SM and HSSK systems ($N_r = 4$).**

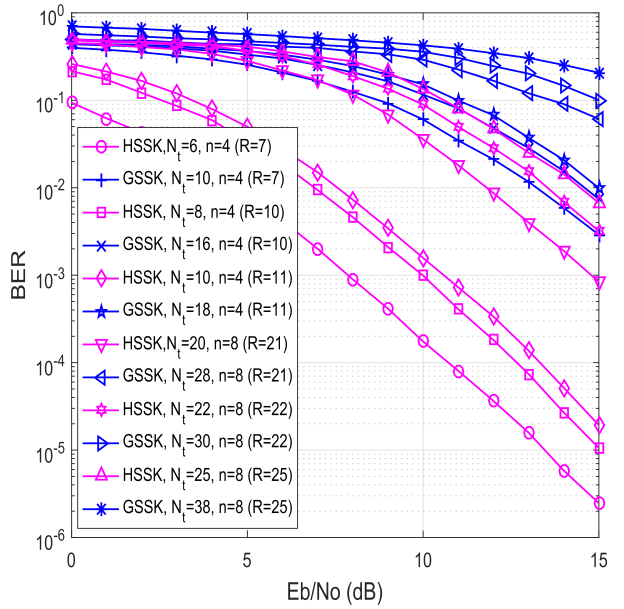

FIGURE 8. **BER performance comparison of the HSSK and GSSK systems ($N_r = 4$).**

the same transmission rate, the MA-SM system is equipped with Binary Phase Shift Keying (BPSK) and the number of activate antennas n is adjusted. It can be seen from Fig. 6 that, the HSSK systems achieve a better BER performance than that of the MA-SM systems.

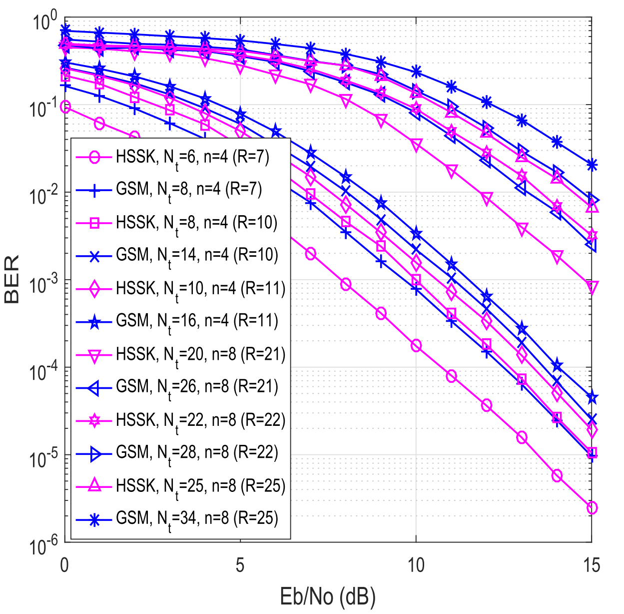

FIGURE 7. **BER performance comparison of the GSM and HSSK systems ($N_r = 4$).**

Fig. 7 also compares the BER performance of the HSSK system to that of the GSM system with the same transmission rate γ and BPSK modulation. In order to achieve the same transmission rate, the number of transmit antennas N_t is adjusted for GSM system. It can be seen from Fig. 6 that, the HSSK systems also achieve better BER performance than that of the GSM systems.

Fig. 8 compares the BER performan of the HSSK systems and the GSSK systems with the same transmission rate γ. In order to achieve the same transmission rate, the number of transmit antennas N_t is adjusted for GSSK system. It can be seen from Fig. 8 that, the BER performance of HSSK system is better than that of the GSSK system with the same transmission rate.

VI. CONCLUSION

In this paper, we introduced the complex Hadamard matrix-aided GSSK, which is referred to as the HSSK. The HSSK system applied complex Hadamard based mapping operations at the transmitter to increase the transmission rate, but with low detection complexity due to the fast complex Hadamard transform. In order to further improve the BER performance, we also proposed the optimized mapping for the HSSK system based on the Lee distance of the symbol vectors. The simulation results showed that HSSK systems with optimized mapping performed better than HSSK systems with sequential mapping. Furthermore, we also provided the upper bound on BER of the HSSK system. This analytical results agreed well with the simulation results. Finally, we compared HSSK systems to MA-SM, GSM and GSSK systems. It was shown that the HSSK systems have better BER performance than the MA-SM, GSM, GSSK systems with the same transmission rates.

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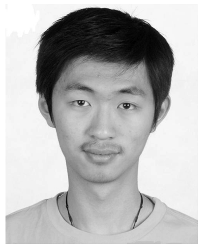
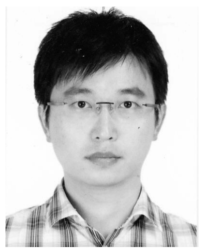


HAN HAI received the B.S. degree in information and computing sciences from South Central University for Nationalities, China, and the M.S. degree in electronics engineering from Chonbuk National University, South Korea. He is currently pursuing the Ph.D. degree in electronics engineering with Chonbuk National University, South Korea. His research interests include wireless communications, MIMO, and spatial modulation.



XUE-QIN JIANG (M'13) received the B.S. degree in computer science from the Nanjing Institute of Technology, Nanjing, China, the M.S. and Ph.D. degrees in electronics engineering from Chonbuk National university, Jeonju, South Korea. He is currently an Associate Professor with the School of Information Science and Technology, Donghua University, Shanghai, China. He has authored/co-authored over 70 technical papers, several book chapters. His main research interests include LDPC codes, physical-layer security, and wireless communications.

WEI DUAN received the B.S. degree from the Cheng Du University of Technology, Chengdu, China, in 2008, and the M.S. and Ph.D. degrees from Chonbuk National University, Jeonju, South Korea, in 2012 and 2017, respectively. He is currently a Lecturer with the Nantong University, Nantong, China. His research interests include cooperative networks and non-orthogonal multiple access techniques.

JUN LI (S'13–M'17) received the M.S. and Ph.D. degrees from Chonbuk National University, Jeonju, South Korea, in 2011 and 2017, respectively. He has been an Assistant Professor with Guangzhou University, Guangzhou, China. He had participated in the World Class University Project, sponsored by the National Research Foundation of Korea grant funded by the Korean Ministry of Education Science and Technology as a Vice Head Researcher. His research interests include spatial modulation and OFDM index modulation.

MOON HO LEE (S'81–M'85–SM'86) received the B.S. and M.S. degrees in electrical engineering from Chonbuk National University, South Korea, in 1967 and 1976, respectively, the Ph.D. (I) degree in electrical engineering from Chonnam National University in 1984, and the Ph.D. (II) degree in information engineering from The University of Tokyo in 1990. From 1985 to 1986, he held a post-doctoral position with the University of Minnesota. He is also a Professional Engineer, Licensed, in 1981, South Korea.

From 1970 to 1980, he was a Chief Engineer with Namyang Moonhwa Broadcasting Company. He was a recipient of paper prize awards from the Korea Institute of Communication Sciences in 1986 and 1996, the Korea Institute Telematics and Electronics in 1987, Chonbuk Province in 1992, Chonbuk National Engineering in 2000, and Hedong Information Communication in 2007. Also he received the National order of Merits, Doyak Medal in 2007, this Month's Scientist Award by the Ministry of Science and Technology and the Korea Science and Engineering Foundation in 2008, respectively. He also studied at the University of Hannover in 1990 and at the University of Aachen in 1995, Hannova and at the University of Munchen in 1998 and 2002, Kaiserslautern supported by Germany government. Also he was invited Prof. at Nagaoka University of Technology, Japan, in 2000, RMIT in 2003 and University of Wollongong, Australia, in 2004.

He is currently a Professor of Division of Electronic and Information Engineering with Chonbuk National University. His research interests include Image Processing, Mobile Communication and High Speed Communication Networks and Information and Algebra Coding Theory. He is a member of Sigma Xi. He is a Korea Engineering Academy Member and a Guest Editor of the IEEE Communication Magazine in 2007, Quality of Service-based Routing Algorithm for Heterogeneous Networks.

YONGCHAE JEONG (M'99–SM'10) received the B.S.E.E., M.S.E.E., and Ph.D. degrees in electronics engineering from Sogang University, Seoul, South Korea, in 1989 and 1991, respectively. From 1991 to 1998, he was a Senior Engineer with Samsung Electronics. In 1998, he joined the Division of Electronic Engineering, Chonbuk National University, Jeonju, South Korea. From 2006 to 2007, he was with the Georgia Institute of Technology, Atlanta, as a Visiting Professor. He is currently a Professor and a member of the IT Convergence Research Center, Chonbuk National University. He has authored and co-authored over 100 papers in international journals and conference proceedings. He is currently teaching and conducting research in the areas of microwave devices, base-station amplifiers, nonlinear devices and system linearizing technology, and RFIC design. He is a member of the Korea Institute of Electromagnetic Engineering and Science.